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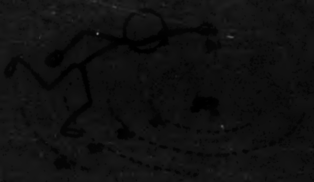
ABSTRACT

Developed by a committee of the National Council of Teachers of Mathematics, this publication is designed to help teachers provide interesting and worthwhile learning opportunities for slow learners in grades five through eight. It employs a variety of teaching strategies, many not commonly known or practiced, which are particularly helpful with slow learners. In particular, the activities suggested one of a "laboratory nature" and encourage participation by all students. The subjects covered include base and place value, renaming numbers in addition and subtraction, physical models for multiplication, units of measure, physical models for fractions, and physical models for decimals. Most of the units are independent of the others and need not be taught in any specified order nor at specified grade levels. The volume includes a "Teaching Package" containing materials which can be duplicated by the teacher for use as overheads, worksheets, or laboratory materials. (Author/CT)



EXPERIENCES IN MATHEMATICAL IDEAS

VOLUME 1

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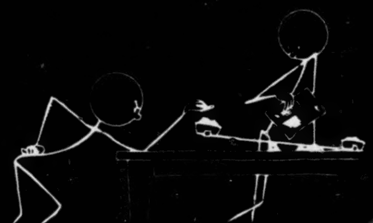

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EXPERIENCES IN MATHEMATICAL IDEAS

2

EXPERIENCES
IN
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3

Volume One

IDEAS

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS
1201 Sixteenth Street, Northwest
Washington, D.C. 20036

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Preface

Curriculum projects during the late fifties and early sixties in all subject fields, but particularly in mathematics and science, produced and made available an abundance of material for average and above-average students throughout the elementary and secondary schools. Many of these curriculum projects employed the development strategy of "teaming." Teams of experts, including classroom teachers, psychologists, and representatives of the relevant subject disciplines, developed materials that were widely accepted and implemented throughout the schools of our nation. The implementation process was enhanced by private corporations and public agencies that expended millions of dollars to provide additional educational experiences for thousands of teachers. During this same period of time, however, little concern, effort, money, or talent was directed toward developing curriculum materials for slow learners and their teachers.

The challenge to develop mathematics materials for slow learners was accepted by the Board of Directors of the NCTM at the 1967 Annual Meeting when the Board approved a proposal presented by the Committee on Mathematics for the Non–College Bound. The proposal evolved into a writing project which has become known and identified as "Experiences in Mathematical Ideas" (EMI).

The successful completion of any writing project is a direct result of the dedication, talents, and enthusiasm of many individuals. The EMI project is no exception.

Members of the NCTM Committee on Mathematics for the Non–College Bound were responsible for the initial concept and beginning blueprint for the prototype units which emerged from this project—units now assembled as chapters of the present publication.

The NCTM Board of Directors, after considerable study of the original proposal, not only approved the concept but agreed to fund the project entirely from Council resources. Marguerite Brydegaard served diligently as the Board liaison representative. James Gates and the staff of the NCTM office helped immeasurably with financial matters and final production procedures. The support thus given is greatly appreciated.

Although the primary leadership and direction of the entire project was the responsibility of the EMI Executive Committee, the heart of the developmental activities was a result of the insights provided by the writing team. The EMI writing team included Charles Allen, Beth Baer, Bonnie Brooks, C. William Engel, Lowell Leake, Jr., Edith Robinson, Carolyn Smith, Donald Wiederanders, and Larry Yarck. The experience of these writers in working with teachers of slow learners and with pupils with learning problems was invaluable. Moreover, each of the writers demonstrated an unusual talent for creating new mathematical experiences. They worked cooperatively as a team throughout the entire project.

An unusual feature of the EMI project was the manner in which materials were edited. While the Executive Committee carried the overall editorial responsibility, two editors, Raphael Wagner and Kenneth Travers, offered their talents at two different stages of the project development in order that fresh points of view might effect a better product.

There are always many individuals who have been associated with a project such as EMI to whom no special recognition is given. For example, teachers in both urban and rural settings tried out preliminary drafts of the EMI materials and fed back information which was utilized in the final writing of each unit. To all these individuals a sincere "Thank you" is expressed.

Finally, special acknowledgment is given to Mary Parks and Aletha Buchert for their secretarial assistance. Without them many deadlines could not have been met.

The Executive Committee of EMI is truly indebted to and appreciative of the talents and excellent contributions of all these individuals. The ultimate reward to all who have been associated with this project will be the degree to which the basic ideas and spirit of EMI are transmitted into improved opportunities for slow learners throughout our nation.

Executive Committee

Arnold M. Chandler, *Director*
George Immerzeel
Harold C. Trimble

Introduction

Experiences in Mathematical Ideas is designed to help teachers provide interesting and worthwhile learning opportunities for those students in grades five through eight who have had little success in mathematics. It will help those teachers whose devotion to slow learners has made them sensitive to the shortcomings of present curriculum materials and methods of presentation. The EMI project materials are designed so that teachers may implement them in conventional self-contained classrooms as well as in other organizational patterns—for example, in team teaching, multi-unit programs, nongraded structures. Hence, EMI materials are dependent only on the willingness of teachers to offer pupils opportunities and freedom to learn those basic mathematical concepts that have been provided. Teachers are a crucial and central element in implementing the student-centered and activity-oriented EMI materials. They must be exceptionally kind, warm, and fair, displaying a sincere empathy for pupils with learning problems.

The EMI project, like other projects, has been carefully structured within a philosophical and psychological framework which includes certain assumptions and beliefs. The assumptions and beliefs come from what is generally known and accepted about human growth and development as it pertains to slow learners. For example, it is assumed that pupils identified as slow learners *can* learn if experiences for them grow out of their physical environment. Further, it is assumed that there are a variety of teaching strategies, not commonly known and practiced, which will aid and abet the learning process of slow learners. There is a fervent belief that objectives for pupil achievement can be stated and should serve as a focus for most mathematical learning activities. It follows naturally that certain evaluations of pupils can then be made in terms of these objectives.

The following additional assumptions of the EMI project give further insight into the orientation and purpose of the materials:

1. *Activity*. Learning is an active process. Slow learners learn mathematics by reacting to and interacting with their environment. Too often mathematical concepts have been treated initially at the symbolic level by authors in textbooks, by teachers on chalkboards, and by students in notebooks. This approach has been unsatisfactory for slow learners. The EMI materials suggest many ways in which the same mathematical concepts can be learned by these students when initiated through experiences. Only after the experiences is attention given to associating these activities with appropriate mathematical symbols.
2. *Individuality*. Learning is an individual process. Each learner must actively participate in the ongoing classroom situation. EMI materials are developed so that each pupil has an opportunity to observe, to explore relationships, to collect information, to hypothesize solutions to problems, and to test tentative hunches. Typically the classroom is busy with teacher activity. EMI materials are structured so the classroom will be dominated by pupil activity. When learning mathematics, it is not enough to sit and watch the teacher "do mathematics." Each individual must pitch in and get a piece of the action.
3. *Success*. Success breeds success. Yet too many mathematics classes offer many students little else but experiences of failure. EMI materials are designed to provide activities that will give even the slower students successful encounters with mathematics. This is done by breaking down concepts into very small components and devising simple tasks that will carefully lead the student to a grasp of the basic ideas. The teacher using EMI material will have to resist the urge to move too quickly through the experiences because they look "too easy."
4. *Meaning*. Classroom experiences must have meaning for the learner. Most students readily pick up catch phrases or rules of thumb like "Invert the divisor and multiply," but many have little understanding of what they are talking about. It is not surprising when these individuals deal with mathematics in utterly nonsensical ways, if they deal with it at all. EMI materials attempt to bring meaning to mathematics by drawing largely from situations with which the student is familiar, or in which he can readily imagine himself to be an active participant.

5. *Novelty.* A change is as good as a rest. Perhaps more than any other subject in the curriculum, mathematics poses the risk of becoming routine and dull in the classroom. The prospects are particularly bleak for the youngster who day after day faces the same kind of tasks that have repeatedly meant failure for him in the past. EMI materials attempt to approach familiar mathematics topics from a fresh point of view. At the same time, the ultimate objective is to develop a more complete understanding of the fundamental mathematical concepts included in the set of units.

The EMI project materials comprise thirteen prototype units: (1) Base and Place Value, (2) Renaming Numbers in Addition and Subtraction, (3) Physical Models for Multiplication, (4) Units of Measure, (5) Physical Models for Decimals, (7) Tables and Change, (8) Using Tables to Solve Problems, (9) Ratio, (10) Graphs, (11) Organizing Data, (12) Dealing with Uncertainty, and (13) Geometry. The first six units are bound in volume 1, and the remaining seven units are bound in volume 2.

The units in the EMI project do not compose a complete mathematics program for low achievers in grades five through eight, and they are not intended to be used as such. Neither are the units intended to be used solely for supplementary or enrichment purposes. Rather, the units offer a model for teaching representative segments of mathematics—segments that are usually taught because they are needed by all students. Most of the units are independent of the others and need not be taught in any specified order nor at specified grade levels within the scope of this project.

Each unit contains a sequential development of a specific mathematical topic through a series of "experiences." A brief overview of these experiences is outlined at the beginning of each unit. This overview gives the teacher a quick and understandable preview of what the entire unit is about. Generally there are five experiences in a unit. Some may take as long as three days, but many will require only one day.

The description of each experience includes (1) a statement of a pupil objective, (2) a list of materials to be used, (3) a strategy that may be employed by the teacher, (4) reproductions of student activity cards or worksheets, and (5) a suggestion about evaluation.

Another special feature of the EMI project is the "Teaching Package." A teaching package has been developed for each volume, with materials that are closely correlated with individual activities within each experience. These materials are printed on 8½-by-11-inch sheets so they can be duplicated by the teacher to make (1) overhead transparencies, (2) labels and other software needed at activity stations, and (3) copies of worksheets for students. The teaching-package materials have been punched and numbered so the teacher can keep them in order in a notebook.

The two volumes describing experiences and the associated teaching packages in the EMI project have been structured in such a way as to help many teachers provide new and different learning experiences in mathematics for students who have not achieved from teaching materials and patterns commonly employed. The Council hopes that success with some or all of the prototypes will encourage teachers to provide similar experiences throughout grades five to eight for students whose mathematical needs are not now being met.

1

EXPERIENCES WITH

Base and Place Value

The purpose of this unit is to bring out some basic properties of the decimal numeration system. An understanding of base and place value is essential if the algorithms for addition, subtraction, multiplication, and division are to be developed in a meaningful way.

In order to achieve the objectives of this unit, a laboratory approach is utilized, with various activities designed to aid the student in grasping a particular mathematical concept. The experiences are planned so that the amount of teacher direction is at a minimum, and the Teaching Package provides help for preparing the necessary materials.

Introduce each experience by describing the materials and the procedures; then assume a supervisory role similar to that of the chemistry teacher in a chemistry laboratory. When the students have finished a set of activities, lead a discussion that emphasizes the major mathematical ideas of the experience. The discussion may also serve as a means of evaluating student progress.

OVERVIEW

Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective for his class.

2 BASE AND PLACE VALUE

The following brief summary will help you decide whether these experiences suit the purposes of your particular class. It will also give you some indication of the amount of preparation involved for the experiences. Student worksheets and other "handout" materials are provided. It is assumed that duplicating facilities for providing copies for each student are available.

Experience 1: Focus on "Ten"

This experience develops the basic concept that the number of elements in a set can be approximated by grouping the members of the set into subsets of size ten. A contest that requires students to guess the number of beans in a box is introduced in order to achieve this objective.

Materials needed: cigar box, lima beans, envelopes, paper clips, rubber bands, Krazy Kontest slips.

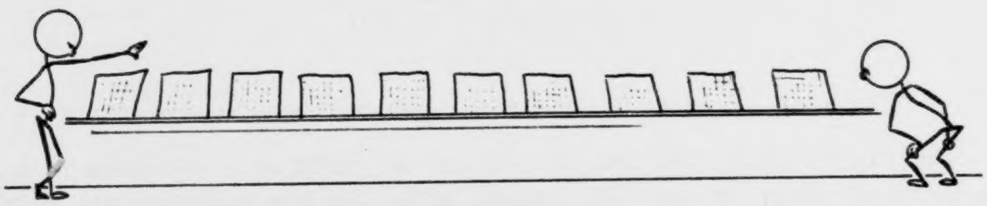
Experience 2: The Base Property

Various physical models for ten, one hundred, and one thousand are introduced and the *ten*ness of our numeration system is emphasized. Six stations are established in this experience. The students are required to guess which sets number 1, 10, 100, and 1,000.

Materials needed: index cards, lima beans, unpopped popcorn, Popsicle sticks, washers, poster board, rubber bands, plastic bags, boxes, paper bags, worksheets.

Experience 3: Place Value

Five stations provide a variety of activities illustrating the place-value property of the decimal numeration system.



Materials needed: poster board and matching crayons in four different colors, paper bags, lima beans, rubber bands, graph paper, worksheets.

Experience 4: Expanded Notation

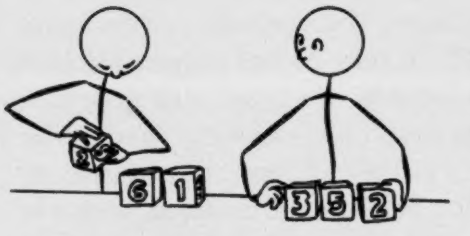
The base and place-value properties are used to express numbers in expanded form. Four game stations are established in this experience. The game activities are laboratory-oriented and provide a change of pace from the activities of the previous experiences.

Materials needed: index cards, poster board, winners report forms, worksheets.

Experience 5: Comparing Numbers

The student's understanding of decimal numeration is tested by having him compare numbers and find the larger. Four stations are established here.

Materials needed: index cards, rubber bands, small wooden cubes, worksheets.



EXPERIENCE 1
Focus on "Ten"

OBJECTIVE

The student should be able to estimate the number of elements in a large set of objects by mentally forming subsets of tens and ten tens.

MATERIALS

Lima beans
Cigar box
60 envelopes
Paper clips
10 large rubber bands
Krazy Kontest slips (reproduced from Teaching Package)

Krazy Kontest

My Guess ____________________

Name ____________________

TEACHER STRATEGY

In many cases students are required to work with decimal numerals in symbolic form before they have had sufficient experience with physical objects. It is the purpose of this experience and the next one to provide students with abundant physical models for the decimal numeration system.

I like to introduce the experience by showing the class a cigar box full of lima beans. Then I lead a discussion about contests based on guessing the number of objects in a large container. Quite often television shows have contests that require entrants to guess, for example, the number of marbles in a fishbowl or the number of nails in a keg.

After the initial discussion I ask the students, "How would you go about guessing the number of beans in this box?" If they do not suggest the idea of mentally subdividing the set into smaller sets, I suggest this approach by asking questions such as "What size set would you estimate?" and "Why might ten be a better size than nine or twelve?" I lead the students to realize that ten is a good choice because ten tens are one hundred, and it is quite easy to count by tens. We could count by twos, but this group would be too small. One hundred might be a good group to select, but in order to estimate one hundred we would have to think in terms of ten anyway.

When students have accepted the fact that grouping by tens is an easy method of estimating the total number of beans, I count out ten beans from the box and arrange them as shown in figure 1. I find that students learn to recognize a group of ten objects quickly if I place them in this triangular form.

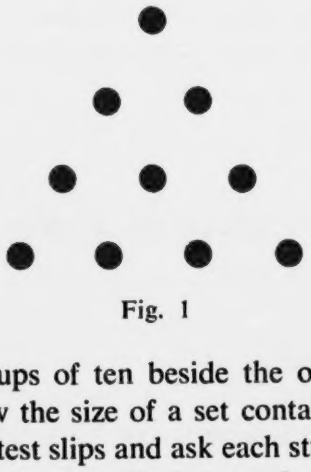


Fig. 1

I place nine more groups of ten beside the original group, then count the beans by tens to show the size of a set containing one hundred. After this I pass out Krazy Kontest slips and ask each student to guess the number of beans in the box and to write his guess, along with his name, on the contest slip.

While the students are making their guesses, I walk around the class with the box of beans so that the students may take a closer look. When the guessing is finished, slips are collected and turned over to a committee of two students, who are asked to organize the answers in such a way that it will be easy to find the winner when the answer is known. I help the students with this task, if they need to be helped, while the rest of the class is counting the beans to learn what the total number really is.

The class should package the beans, by tens, in small envelopes. Each student is given a handful of beans and two envelopes, each with a paper clip. I direct the students to put ten beans into an envelope and then replace the paper clip to close it. Students may work together, since each will have a different number of beans. In order to make groups of ten, anyone who has a few beans left over after the packaging will have to give them to someone who has more. I circulate around the room to help organize the packaging. As it nears completion, I collect the full envelopes and any excess beans. If the number of excess beans is ten or greater than ten, I have the students package them.

Now I group the envelopes by tens and place a rubber band around each set of ten envelopes, saying, "This makes one hundred." When the packaging is completed, we determine the total number of beans and write the total on the board. I emphasize the relation between the number and the type of packaging. For example, if the number is 643, there will be six bundles of ten tens and four packages of ten, with three beans (ones) left over.

The committee that organized the slips then announces the winner. Sometimes I supply prizes for the three students who made the best guesses.

In discussing and summarizing this experience, I use the envelopes of beans as visual aids. For example, I hold up three envelopes and ask:

How many envelopes do I have?
How many tens do I have?
How many beans do I have?

Similarly, I hold up three bundles of ten envelopes and ask:

How many hundreds do I have?
How many envelopes?
How many tens?
How many beans?

I might then hold up two hundreds and three tens and ask similar questions.

As another kind of exercise, I sometimes place a number such as 142 on the board and have a student form the amount by using beans.

EVALUATION

The summary discussion will serve as a means of evaluating the effectiveness of this experience. It is not expected that all students will have developed a full understanding of the meaning of base ten at this time. Students should, however, realize that ten tens are one hundred and that using some systematic procedure makes it easier to determine the number in a set.

EXPERIENCE 2
The Base Property

OBJECTIVE

The student should realize that a set of one hundred contains ten tens and also that a set of one thousand contains ten hundreds.

MATERIALS

500 index cards
Uncooked lima beans
Unpopped popcorn
250 Popsicle sticks
150 metal washers
2 large pieces of poster board and reproductions of dot patterns from Teaching Package
25 rubber bands
5 plastic bags
5 boxes
5 paper bags
1 worksheet for each student

Stations. Six stations are established for this unit, and at each station there are five sets of similar objects. The sets are labeled A through E. If the materials suggested are not available, substitutions may be made. Possible substitutes include playing cards, rice, peas, nuts, bolts, marbles, buckshot, BBs, coins, toothpicks, buttons, beads.

Station A has sets of index cards (held together by rubber bands) labeled A through E and containing, respectively, 300, 50, 5, 100, and 10 cards.

Station B has sets of lima beans in plastic bags, labeled A through E and containing 1,000 (approximately), 100, 40, 200, and 10 beans.

Station C has sets of unpopped popcorn in sealed boxes, labeled A through E and containing 10, 20, 400, 100, and approximately 5,000 kernels.

Station D has sets of Popsicle sticks held together by rubber bands, labeled A through E and containing 10, 20, 100, 40, and 70 sticks.

Station E has sets of washers in sealed paper bags, labeled A through E and containing 100, 3, 37, 10, and 1 washer(s).

Station F has dot patterns reproduced from the Teaching Package and arranged on poster board as seen in figure 2, the number of dots being 100, 10, 1,000, 500, and 120 for Sets A through E, respectively.

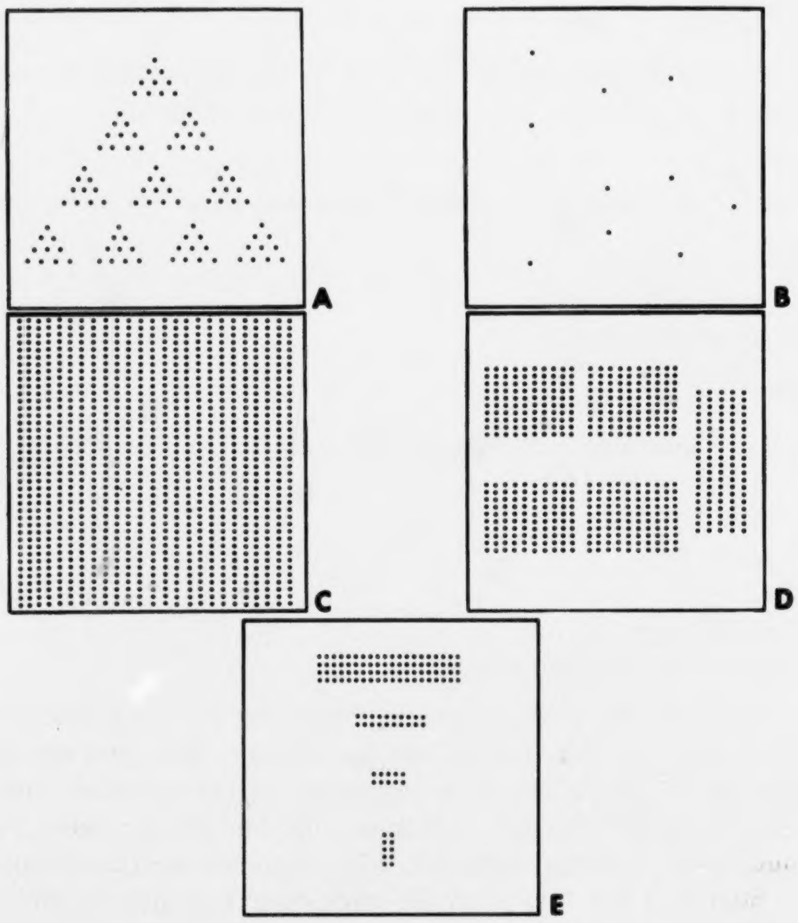


Fig. 2

TEACHER STRATEGY

In order to show how ten plays a major role in the decimal numeration system, students are given concrete experiences with sets of size ten, one hundred, and one thousand. The six stations already described are set up around the classroom in order to achieve this objective. The students are asked to visit the stations with a worksheet and select the sets containing the number of objects asked for. Wherever the sets are enclosed in bags or boxes, the students have to rattle the contents or judge by weight in order to make a guess at the size of the set.

I start this experience with a review of the methods that were used in Experience 1 to guess the number of elements in a set. I emphasize that by estimating a set of ten and then a set of ten tens it is easier to estimate the total number. Now I extend to one thousand the concept of grouping by tens; I ask the questions "How many ones in ten hundreds?" and "How many tens in one thousand?"

After this brief review and discussion I pass out the worksheet and go over the directions with the students until it is clear what they are to do. Then I divide the class into groups and assign each group to one of the stations. To avoid confusion (it would not matter otherwise), I have the students visit the stations in sequence after this initial assignment.

I assure the students that I will not grade them on their answers and tell them that if they want to look at other students' guesses, they may. An incorrect guess is not to be thought of as a wrong answer. I keep the atmosphere of this experience gamelike.

After a student has visited all six stations and completed his worksheet, he returns to his seat. When most of them have finished, I ask them all to return to their seats. Then I visit each station and hold up the sets that correspond to the ones asked for on the worksheet. It is not expected that the students will name all of the sets correctly, but it is expected that they will make reasonable guesses.

EVALUATION

A brief discussion following the checking of the work should include questions such as these:

Is it harder to guess cards or beans?
How many tens in one hundred?
How many hundreds in one thousand?
How many tens in one thousand?
Which is larger, ten one-hundreds or one hundred tens?

Responses to these questions should serve to measure the success of the experience.

Observe the methods that students use to determine which sets contain ten, one hundred, and one thousand. Students should try to visualize one hundred as a set of ten tens and one thousand as a set of ten one-hundreds.

THE BASE PROPERTY **UNIT 1, EXPERIENCE 2**

Name ______________________________

Go to each station and find the sets equivalent to the numbers given, then write the letter for each of these sets in the proper space. As an example, the answer has already been given for 10 at Station A.

Visit the stations in the order indicated by your teacher.

Station A
10 .... Set E
100 .... Set C

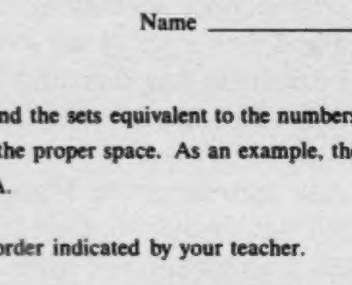
Station B
10 .... Set E
100 .... Set F
1,000 .... Set A

Station C
10 .... Set A
100 .... Set D

Station D
10 .... Set A
100 .... Set C

Station E
1 .... Set E
10 .... Set D
100 .... Set A

Station F
10 .... Set B
100 .... Set A
1,000 .... Set C



EXPERIENCE 3
Place Value

OBJECTIVE

The student should be able to recognize "123" as a symbol that represents a set made up of one hundred, two tens, and three ones. He should also be able to express any number of objects less than 1,000 in decimal notation.

MATERIALS

Grids and patterns (reproduced from Teaching Package)
Poster board in 4 different colors
Crayons to match poster board, 5 or 6 of each
Lima beans
20 paper bags
10 rubber bands
Graph paper
3 worksheets for each student

Station A. From the Teaching Package, reproduce a copy of the unmarked 10 × 10 grid for each student, also a copy of the grid marked with a few numbers, as shown in figure 3. Make four copies of the grid, showing the path of a submarine (fig. 4), mount them on poster board, and place them face down.

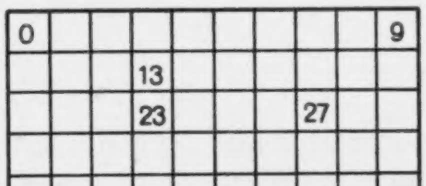


Fig. 3

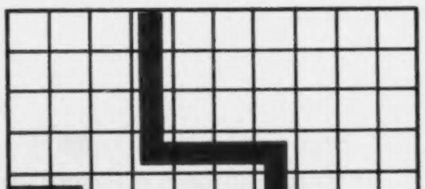


Fig. 4

Station B. Mount ten unmarked 10 × 10 grids on poster board, to be placed side by side to display a total of 1,000 squares. Add number identifications to the cells as diagramed in figure 5, according to the following plan for assigning values: top row of the grid on the left, 0, 1, 2, . . . , 9; bottom row of that grid, 90, 91, 92, . . . , 99; top row of second grid, 100 through 109; and so on. The second grid would end with 199, and the grid on the far right would end with 999. In addition to the numerals shown in figure 5 add a few at random, but do not use any that are shown in figure 6.

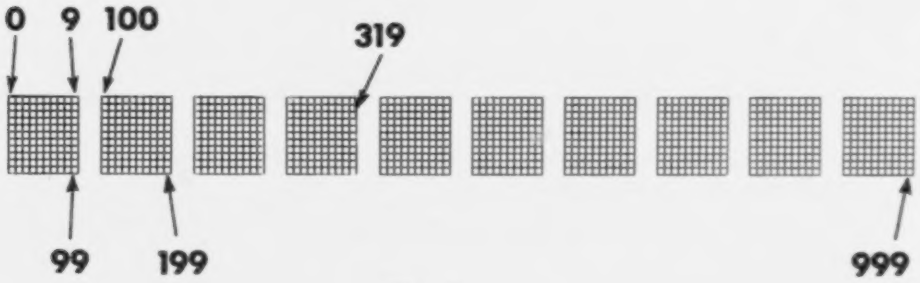


Fig. 5

Letters should now be added to the grids in the cells that have the values indicated by the numerals over the letters in figure 6, so that a "secret message" can be revealed as students do the worksheet activity for this station. Twenty or so additional letters should be placed at random throughout the cells to help conceal this secret message.

35	83	100	101	104	205	285	297	333	346
I	O	R	Y	T	A	E	U	E	Y
401	459	490	504	582	639	718	942	937	
H	O	R	R	V	O	D	G	G	

Fig. 6

Station C. Four small poster boards are required for this station. Each should be a different color, and in this discussion we are assuming the use of red, orange, yellow, and white. Dots and labels are placed on the boards as shown in figure 7 on the next page. Dot patterns may be duplicated from the patterns in the Teaching Package and colored with crayons.

These boards can be hung on the wall or bulletin board, or you may wish to support them with stands and place them on a table. Five or six crayons in matching colors should be supplied to each student.

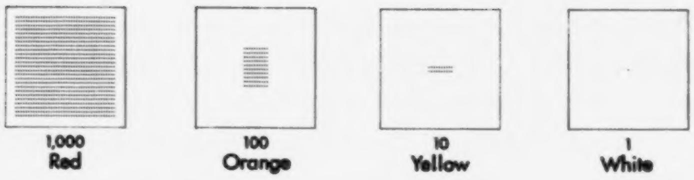


Fig. 7

Station D. For this station twenty paper bags are required. Two sets of ten bags each are labeled 1, 2, 4, 10, 20, 40, 100, 200, 400, and 1,000. Into each bag place the corresponding number of beans, seal the bags, and place them in full sight. (For the larger numbers, approximations are sufficient.)

Station E. Twenty patterns cut from graph paper and mounted on poster board for added firmness are needed for the activity at this

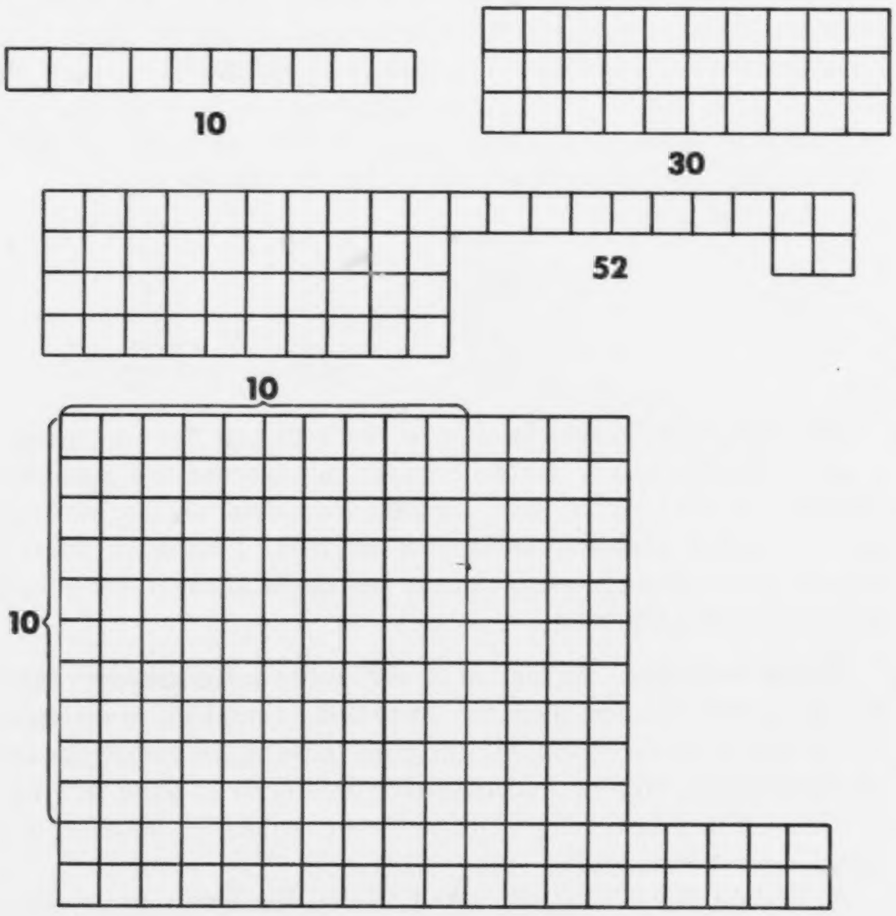


Fig. 8

station. They should be labeled from 1 through 20. The patterns should include groupings of ten and one hundred, and they should also include totals such as 102, 201, 43, 34, and so on, to reinforce the concept of place value. Patterns suggested as examples are shown in figure 8.

TEACHER STRATEGY

In this experience you will develop the concept of place value. You will also reinforce the *ten*ness of the decimal system.

I introduce this experience by writing 7,777 on the board and asking what each 7 represents. In the example each 7 represents a different value, depending on its place. I point out that if a 7 is moved one place to the left, its value increases to ten times its previous value.

I like to illustrate the importance of place value by having students express the different numbers that can be made using the digits 1, 2, and 3 without repeating any digit. The same exercise can be repeated using the digits 1, 2, 3, and 4. Six different numbers can be expressed with three digits and twenty-four with four digits, without repeating the digits in any one number. The two sets are shown here in random order.

231
132
123
213
321
312

2,314
4,312
3,421
3,142
4,132
3,214

3,124
2,431
2,413
4,321
4,213
3,412

1,324
2,143
2,341
2,134
4,123
1,432

1,243
1,342
4,231
1,234
1,423
3,241

After this brief introduction I pass out copies of the worksheets to each student, then go to the various stations and describe the procedure to be followed. I have the students read the directions on the worksheets and make sure they understand what is to be done. I divide the class into five groups and have each group start at one of the stations, continuing to the next as work is completed.

Station Activities. At Station A the students are asked to take one of the unmarked grids and place ten Xs in cells at random, to represent mines. When they have completed this, the grids showing the path of the submarine are turned over, and the students check their grids to see if they have made any hits. If they have, they write on the worksheet the numbers of the cells where the hits were made.

At Station B, students are asked to find the letters in certain squares, following the pattern of assigned values to which the ten marked grids furnish clues, and to write the letters on the worksheet to "break the code" and receive a "secret message."

Station C has four poster boards with dot patterns in different colors, with sets of crayons to match the colors. The students are asked to color squares on their worksheet to represent certain numbers, following the diagramed example shown in figure 9.

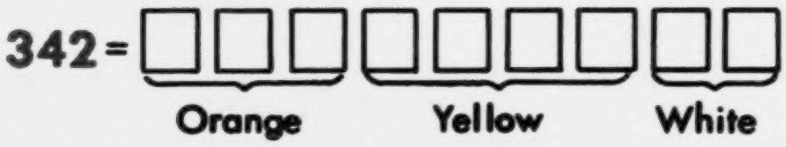


Fig. 9

The number of blank squares next to each number on the worksheet is the sum of the digits; therefore, since no square should be left over, the student will always have a method of checking his work.

At Station D students are asked to indicate the least number of bags it would take to equal a given number of beans. For example, a collection of 167 beans could be formed with the following bags: 100, 40, 20, 4, 2, and 1. This would be six bags. Any other combination that totals 167 would require more than six bags; one such combination would be eight bags of 100, 20, 20, 10, 10, 4, 2, and 1.

At Station E students are to write down on their worksheets the number of squares in each of the twenty patterns that were prepared.

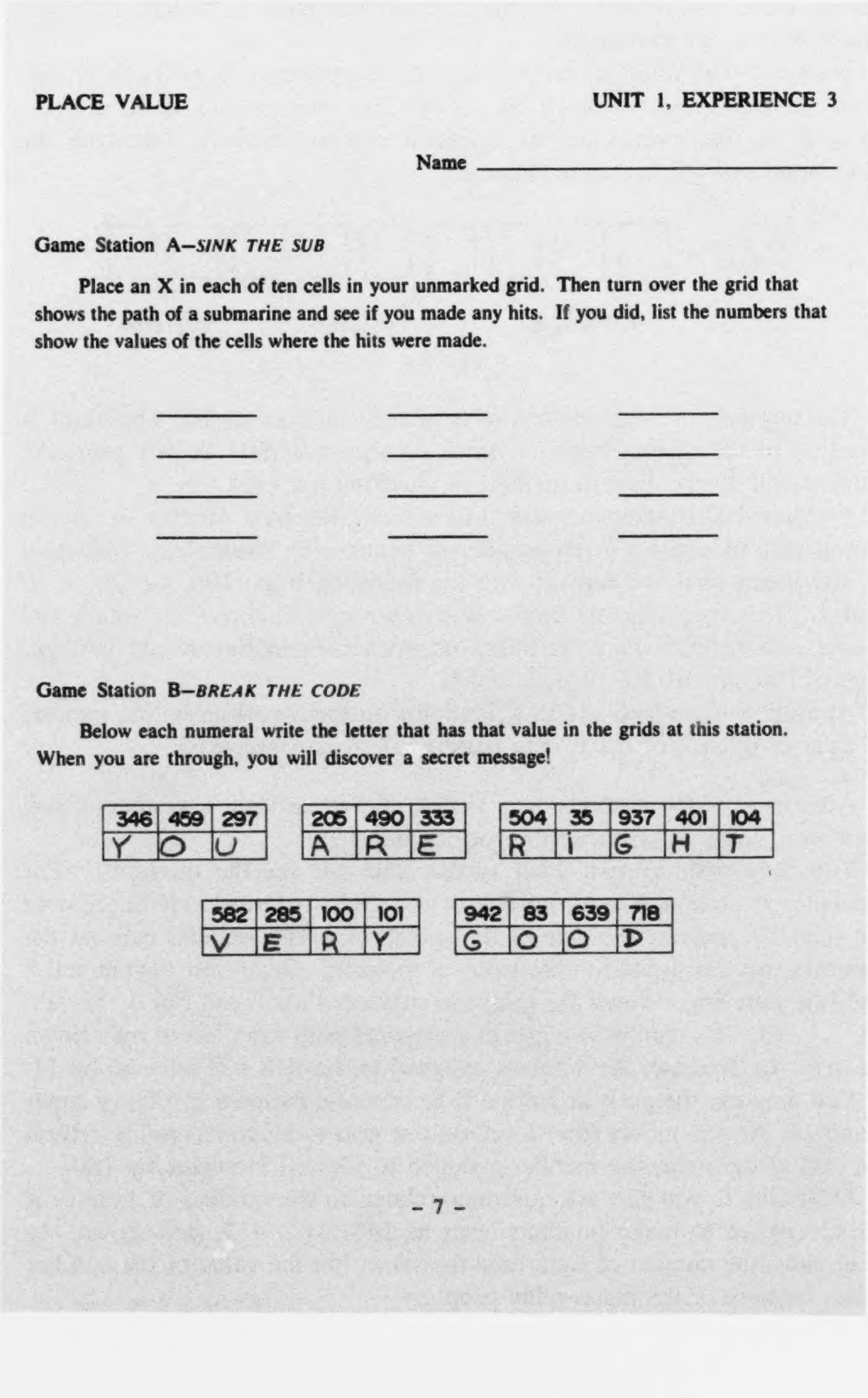
After most of the students have visited all five stations, have them check their worksheets as you read the correct answers.

You may wish to visit each station and ask specific questions. For example, at Station A hold up one of the grids and run your finger over the cells that indicate the path of the submarine. The students call out the numbers that correspond to the value of the cells. Or, if you start at cell 3 and run your finger down the grid, the students should call out 3, 13, 23, 33, . . . , 93. Also run your finger in a diagonal path from left to right down the grid. In this case, the numbers assigned to the cells will increase by 11.

You may use the grids at Station B to generate patterns involving larger numbers. As one moves from a cell on one grid to a corresponding cell on the grid at the right, the number assigned to the cell increases by 100.

At Station C you can ask questions related to the number of squares it would require to make numbers such as 143, 341, 413, and so on. In these cases the number of squares is the same, but the value of the number varies because of the place-value property.

[*Continued on page 19*]



PLACE VALUE **UNIT 1, EXPERIENCE 3**

Name ______________________________

Game Station A—*SINK THE SUB*

Place an X in each of ten cells in your unmarked grid. Then turn over the grid that shows the path of a submarine and see if you made any hits. If you did, list the numbers that show the values of the cells where the hits were made.

________ ________ ________

________ ________ ________

________ ________ ________

________ ________ ________

Game Station B—*BREAK THE CODE*

Below each numeral write the letter that has that value in the grids at this station. When you are through, you will discover a secret message!

346	459	297
Y	O	U

205	490	333
A	R	E

504	35	937	401	104
R	i	G	H	T

582	285	100	101
V	E	R	Y

942	83	639	718
G	O	O	D

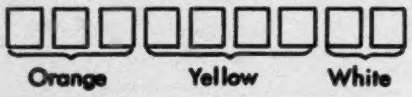
- 7 -

PLACE VALUE UNIT 1, EXPERIENCE 3

Name ______________________________

Game Station C–*HOW MANY DOTS?*

Study the four cards with dots. Color the squares below so that you will have the right number of dots, following the example below:

342 =


162 = O Y Y Y Y Y Y W W

359 = O O O Y Y Y Y Y W W W W W W W W W

902 = O O O O O O O O O W W

209 = O O W W W W W W W W W

1,234 = R O O Y Y Y W W W W

1,342 = R O O O Y Y Y Y W W

Game Station D–*HOW MANY BEANS?*

Find the smallest number of bags of beans for each number below. Following the example given, show the number of beans in each bag and the number of bags.

Number	Bags	Number of Bags
167	100, 40, 20, 4, 2, 1	6
132	100, 20, 10, 2	4
483	400, 40, 40, 2, 1	5
461	400, 40, 20, 1	4
400	400	1
306	200, 100, 4, 2	4
1,301	1,000, 200, 100, 1	4

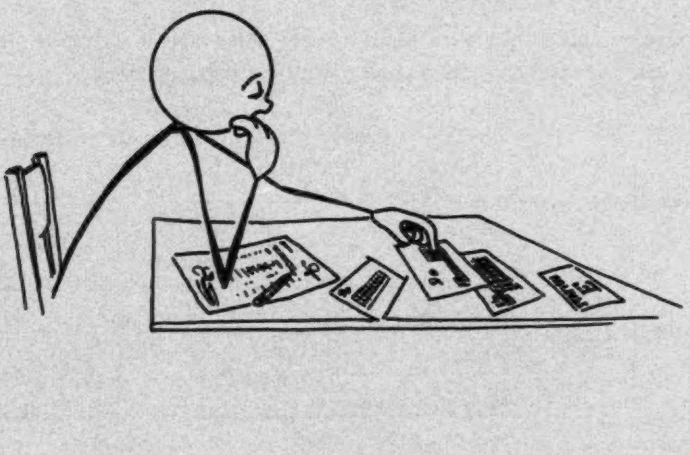
PLACE VALUE **UNIT 1, EXPERIENCE 3**

Name ______________________________

Game Station E—*HOW MANY SQUARES?*

Fill in after the number of each pattern the number of squares it contains.

Pattern	Number of Squares	Pattern	Number of Squares
1	________	11	________
2	________	12	________
3	________	13	________
4	________	14	________
5	________	15	________
6	________	16	________
7	________	17	________
8	________	18	________
9	________	19	________
10	________	20	________



While visiting the stations, ask students to explain how they arrived at the solutions to the various problems presented.

EVALUATION

By observing the students as they work at the stations you can determine the degree of difficulty that students are having in recognizing 123 as a set of one hundred, two tens, and three ones.

In the discussion following the visits to the stations ask questions such as the ones in the following sequence:

If the units digit in a decimal is increased by one, how much does the number increase?

If the tens digit in a decimal is increased by one, how much does the number increase?

If two adjacent digits are switched, will the number represented get bigger or smaller?

What is the largest number expressed by a three-digit decimal?

What is the smallest number expressed by a three-digit decimal?

EXPERIENCE 4

Expanded Notation

OBJECTIVE

The student should be able to express numbers in expanded notation ($123 = 100 + 20 + 3$). The student should also be able to recognize $100 + 20 + 3$ as a name for 123.

MATERIALS

124 index cards
Poster board
Labeled triangles (reproduced from the Teaching Package), 1 for each student at Game Station B
Lucky Number listing (Teaching Package)
4 sets of Games Rules and Report of Winners

Game Station A. For this game a deck of thirty-two cards is required. The deck consists of eight sets of four cards each, the four cards naming the same number, as shown in figure 10. The cards may be labeled with a felt-tip pen.

32	30 + 2	2 + 30	3 tens 2 ones
104	100 + 4	4 + 100	10 tens 4 ones
401	400 + 1	1 + 400	40 tens 1 one
632	600 + 30 + 2	30 + 2 + 600	6 hundreds 3 tens 2 ones
326	300 + 20 + 6	20 + 6 + 300	3 hundreds 2 tens 6 ones
623	600 + 20 + 3	3 + 20 + 600	6 hundreds 2 tens 3 ones
14	10 + 4	4 + 10	1 ten 4 ones
100	10 tens	100 tens	1 hundred

Fig. 10

Game Station B. A similar deck of cards, representing different numbers, is needed for this station.

Game Station C. Labeled triangles, reproduced from the Teaching Package and mounted on poster board, are needed for this station. Each triangle has a 1, a 10, and a 100, as shown in figure 11.

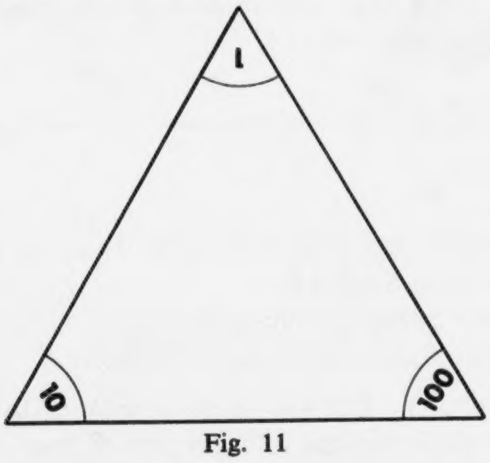


Fig. 11

Game Station D. A deck of sixty cards is required for this station, three sets of twenty cards each, labeled with a 1, a 10, and a 100. A listing of lucky numbers (fig. 12) is also needed. This is found in the Teaching Package. You might want to make enough copies so that each student at the game station will have his own list.

LUCKY NUMBERS

5
13
24
33
42
50
65
72
104
111
115
123
132
141
145
150
160
165
202
211
226
230
231
240
241
301
305
314
320
321
330
331
334
415
422
423
441
442
453
500
510
512
513
524
525
535
545
552
563
571
572
573
574
580
581
583
585
600
601
621
624
625
630
642
645
652
700
701
702
724
842
851
921

Fig. 12

Copies of the set of Game Rules and Report of Winners are also needed.

TEACHER STRATEGY

This experience differs from Experiences 2 and 3 in that a game that can be played by a small group of students is presented at each of the four game stations.

I like to introduce this experience by having students name a three-digit number in several different ways. I direct their attention to those names that illustrate the base property of the numeration system. For example,

$$\begin{aligned} 362 &= 3 \text{ hundreds} + 6 \text{ tens} + 2 \text{ ones} \\ &= 300 + 60 + 2. \end{aligned}$$

After this brief introduction divide the class into four groups and assign each group to a specific game station. The groups should be heterogeneous with respect to mathematical achievement. A Game Rules and Report of Winners form is given to one member of each group, who is called the captain and is responsible for recording the names of the winners. Students should remain at each station until you tell them to rotate to a new station. Call for a rotation when all groups have played their game at least once. Some games are shorter than others and may be played several times.

If your class is large you may wish to have two of each of the games in order to have smaller groups and more opportunity for each student to participate.

After the teams and their captains are selected, you should visit each game station, describe the game, and show the materials to the entire class. It will probably be necessary for you to visit the stations during the play of the games to clarify the rules.

Since the game rules given on the forms are necessarily brief, some additional information is given here.

Game A. The player who turns up two cards need not show the card faces to the other players unless they form a pair. The winner is the person who has the most cards when they have all been turned up and collected or who has the most cards at the time you interrupt a game by calling for a rotation to the next game station. In case of a tie of two or more players count each of the tied players as a winner.

Game B. The players, in rotation, turn up one card and place it face up on the table. When a player turns up a card that has the same value as a card that has already been turned up, he takes the matching card and puts the pair on the table before him. If any player fails to recognize that the card he has turned up matches one that has already been turned up, the next player to his left can claim the matching pair and then make his own play of turning up a card. The winner is the person with the most cards when all cards have been collected or who has the most cards at the time you call for a rotation to the next game station. In case of a tie the tied players are all counted as winners.

Game C. Each student holds his triangle upright so that its face is concealed from the other players and rotates it until the others do not know which numeral is at the top vertex. Students record on scratch paper their guesses about the sum of the numbers at the top, then they place the triangles on the table with the tops pointed inward. One result is seen in figure 13, in which the sum is

$$1 + 1 + 10 + 10 = 22.$$

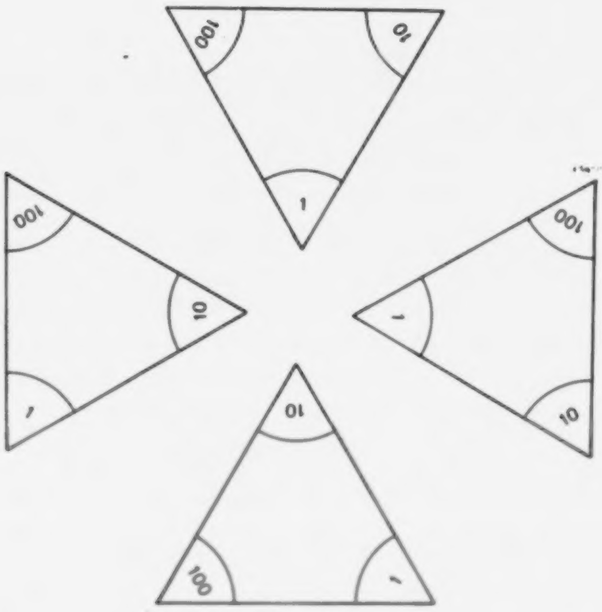


Fig. 13

The winner is the student whose guess is most nearly correct. If two or more players have guesses that are equally close to the correct sum (26 and 18, for instance, in the example given), they are all listed as winners.

You may wish to ask the players, after they have had some experience in playing the game, questions such as these:

What is the largest possible sum?
What is the smallest possible sum?
Is every whole number between these possible as a sum?

Game D. Cards are drawn in the usual order with the play passing from one person to the person at his left. The winner is the first player who recognizes that the sum of the numbers on *all* the cards he is holding is one of the lucky numbers listed. He calls out, "Lucky Number!" and puts his cards down so that others can verify his addition. If the player is in error, he must pick up his cards, and the play resumes. The students will find that it is easier to keep track of their cumulative sum by making separate piles for ones, tens, and hundreds.

When a lucky number is represented, the cards are shuffled and the game starts again.

[*Continued on page 27*]

EXPANDED NOTATION **UNIT 1, EXPERIENCE 4**

Recorder's Name ______________________

GAME RULES AND REPORT OF WINNERS

Game A

Mix the cards and lay them face down on the table in neat rows. The first person turns over any two cards. If the cards match in value, the player keeps them. If the cards do not match, he turns them face down again. Then the player to the left turns up two cards. Play continues until there are no more unclaimed cards. The player with the most cards at the end wins.

WINNERS

Round 1 ______________________

Round 2 ______________________

Round 3 ______________________

Round 4 ______________________

Game B

Mix the deck of cards and spread them out face down on the table in neat rows. One player turns a card face up. Then the next person on the left does the same thing. If this makes a pair of cards that match in value he can claim both cards; if it does not, both cards are kept face up and again play passes to the left. The winner is the person who has the most cards when the cards have all been claimed.

WINNERS

Round 1 ______________________

Round 2 ______________________

Round 3 ______________________

EXPANDED NOTATION **UNIT 1, EXPERIENCE 4**

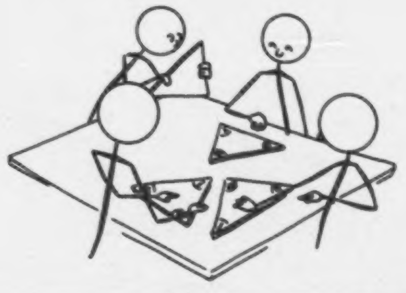
Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

Game C

Each person takes one of the triangles and holds it in front of him with the numbers hidden from the other players. He turns it around several times so no one knows which number is at the top.

Each player then guesses the sum of the numbers at the top and writes the guess on a piece of paper. After everyone has recorded his guess, players hold their triangles at the lower corners and place them face up in the middle of the table. The person who has the best guess about the sums of the corners in the middle is the winner of that round. If there is a tie, both students are winners.



WINNERS

Round 1	____________	____________	____________
Round 2	____________	____________	____________
Round 3	____________	____________	____________
Round 4	____________	____________	____________
Round 5	____________	____________	____________
Round 6	____________	____________	____________
Round 7	____________	____________	____________
Round 8	____________	____________	____________

EXPANDED NOTATION **UNIT 1, EXPERIENCE 4**

Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

Game D

Mix the deck of cards and place the cards face down in a pile in the center of the table. Each player draws a card in turn. The drawing of cards continues until someone can match the values of the cards he has drawn to one of the lucky numbers.

When drawing cards you should make piles of ones, tens, and hundreds.

WINNERS

Round 1 ______________________________

Round 2 ______________________________

Round 3 ______________________________

Round 4 ______________________________

Round 5 ______________________________

Round 6 ______________________________

EVALUATION

In this experience you have introduced the idea of extended notation. The winners of the games are not necessarily the ones who best understand this concept, since the games involve some luck. In order to evaluate the students' understanding of extended notation, lead a group discussion after all students have played all of the games. You may wish to place on the chalkboard, for discussion, problems such as those listed below.

1. 362 = 300 + ________ + 2.
2. 783 = ________ + 80 + 3.
3. 1,870 = ________ + ________ + ________.
4. 1,201 = ________ + ________ + ________.
5. 7,003 = ________ + ________.
6. 100 + 20 + 4 = ________.
7. 200 + 40 = ________.
8. 1,000 + 200 + 30 + 4 = ________.

EXPERIENCE 5
Comparing Numbers

OBJECTIVE

The student should be able to determine which of two whole numbers is the greater.

MATERIALS

266 or more index cards
Rubber bands
12 small wooden cubes
3 worksheets for each student

Station A. Four identical decks of cards are placed at this station. (More decks may be required with a large class.) Each deck consists of fourteen index cards marked on front and back as shown below:

Front	Back	Front	Back
35	T	801	R
72	H	810	I
173	I	811	G
617	S	907	H
671	/	970	T
761	I	978	•
796	S		
800	/		

The cards should be thoroughly mixed in each pack, and each pack should be held together with a rubber band.

Station B. Four or more sets of index cards are used in Station B. Each set consists of fifty cards with pairs of numbers like those shown below:

6 and 7
24 and 42
104 and 401
89 and 98
64 and 78
1,623 and 1,632

In order to emphasize the importance of place value, many of the pairs should have the same digits. Thirty of the cards should have the greater number on the left; the remaining twenty would have the greater number on the right.

Station C. Ten index cards are cut in fourths, making a total of forty small cards. The cards are labeled on the front and back as shown below, the letters on one side and the numerals on the other. Each student should have his own deck of cards.

6 L
7 L
24 L
115 L
126 L
178 L
42 L
943 G
68 L
978 G

78 L
41 L
104 L
140 L
68 L
984 G
381 L
362 L
174 L
789 G

682 G
794 G
547 G
571 G
617 G
981 G
683 G
372 L
84 L
79 L

541 L
569 G
983 G
741 G
641 G
394 L
542 E
78 L
848 G
681 G

Station D. Four sets of wooden cubes are required for this station. Each set contains three cubes that are labeled differently. The faces on one are 1, 2, 3, 4, 5, 6. The faces on the second are 4, 5, 6, 7, 8, 9, and on the third they are 1, 2, 3, 7, 8, 9. Each set should be kept in a small bag so that it does not get mixed up with the other sets.

TEACHER STRATEGY

The major purpose of this experience is to reinforce the idea of place value by having students order sets of numbers.

I like to introduce this experience by playing a high-low game with the students. I select a number between 1 and 1,000 and write it on a piece of paper. A student makes a guess, and I tell him whether the guess is too high or too low. The students continue guessing until someone guesses the correct number. You may wish to have students respond in sequence around the room in order to give everyone a chance.

This introduction should be brief, in order to allow time for the laboratory activities and the discussion following them.

After the introduction pass out the worksheets. Visit each station in order to read the directions aloud and describe the materials and the procedure. Emphasize that at Stations A and B the students should mix their decks of cards thoroughly and replace the rubber bands. Show the students how to tally the number of cards turned over as indicated in figure 14, which stands for eleven cards turned over.

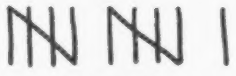


Fig. 14

At Station C the student should learn quickly how to avoid picking up cards unnecessarily. If he turns over a card with 172, which is "less than," he should know that the card with 163 would also be a "less than," so that he would not want to turn it over. If you have a deck of cards for each student working at this station, instruct the students not to give away the fact when they have found the card with an E label. Tell them to return it to the pile quietly and to continue picking up other cards but only pretend they are still marking tallies.

At Station D the student is asked to make different numbers by lining up the cubes. For example, they are asked what is the greatest number they can make by lining up the cubes; the answer is 996. The smallest number is 114. The student is also asked to find numbers such as the greatest number with a 7 in the tens place. This would be 976.

When most of the students have visited all of the four stations, lead a discussion to determine the extent to which the objective of this experience has been met.

EVALUATION

You can evaluate the success of this experience by having students place in increasing order all of the four-digit numbers that can be formed by using 1, 2, 3, and 4 without repeating any of the digits. This set of numbers was generated in Experience 3 without regard to order.

Another type of game can be used to evaluate the student's ability to compare numbers. Give each student in the room a card with a different numeral on it. Ask all students who have numbers larger than 63 to hold up their cards; have all students who have numbers between 143 and 467 hold up their cards; and so on.

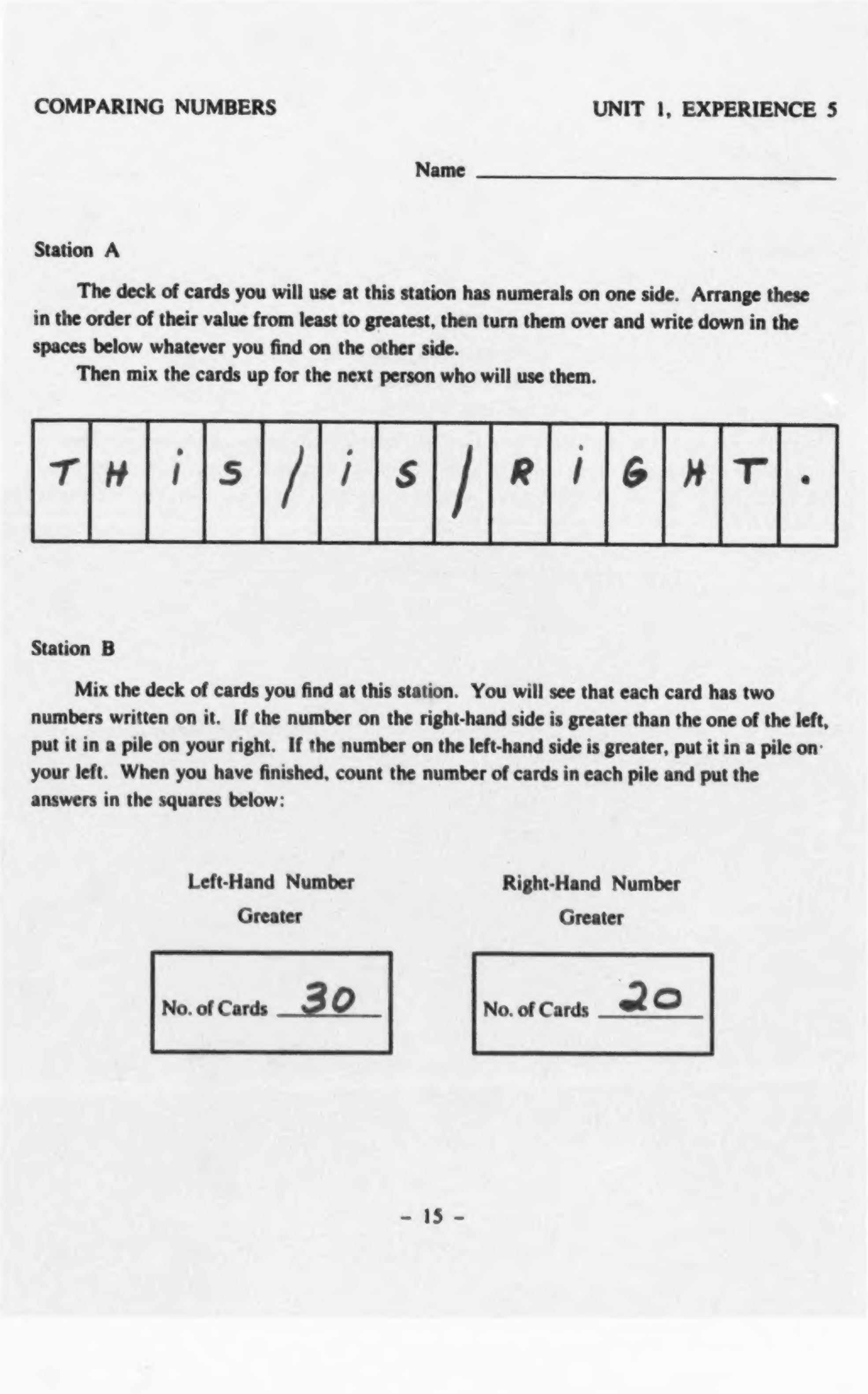
You may lead a further discussion by posing questions such as these:

Which number is greater, 345 or 354?

What is the greatest number that can be formed using the digits 1 through 9 once and only once?

What is the smallest number that can be formed using the digits 1 through 9 once and only once?

What is a systematic way of comparing two numbers?



COMPARING NUMBERS **UNIT 1, EXPERIENCE 5**

Name ______________________

Station A

The deck of cards you will use at this station has numerals on one side. Arrange these in the order of their value from least to greatest, then turn them over and write down in the spaces below whatever you find on the other side.

Then mix the cards up for the next person who will use them.

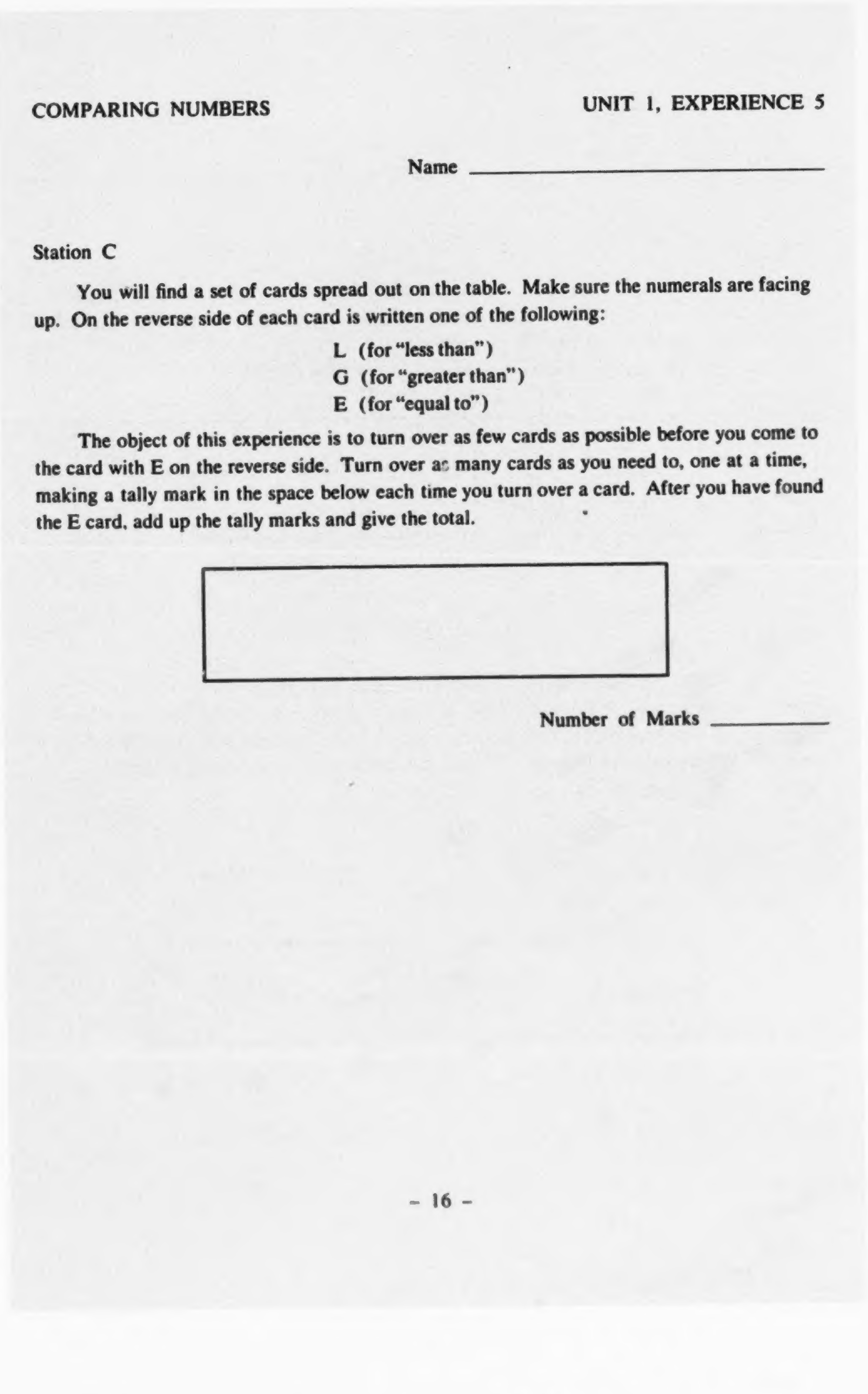
T	H	i	S	/	i	S	/	R	i	G	H	T	.

Station B

Mix the deck of cards you find at this station. You will see that each card has two numbers written on it. If the number on the right-hand side is greater than the one of the left, put it in a pile on your right. If the number on the left-hand side is greater, put it in a pile on your left. When you have finished, count the number of cards in each pile and put the answers in the squares below:

Left-Hand Number Greater	Right-Hand Number Greater
No. of Cards 30	No. of Cards 20

- 15 -



COMPARING NUMBERS

UNIT 1, EXPERIENCE 5

Name ______________________________

Station C

You will find a set of cards spread out on the table. Make sure the numerals are facing up. On the reverse side of each card is written one of the following:

L (for "less than")
G (for "greater than")
E (for "equal to")

The object of this experience is to turn over as few cards as possible before you come to the card with E on the reverse side. Turn over as many cards as you need to, one at a time, making a tally mark in the space below each time you turn over a card. After you have found the E card, add up the tally marks and give the total.

Number of Marks __________

COMPARING NUMBERS **UNIT 1, EXPERIENCE 5**

Name ______________________________

Station D

You will find three cubes with numerals on them at this station. Each cube is different. The numerals on them are shown below:

1, 2, 3, 4, 5, 6
3, 5, 6, 7, 8, 9
1, 2, 3, 7, 8, 9

1. What is the greatest number you can make by lining up the cubes next to each other? 9 9 6
2. What is the greatest number with 7 in the tens place? 9 7 6
3. What is the greatest number with 1 in the tens place? 9 1 9
4. What is the smallest number with 9 in the tens place? 1 9 1
5. What is the smallest number with 7 in the tens place? 1 7 1
6. What is the smallest number with 1 in the tens place? 1 1 3

2

EXPERIENCES WITH

Renaming Numbers in Addition and Subtraction

The purpose of this unit is to develop physical models for the renaming of numbers and to show how this renaming property can be used in the addition and subtraction algorithms.

The word "exchange" is used to describe what happens in a concrete model for renaming numbers. In addition it is often necessary to exchange ten ones for one ten. This is traditionally known as carrying. In subtraction the exchange property appears when it is necessary to exchange one ten for ten ones. This is traditionally known as borrowing. Exchange better describes what happens in both cases. The word "exchange" becomes very natural, for example, when using the abacus to illustrate addition and subtraction. Some teachers refer to this concept as "regrouping."

A laboratory approach is utilized to achieve the objectives of this unit. The activities are designed to help the student develop particular mathematical concepts with a minimum of teacher direction. You should assume a supervisory role similar to that of the chemistry teacher in the chemistry laboratory. You will need to prepare the necessary materials and to introduce each experience by describing the materials and procedures. The discussion you lead when students have finished the activities can serve both as a summary emphasizing the major mathematical ideas and as a means of evaluating student progress.

OVERVIEW

Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether the unit is suitable for your class. It also lists the materials you will need for each experience. Master copies of student worksheets and teacher materials are given in the Teaching Package. It is assumed that you have access to duplicating facilities.

Experience 1: Exchanging Groups of Ten

Experience 1 provides some basic activities in renaming numbers (exchanging). Students compete in small groups at the four game stations. After all the games have been played at least once, each group moves on to another station.

Materials needed: white, red, green, and yellow index cards, board, Popsicle sticks, nails, washers, poster board, sets of "Game Rules and Report of Winners."

Experience 2: Recording Names for Groups of Ten

A "dart" game is used to motivate this experience. The students work with paper, pencil, and overhead projector transparencies.

Materials needed: overhead projector transparencies, worksheets.

Experience 3: Renaming Numbers

The dart game introduced in Experience 2 is extended here. Emphasis is placed on the exchange property (changing names of numbers).

Materials needed: worksheets.

Experience 4: Exchanges in Column Addition

The algorithm for column addition is developed by means of a lima-bean device constructed by each student.

Materials needed: paper cups, poster board, lima beans, worksheets.

Experience 5: Exchanges in Subtraction

The lima-bean calculator of Experience 4 is used here to develop the subtraction algorithm. In both experiences, the student is encouraged to use the exchange property without the aid of the lima-bean device.

Materials needed: lima-bean calculators and beans used in Experience 4, worksheets.

EXPERIENCE 1
Exchanging Groups of Ten

OBJECTIVE

The student should be able to recognize that ten ones can be exchanged for one ten and that one ten can be exchanged for ten ones. He should also be able to make exchanges involving ones, tens, hundreds, and thousands.

MATERIALS

3-by-5-in. index cards (190 white, 30 red, 30 green, 30 yellow)
8-by-12-by-¼-in. board (approximate dimensions)
2 Popsicle sticks
2 small nails
2 washers
1 heavy poster board
4 sets of Game Rules and Report of Winners

Game Station A. Eighty index cards labeled 1, 10, 100, and 1,000 (twenty of each) are required for this game. Identification is easier if a different color is used for each denomination.

Two spinners must be constructed. The two dials, duplicated from the originals in the Teaching Package, are mounted on the same board (see fig. 1). The pointers are Popsicle sticks filed to a point at one end and nailed to the board. A washer might be needed between the

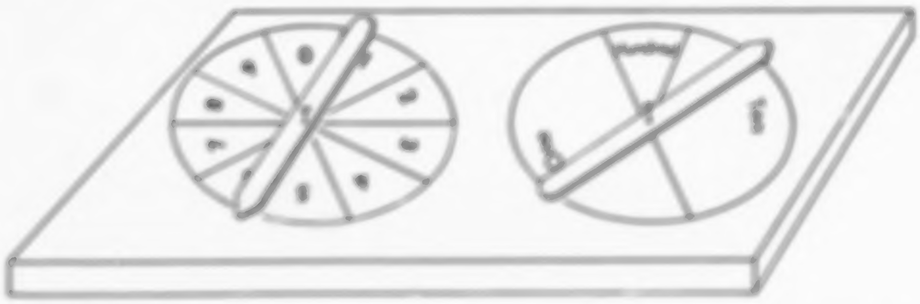


Fig. 1

board and the stick. A copy for each team of the Game Rules and Report of Winners for this game should also be made.

Game Station B. A deck of sixty-five white index cards is required for this game. The deck should consist of twenty cards labeled 1, twenty cards labeled 10, twenty cards labeled 100, and five cards labeled with a star. For each team duplicate the copy of Game Rules and Report of Winners for this game.

Game Station C. This game requires a deck of fifty-four white index cards: two sets of cards labeled 1, 2, . . . , 9; 10, 20, . . . , 90; and 100, 200, . . . , 900. Thirty 3-by-5-inch score cards, labeled as shown in figure 2, are also required. The three scores should be identified by three different colors, and there are ten cards for each score. A copy of the Game Rules and Report of Winners for this station is needed for each team.

10 1 point	100 2 points	1,000 3 points

Fig. 2

Game Station D. This game is not concerned directly with the concept of exchange, but it does illustrate the *ten*ness of the decimal numeration system.

Make a racetrack (about 3 ft. by 2 ft.) on heavy poster board and divide it into exactly 100 spaces (see fig. 3).

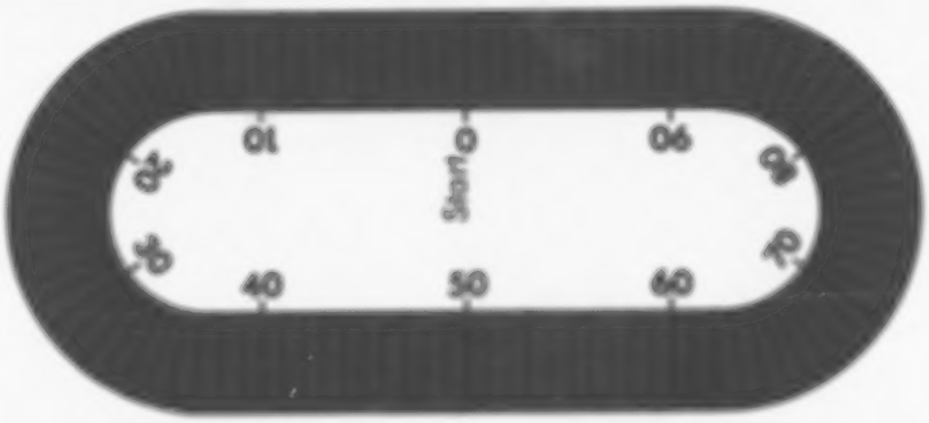


Fig. 3

You should supply each player with a small marker representing a race car and with a counter enabling him to record the number of complete circuits he makes around the track. Counters for this purpose can be made, as shown in figure 4, using the master in the Teaching Package.

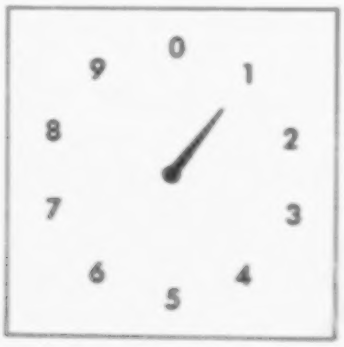


Fig. 4

A deck of fifty index cards is required for this game. The numbers on the cards should range from 20 to 300, with most of them close to 150.

TEACHER STRATEGY

In this experience students use the exchange property to exchange a group of ten for one in another group. This gives them a concrete example for the more abstract work related to renaming numbers in the latter part of the unit.

The experience is game-oriented. Four groups of students are formed, and a team captain is chosen for each. The captain is responsible for recording the names of the winners.

I usually introduce the activities by asking students to consider how they would exchange a dollar bill for dimes and a dime for pennies. I try to draw out from them the fact that one dime and ten pennies both name the same amount of money.

After the teams are selected, I visit each station to describe the rules of the games and to show the materials.

Each team is assigned to a game station. After all four games have been played at least once, the teams rotate to new stations.

While the students are playing the games, I visit the stations and clarify the rules. You may wish to set up two stations for each game so that every student will have an opportunity to participate.

The games are played as follows, with the materials already described.

Game A. The purpose of this game is to give the students experiences in exchanging. The cards should be arranged in four piles according to denomination to serve as a bank. Each player takes a turn, spinning both pointers. If one points to 2 and the other points to Ten (see fig. 1), the player takes two cards labeled 10. He places the cards in front of him and the play passes to the left.

When a player gets ten or more cards of the same denomination, he *must* exchange them for one card that is equivalent. For example, ten ones should be exchanged for one ten. If after any spin there are not enough of the required cards in the bank, the player spins again—without this agreement many more cards would be required for the game.

The first person to get a total score of 1,000 points is the winner. The team captain should record the winner's name. If you have not asked the students to move around to a new game, they should start the game again.

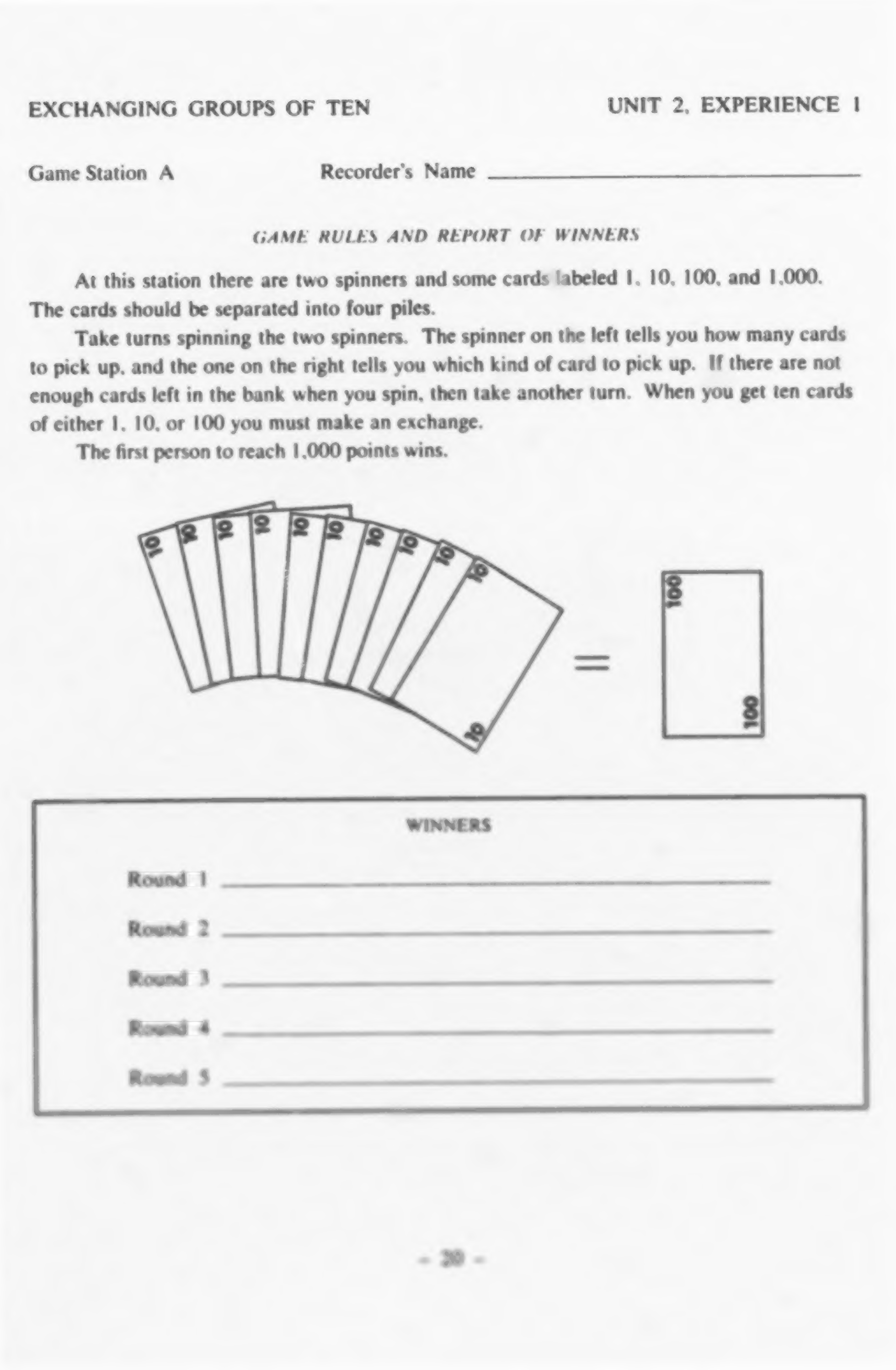
Game B. The deck is shuffled and placed in the center of the table. Students take turns drawing one card at a time. When a star is drawn, each student adds up the values of the cards in his hand. In some cases he can determine his score more quickly by grouping by tens. The student with the highest score is the winner, and his name is recorded by the team captain. The game is repeated as time permits.

Game C. The white cards are shuffled, then dealt out until the deck is gone. Players should conceal their cards. Any player who can form a sum of 10, 100, or 1,000 with two or more cards should exchange these cards for a colored score card. Thus cards labeled 70, 20, and 10 are exchanged for a 100 score card, which is worth two points. Students are required to "trade up"; for example, they cannot exchange a card labeled 400 for four cards labeled 100.

After all exchanges are made, each player must pass any one card to the person on his left. Again the players look for exchanges that will produce score cards. They continue passing to the left and making exchanges until no more exchanges can be made. At that point the game ends, and the student with the greatest number of points wins. The game is repeated as time permits.

Game D. Players place their cars on "Start" and adjust their counters to show 0 laps. The cards are shuffled and placed face down in the center of the table. Each player, in turn, draws a card and moves his car the

[*Continued on page 45*]



EXCHANGING GROUPS OF TEN UNIT 2, EXPERIENCE 1

Game Station A Recorder's Name ____________________

GAME RULES AND REPORT OF WINNERS

At this station there are two spinners and some cards labeled 1, 10, 100, and 1,000. The cards should be separated into four piles.

Take turns spinning the two spinners. The spinner on the left tells you how many cards to pick up, and the one on the right tells you which kind of card to pick up. If there are not enough cards left in the bank when you spin, then take another turn. When you get ten cards of either 1, 10, or 100 you must make an exchange.

The first person to reach 1,000 points wins.

WINNERS

Round 1 ____________________

Round 2 ____________________

Round 3 ____________________

Round 4 ____________________

Round 5 ____________________

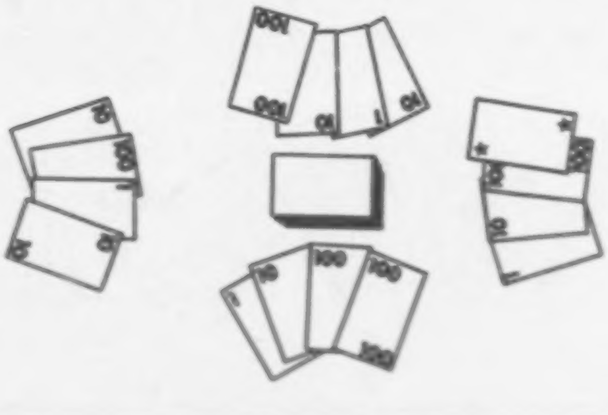
EXCHANGING GROUPS OF TEN **UNIT 2, EXPERIENCE 1**

Game Station B Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

At this station there is a deck of cards labeled with 1s, 10s, 100s, and stars. Shuffle the deck and put it in the center of the table. Take turns drawing one card at a time. When a card with a star is drawn, each student adds the numbers on his cards to find his score.

The student with the highest score wins.



WINNERS

Round 1 ______________________________

Round 2 ______________________________

Round 3 ______________________________

Round 4 ______________________________

Round 5 ______________________________

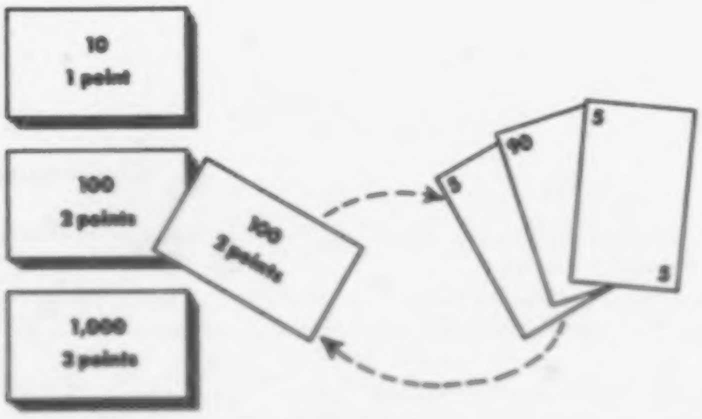
EXCHANGING GROUPS OF TEN **UNIT 2, EXPERIENCE 1**

Game Station C Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

At this station there is a deck of white cards and a set of colored score cards. The score cards should be divided into three piles according to color. The white cards are mixed up and dealt to the players until the deck is gone. Hold your cards so other players cannot see them.

If you can form a sum of 10, 100, or 1,000 by using two or more cards, you should exchange them for the score card with the correct value. Always exchange a group of white cards for one colored score card. After each player has made all the exchanges he can, everyone must pass one card to the person on his left. Again make any possible exchanges, then everyone pass one card to the person on his left. Again make exchanges. Continue the game until no more exchanges can be made. The player with the greatest number of points wins.



WINNERS

Round 1 ______________________________

Round 2 ______________________________

Round 3 ______________________________

Round 4 ______________________________

EXCHANGING GROUPS OF TEN UNIT 2, EXPERIENCE 1

Game Station D Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

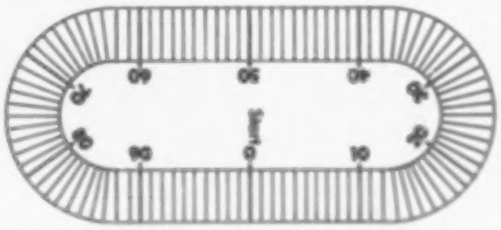
At Station D you will find the following materials:

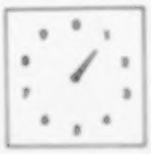
a racetrack

markers

a deck of cards

lap counters





Each player takes a marker and a counter. Put the markers on Start. Shuffle the deck and put it in the middle of the table.

Take turns drawing cards. Move your marker the number of spaces indicated on the card you draw. Every time you pass Start, move the pointer on your counter to show one more lap.

The first player to complete ten laps wins.

WINNERS

Round 1 ______________________________

Round 2 ______________________________

number of spaces indicated on the card. In most cases a player will have to adjust his lap counter. For example, assume his first turn takes him to space 73 and his second draw is 102. He must move his counter from 0 to 1 and move his car ahead two spaces, to space 75. Since most of the cards are labeled with numbers near 150, it takes about seven draws to complete ten revolutions.

The first car to complete ten laps is the winner, and the owner's name is recorded. If time permits, the game can be started again; if it must be stopped, the car out in front is considered the winner.

EVALUATION

This experience develops the concept of exchange. After all students have played each game at least once, lead a group discussion. Questions like the following may be posed:

How many cents in a dollar? In a dime? In a ten-dollar bill?
How many tens are there in one hundred?
How many dimes could be exchanged for a dollar? For a ten?
How many hundreds are there in one thousand?
How many dimes could be exchanged for a hundred-dollar bill?
How many pennies could you get for a ten-dollar bill?

EXPERIENCE 2
Recording Names for Groups of Ten

OBJECTIVE

The student should be able to place letters representing darts on a target to form a specific score. He should also be able to determine the score in a round of darts. Both activities require some exchanges.

MATERIALS

20 overhead-projector transparencies (a projector is not needed)
2 worksheets for each student

Stations. Stations A, B, C, D, and E should be set up around the room. Each station should have four identical transparencies reproduced from the appropriate sheet in the Teaching Package (see fig. 5). Each student will need a copy of the worksheet for Activity 2 to use at all five stations.

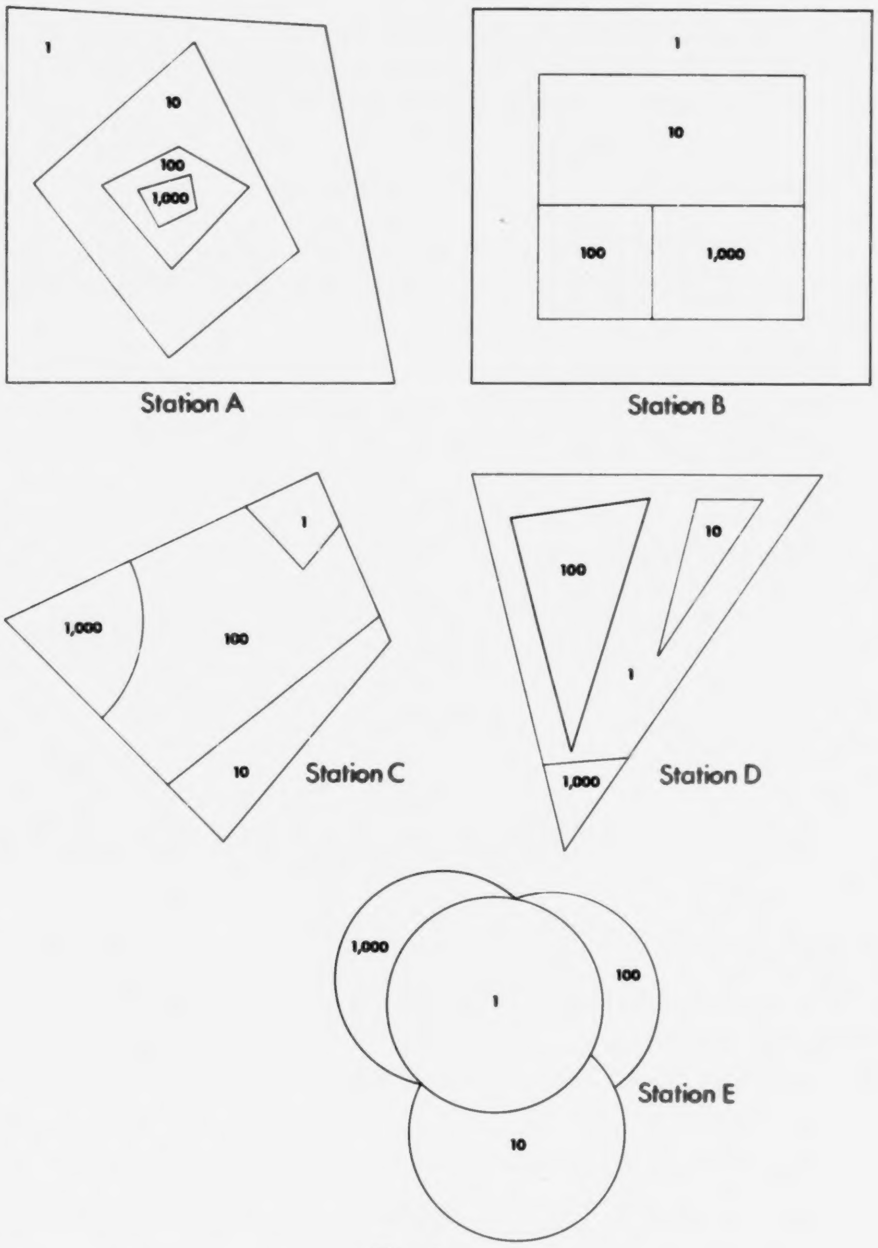

1
10
100
1,000
Station A
1
10
100
1,000
Station B
1
1,000
100
10
Station C
10
100
1
1,000
Station D
1,000
100
1
10
Station E

Fig. 5

TEACHER STRATEGY

In this experience students are introduced to a dart game in which ten "darts" in one ring of a target are exchanged for one "dart" in another ring in such a way that the total score remains the same. In Experience 3, symbolism will be introduced to represent these exchanges.

I begin the activities by distributing the worksheet for Activity 1 and giving the students a practice example. If I use the letter x to get a score of 428 with as few darts (x's) as possible, the only correct answer would be that shown in figure 6. I show the students that another way to get a score of 428 would be to place four darts in the 100 ring and twenty-eight in the 1 ring; however, this arrangement would not represent the minimum number of darts.

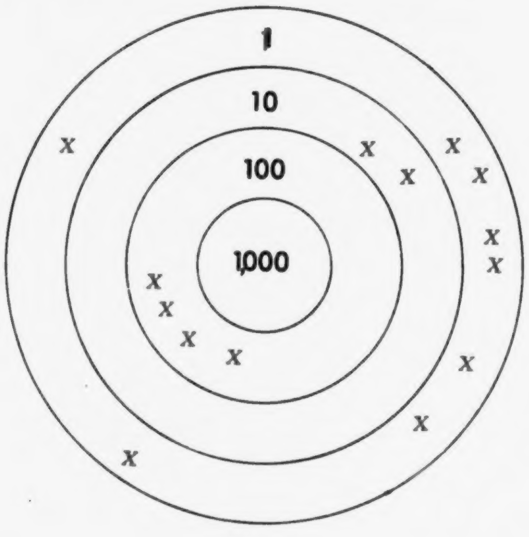


Fig. 6

When the students have finished the first worksheet, I pass out the second and describe the activities at the five stations. To illustrate, I draw twenty x's on a sheet of paper and show that the score is determined by placing the transparency over the paper. It is a good idea to have more than ten x's in one of the rings in order to illustrate the concept of exchange.

After the students place twenty x's in the large box on the worksheet, they determine their score at each station by recording the number of darts that fall into each band. They also record their total score from all stations at the bottom of the worksheet.

You may want to attach each transparency to a desk, table, or wall with

[*Continued on page 50*]

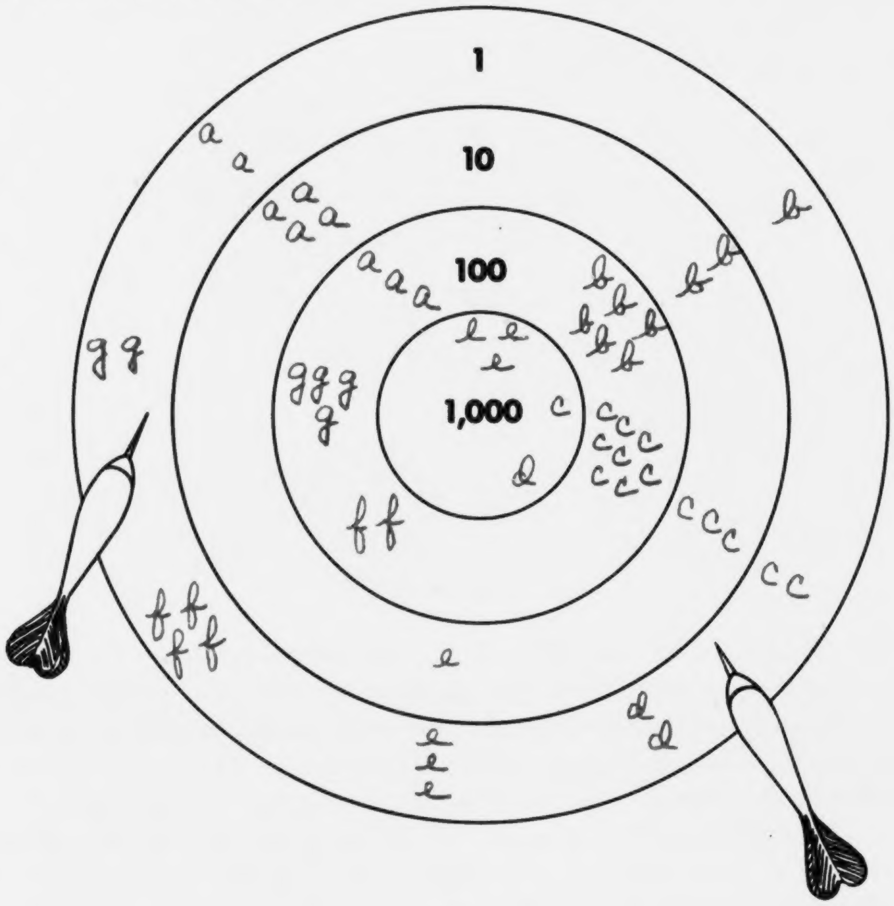
RECORDING NAMES FOR GROUPS OF TEN — UNIT 2, EXPERIENCE 2

Activity 1 — Name ______________________

Form the following scores by placing the letters on the target:

342 with *a*'s	3,013 with *e*'s
621 with *b*'s	204 with *f*'s
1,832 with *c*'s	402 with *g*'s
1,002 with *d*'s	

Use as few letters as possible.


1
10
100
1,000
a a
a a a a
a a a
b
b b
b b
b b b b
c
c c c c c c c
c c c
c c
d
d d
e e e
e
e e e
f f
f f f f
g g
g g g g

RECORDING NAMES FOR GROUPS OF TEN UNIT 2, EXPERIENCE 2

Activity 2 Name ______________________________

Draw twenty Xs in the box. At each station place the transparency over this sheet and determine your score. Record your scores below and then find the total score.

	Thousand	Hundred	Ten	One	Score
Station A					
Station B					
Station C					
Station D					
Station E					
				TOTAL SCORE	

a piece of string so that it is not removed from the station. Students should not change the position of any of the *x*'s once they are on the worksheet. Those who finish quickly may wish to try again for a higher score.

EVALUATION

A student's understanding of the exchange property cannot be judged by the numerical value of his score, since this is partly a matter of luck; his understanding can be judged only by the accuracy of his score. The dart game is used as an entertaining and interesting way of learning the exchange concept.

In the discussion following the visits to the stations you might ask questions such as the following:

What is the highest score you can get at each station?

What is the highest possible total score?

Can the target be made with more rings? What would be the values of additional rings?

Can ten darts always be exchanged for one dart?

EXPERIENCE 3

Renaming Numbers

OBJECTIVE

The student should be able to show several different ways in which a given score can be achieved in the dart game. For example, if slash lines are used to separate the places in a decimal numeral, he could compare a 3 in the 100 ring, a 4 in the 10 ring, and a 2 in the 1 ring with another way of achieving the same score as follows: 3 / 4 / 2 = 2 / 14 / 2.

MATERIALS

4 worksheets for each student

TEACHER STRATEGY

The purpose of this experience is to develop concrete representations for the renaming of numbers. Students work individually on the worksheets.

I introduce the experience by discussing the various ways in which 100 can be named. The students should be able to list the different ways on the chalkboard as follows:

Hundreds	Tens	Ones
1	0	0
0	10	0
0	9	10
0	8	20
⋮	⋮	⋮
0	0	100

After they have demonstrated their ability to make such a list, I choose a number and, using slashes to separate the places, show the different ways of interpreting it. For example, 342 = 34 / 2 = 3 / 4 / 2 = 2 / 14 / 2 = ... Thus 342 can be interpreted as 34 tens and 2 ones; as 3 hundreds, 4 tens, and 2 ones; as 2 hundreds, 14 tens, and 2 ones; and so on.

Next I distribute a copy of the first worksheet to all students. I read the directions with them and give a few examples so they know what to do. While they are working on Activity 1, I draw six targets on the chalkboard. When most of the students have finished the worksheet, I ask some of them to come to the board and illustrate their solutions. Six of the twenty possible ways to represent 142 are the following:

Hundreds	Tens	Ones
0	0	142
0	14	2
0	13	12
1	3	12
1	4	2
1	0	42

[*Continued on page 56*]

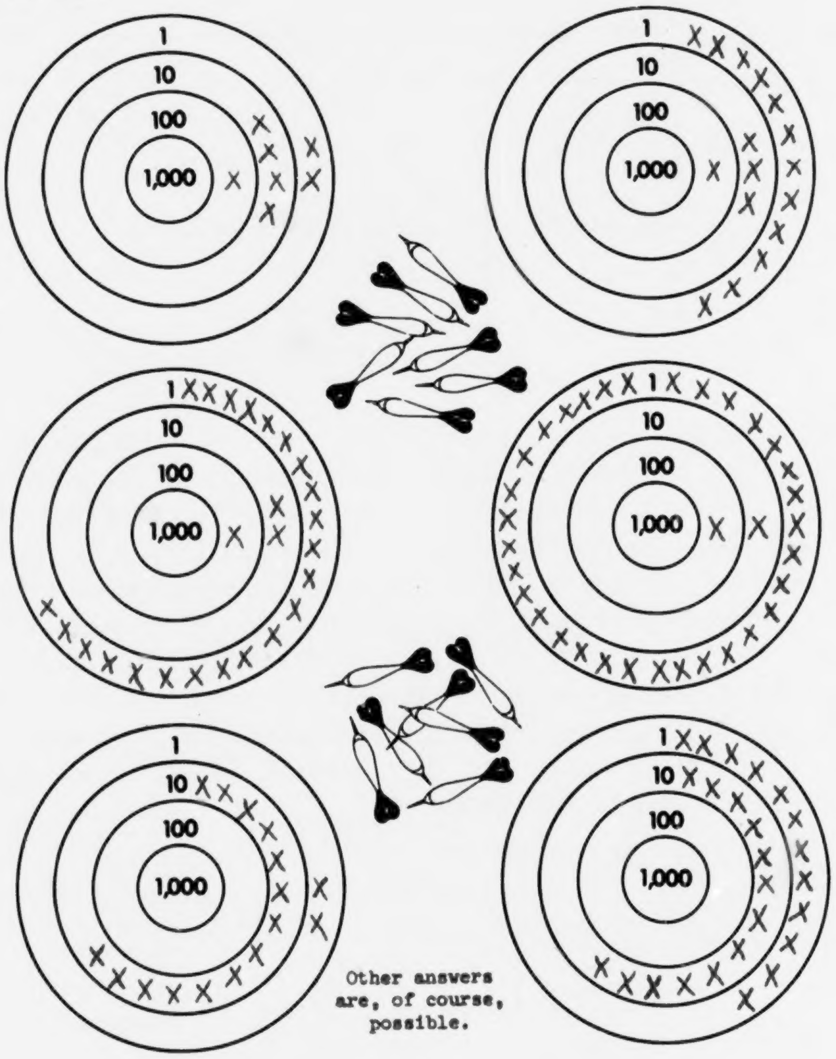
RENAMING NUMBERS

UNIT 2, EXPERIENCE 3

Activity 1

Name ____________________

Show how a score of 142 can be formed in different ways. Use small *x*'s to show where the darts land.



Other answers are, of course, possible.

RENAMING NUMBERS UNIT 2, EXPERIENCE 3

Activity 2 Name ____________________

Fill in the scores.

	Thousand	Hundred	Ten	One	Score
1.	2	3	1	4	2,314
2.			2	0	20
3.		2	0	1	201
4.		1	10	0	200
5.			1	12	22
6.		2	10	3	303
7.		10	2	3	1,023
8.			23	1	231
9.		1	25	2	352
10.		10	10	1	1,101
11.		10	20	30	1,230
12.		0	20	40	240
13.		1	26	42	402
14.		1	73	17	847

RENAMING NUMBERS UNIT 2, EXPERIENCE 3

Activity 3 Name ______________________

Fill in the blanks.

	Thousand	Hundred	Ten	One	Score
1.			2	4	24
2.		6	7	13	683
3.		1	5	14	164
4.		1	14	8	248
5.		6	18	1	781
6.		1	2	43	163
7.			22	2	222
8.			67	13	683
9.		17	2	114	1,834
10.		14	28	8	1,688

RENAMING NUMBERS **UNIT 2, EXPERIENCE 3**

Activity 4 **Name** ____________________

Fill in the blanks to show how the score can be achieved in different ways.

	Thousand	Hundred	Ten	One	Score
1.			2	4	24
2.			1	14	24
3.				24	24
4.			5	10	60
5.			4	20	60
6.			3	30	60
7.			17	3	173
8.			16	13	173
9.			15	23	173
10.			14	33	173
11.		1	7	3	173
12.	3	3	0	42	3,342
13.	3	3	4	2	3,342
14.	3	3	3	12	3,342
15.	3	3	2	22	3,342
16.	3	3	1	32	3,342

I handle the other three worksheets in the same manner. After some of the students have presented their answers at the board, I read the correct answers aloud and lead a discussion.

EVALUATION

You can evaluate the students' understanding of the renaming process by having them report their scores after you read the answers for each worksheet. Questions like the following will give you further insight into their understanding.

Could a score of 42 be achieved with exactly five darts? Why not?

How many different ways could you get a score of 100 if all of the darts fell in one ring?

Is 3 / 1 / 4 the same as 31 / 4?

EXPERIENCE 4

Exchanges in Column Addition

OBJECTIVE

The student should be able to add a column of numbers by using an "abacus" constructed of small paper cups on poster board with lima beans as counters. Some students should be able to add a column of numbers without the aid of the device.

MATERIALS

- 4 small paper cups for each student
- 1 poster board for each student
- Large package of uncooked lima beans or equivalent
- 3 worksheets for each student

TEACHER STRATEGY

The purpose of this experience is to apply the concepts of renaming and exchanging to the development of an algorithm for addition. In order to accomplish this, each student should construct a lima-bean calculator. Four paper cups are mounted on a stiff piece of poster board with quick-drying rubber cement. The cups are labeled as shown in figure 7. This device is also utilized in Experience 5.

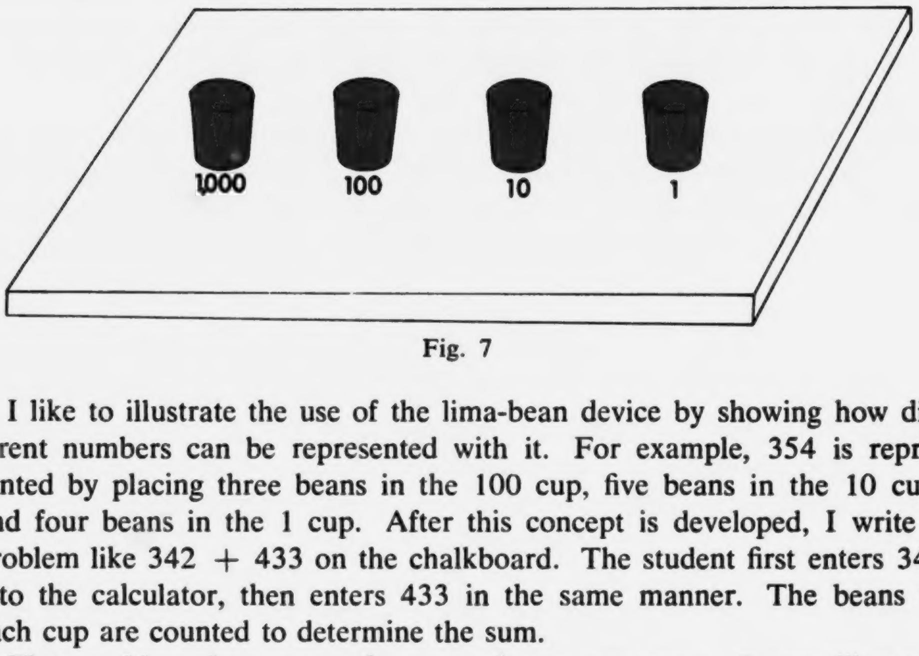


Fig. 7

I like to illustrate the use of the lima-bean device by showing how different numbers can be represented with it. For example, 354 is represented by placing three beans in the 100 cup, five beans in the 10 cup, and four beans in the 1 cup. After this concept is developed, I write a problem like 342 + 433 on the chalkboard. The student first enters 342 into the calculator, then enters 433 in the same manner. The beans in each cup are counted to determine the sum.

That problem does not make an exchange necessary. I next illustrate a problem that requires the use of the exchange principle. For example, 346 + 247 would require a student to exchange ten ones for one ten; although the sum could be expressed as 5 / 8 / 13, 5 / 9 / 3 is much more efficient.

Once students develop the concept that ten in any one cup can be exchanged for one in the cup on the left, I put on the board a problem involving more than two addends, such as the following:

$$\begin{array}{r} 163 \\ 241 \\ 648 \\ +\ 192 \\ \hline \end{array}$$

After each number is entered, students should make sure that all possible leftward exchanges are made.

After the students have worked several problems under my supervision, I hand out the worksheet for Activity 1. Students are to solve the six problems in column addition by using the lima-bean device. Each student should be given about fifty lima beans.

When most have completed the first worksheet, I stop the work and lead a discussion of the idea that any group of ten in a particular column can be exchanged for a one in the adjacent column on the left. To give the students more practice in recognizing sets of numbers whose sum is ten, I pass out the worksheet for Activity 2. They are asked to draw lines through pairs and triplets of numbers that add up to ten.

When students have finished finding sets of numbers whose sum is ten, I illustrate how this principle can be used in adding a column of numbers. If a set of numbers whose sum is ten is crossed out in any column, then one must be added to the column on the left. I illustrate this procedure as follows:

$$\begin{array}{r} 11\cancel{1} \\ \cancel{3}\cancel{2}\cancel{4} \\ \cancel{6}\cancel{1}\cancel{3} \\ \cancel{1}\cancel{6}\cancel{2} \\ 113 \\ +\ 11\cancel{1} \\ \hline 1{,}323 \end{array}$$

Note that the digits in the top row are the ones that have replaced the numbers crossed out in the column to their right.

After illustrating this method of column addition, I distribute the third worksheet. The students are asked to find the sums using the above method and to check their work with the lima-bean device. The problems selected for this worksheet have been designed so that groupings of ten are easy to recognize.

EVALUATION

You may wish to have students come to the chalkboard and explain their work. If students use the word "carry," ask them what they mean. The number of correct answers on the third worksheet is a measure of the success of this experience.

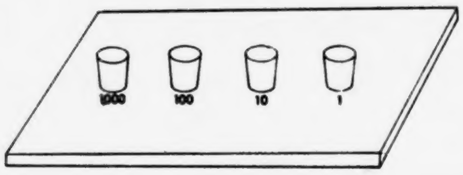
EXCHANGES IN COLUMN ADDITION UNIT 2, EXPERIENCE 4

Activity 1 Name ______________________

Use your lima-bean calculator to find the answers to the problems below.

1\.
163
142
+ 178
483

2\.
113
142
+ 199
454



3\.
113
162
193
184
+ 188
840

4\.
1,162
1,381
1,294
1,163
+ 2,194
7,194

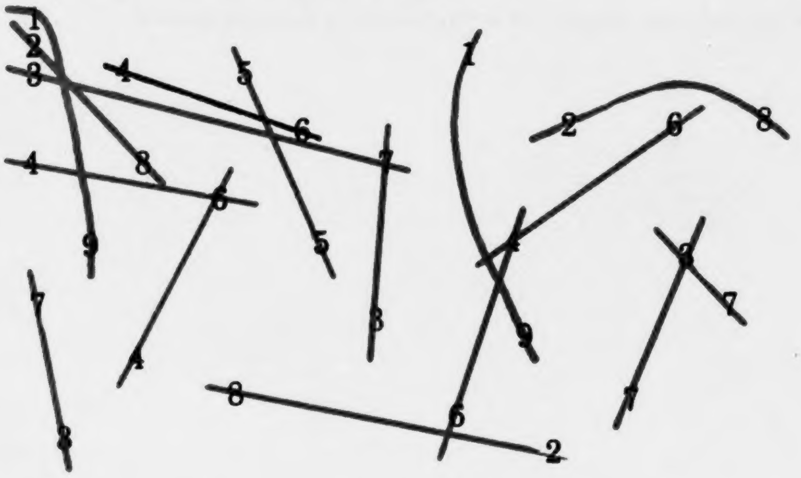
5\.
6,831
1,294
1,193
1,239
4,128
+ 1,188
15,873

6\.
1,138
1,924
1,238
1,949
1,199
+ 1,488
8,936

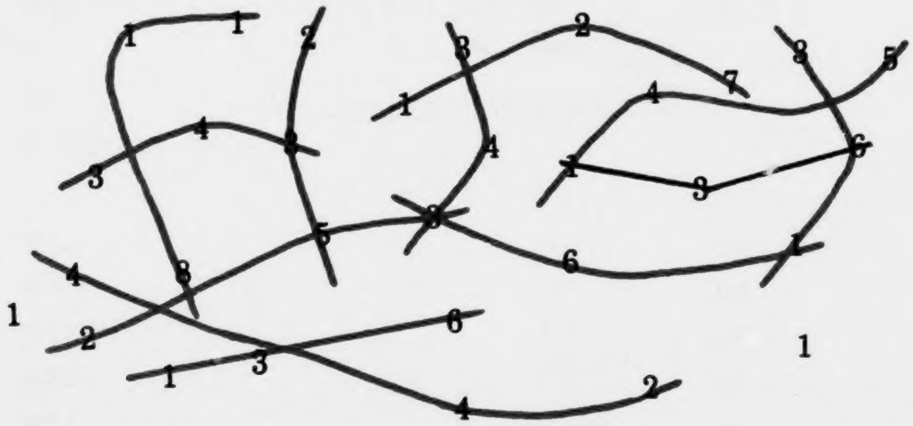
EXCHANGES IN COLUMN ADDITION UNIT 2, EXPERIENCE 4

Activity 2 Name ______________________________

Draw a line through pairs of numbers that add up to ten.


1
2
3
4
5
1
6
7
2
6
8
4
8
6
9
5
4
9
7
7
3
9
4
8
6
7
2

Draw a line through triples of numbers that add up to ten.

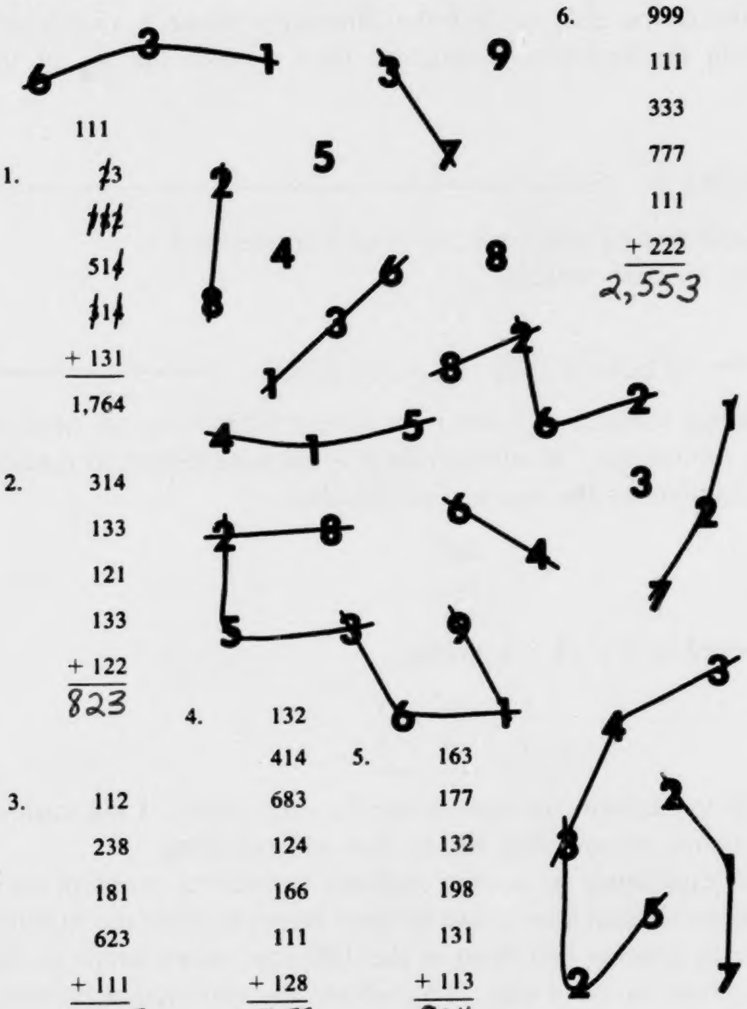

1
1
2
3
2
8
5
1
7
4
3
4
6
1
3
5
8
6
4
1
1
2
6
1
3
4
2
1

Other answers are, of course, possible.

EXCHANGES IN COLUMN ADDITION **UNIT 2, EXPERIENCE 4**

Activity 3 Name ______________________________

Find the answers to the problems below by exchanging ten in one column for one in the next. Check your answers with the lima-bean calculator.



111
1. ~~2~~3
~~782~~
51~~4~~
~~3~~1~~4~~
+ 131
1,764

2. 314
133
121
133
+ 122
823

3. 112
238
181
623
+ 111
1,265

4. 132
414
683
124
166
111
+ 128
1,758

5. 163
177
132
198
131
+ 113
914

6. 999
111
333
777
111
+ 222
2,553

7. 1,342
1,738
1,438
1,122
+ 1,111
6,751

8. 1,138
9,999
1,112
1,181
+ 1,14[illegible]
14,573

9. 1,234
4,321
1,111
2,222
+ 8,888
17,776

EXPERIENCE 5
Exchanges in Subtraction

OBJECTIVE

The student should be able to find the difference between two whole numbers, first with the lima-bean calculator, then without the aid of the device.

MATERIALS

Lima-bean calculators and beans used in Experience 4
2 worksheets for each student

TEACHER STRATEGY

In this experience students use their lima-bean calculators to develop an algorithm for subtraction. In subtraction it is often necessary to rename numbers. For example, in the subtraction problem

$$\begin{array}{r} 483 \\ -191 \\ \hline \end{array},$$

483 may be renamed as 3 / 18 / 3, giving

$$\begin{array}{r} 3\ /\ 18\ /\ 3 \\ -1\ /\ \ 9\ /\ 1 \\ \hline \end{array}.$$

It is now possible to perform the subtraction in each place. I am careful to refer to this process as renaming rather than as borrowing.

I introduce the experience by having students consider a problem such as 172 − 121. Each student should use his lima beans to enter the number 172 in the device by placing one bean in the 100 cup, seven beans in the 10 cup, and two beans in the 1 cup. To perform the subtraction he must take one from the 100 cup, two from the 10 cup, and one from the 1 cup. He can then determine the difference by counting the beans remaining in the cups.

Next I illustrate a problem involving an exchange, such as 572 − 329. To solve the problem, the student enters 572 into the cups. Three beans can be taken from the 100 cup and two from the 10 cup, but it is impossible to take nine from the 1 cup since there are only two beans in it. It is necessary, therefore, to exchange one ten for ten ones. After this exchange it is possible to take nine beans from the 1 cup. The student can now determine the difference. You may wish to have students add this difference to 329 to illustrate the inverse relation between addition and subtraction.

When I am fairly sure that the students understand the method, I distribute the first worksheet. The six subtraction problems are to be worked on the lima-bean calculator. When the students have finished, I discuss the problems with them.

Before I pass out the second worksheet, I put a subtraction problem on the board and ask the students to solve it *without* the aid of the lima-bean calculator. When I feel that they can handle subtraction problems without the calculator, I distribute the worksheet for Activity 2. The ten subtraction problems are set up in order of increasing difficulty. The first two problems require no renaming. In Problems 3–5, the numbers have already been renamed for the student. In Problems 6–10, it is necessary for the student to rename at least one number before the subtraction can be performed.

EVALUATION

In concluding this experience, have students come to the chalkboard and explain their work to the rest of the class. Most students should be able to solve subtraction problems of the type given in the last worksheet. You should not expect them to solve addition and subtraction problems quickly. The objective of this unit has been to develop a basic understanding of the exchange property as it is used in the addition and subtraction algorithms.

This unit is most effective when it is followed by additional practice in addition and subtraction problems.

EXCHANGES IN SUBTRACTION **UNIT 2, EXPERIENCE 5**

Activity 1 Name ______________________________

Carry out these subtractions with the aid of your lima-bean calculator.

1. $\begin{array}{r} 1{,}274 \\ -\ 131 \\ \hline 1{,}143 \end{array}$

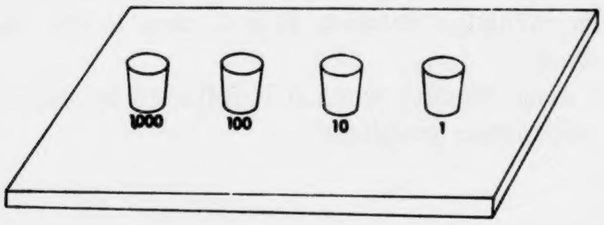
4. $\begin{array}{r} 1{,}631 \\ -\ 480 \\ \hline 1{,}151 \end{array}$

2. $\begin{array}{r} 1{,}263 \\ -\ 172 \\ \hline 1{,}091 \end{array}$

5. $\begin{array}{r} 1{,}631 \\ 1{,}298 \\ \hline 333 \end{array}$

3. $\begin{array}{r} 1{,}621 \\ -\ 118 \\ \hline 1{,}503 \end{array}$

6. $\begin{array}{r} 7{,}823 \\ -\ 938 \\ \hline 6{,}885 \end{array}$



EXCHANGES IN SUBTRACTION **UNIT 2, EXPERIENCE 5**

Activity 2 Name ______________________

Carry out these subtractions *without* the aid of your lima-bean computer. Renaming has already been shown, with slashes, in Problems 3, 4, and 5. Show with slashes any other renaming that you use in solving the problems.

1. 372
   − 161
   211

2. 6,931
   − 811
   6,120

3. 1 '11 '2
   − 8 1
   1 3 1

4. 2 '13 /4
   − 1 8 1
   1 5 3

5. 1 '12/13
   − 8 5
   1 4 8

6. 223
   − 86
   137

7. 1,631
   − 290
   1,341

8. 1,821
   − 489
   1,332

9. 1,611
   − 188
   1,423

10. 1,231
    − 998
    233

3

EXPERIENCES WITH

Physical Models for Multiplication

The purpose of this unit is to help the student develop physical models for multiplication. Because the various applications of multiplication in both real-life and textbook situations require different models, experiences with arrays, cross products, and repeated addition are all provided here.

The low achiever may have memorized many of the basic multiplication facts yet have little or no understanding of the multiplication concept. Manipulating objects and making drawings of simple problems may help this student bridge the gap between the concrete and the abstract. This unit is not intended to provide all the drill a student may need to memorize the multiplication facts or develop the multiplication algorithm. His ability to use these facts, however, can be improved by means of models that make multiplication meaningful.

The experiences include the use of student worksheets. These help you to evaluate the student's progress and his level of mastery of each concept presented.

OVERVIEW

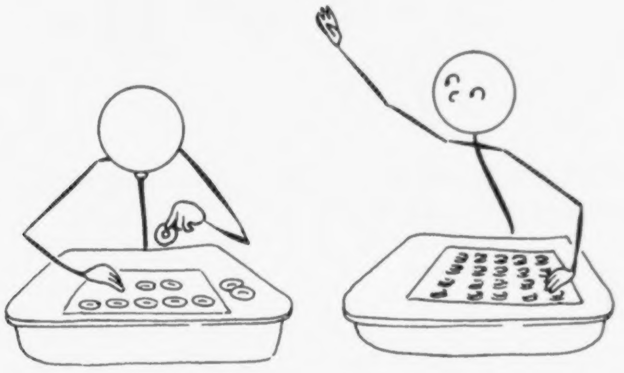
Each of the six experiences in this unit includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

Sample student worksheets and other "handout" materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

Experience 1: Arrays

This is a teacher-directed discovery experience in which the students arrange objects in orderly rows and columns to form arrays. It should help the students "see" multiplication as they observe that the number of rows times the number of columns is the same number as the total number of objects.



Materials needed: set of 20 objects for each student (pennies, washers, bottle caps, poker chips, checkers, or other similar items), an envelope or a box to contain each set of objects, worksheets.

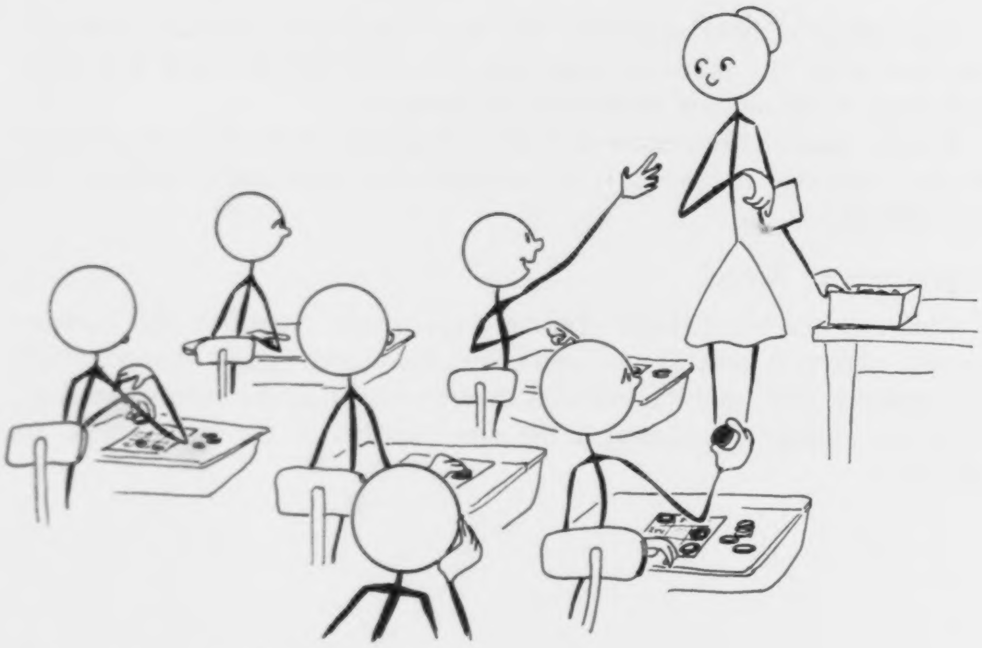
Experience 2: Solving Equations Using Arrays

In this experience students arrange objects in an array to discover the missing factor of an equation.

Materials needed: set of 20 objects for each student as for Experience 1, worksheets.

Experience 3: Multiplication Bingo

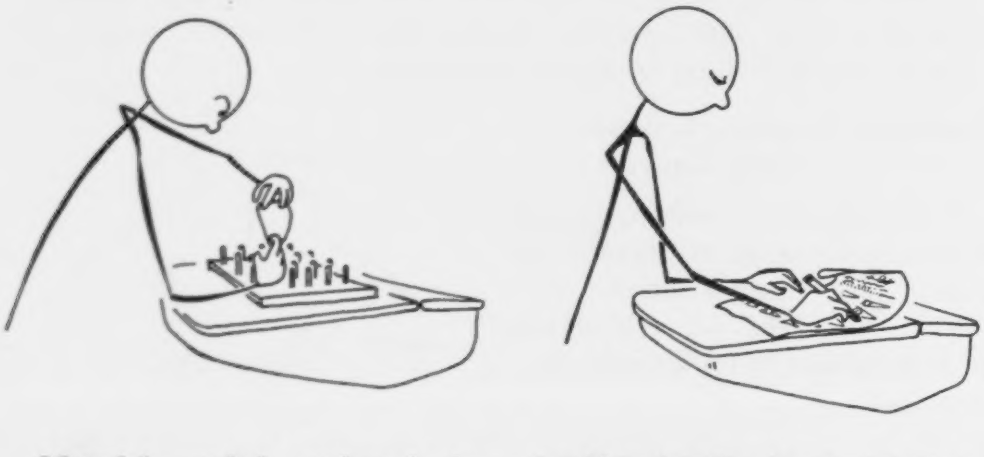
A "multiplication bingo" game provides drill in the basic multiplication facts.



Materials needed: demonstration bingo cards, set of bingo calls, buttons, student bingo cards, worksheets.

Experience 4: Cross Products

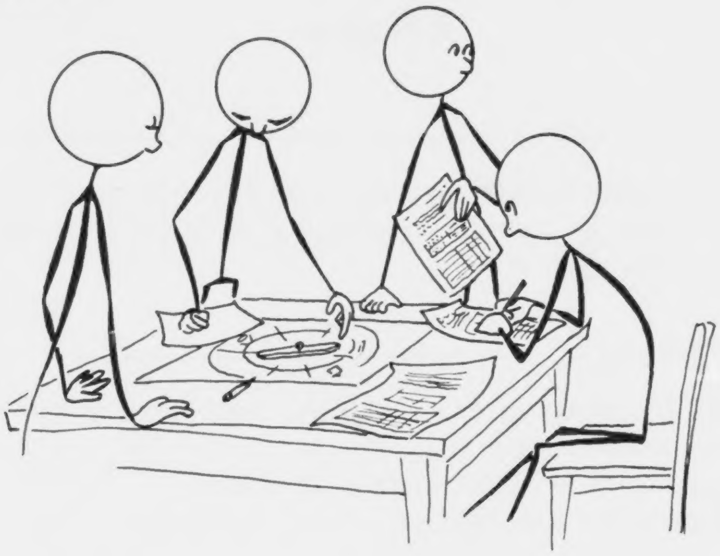
There are three activities in which the student matches each member of one set with each member of another set to find the cross product.



Materials needed: geoboards, box of rubber bands of assorted sizes, red nail polish, white nail polish, construction paper, crayons or colored pencils, worksheets.

Experience 5: Repeated Addition and Multiples

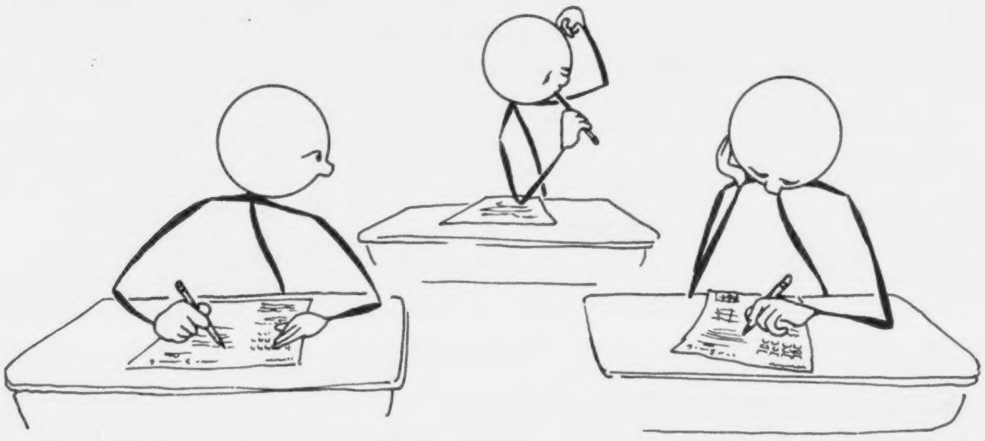
A game helps the students see the connection between multiplication and repeated addition.



Materials needed: spinners; small boxes; objects such as washers, poker chips, beans; worksheets.

Experience 6: Problem Solving

Simple word problems are stated whose solutions require the use of arrays and cross products. The students are asked to draw sketches and write equations to solve the problems.



Materials needed: worksheets.

EXPERIENCE 1
Arrays

OBJECTIVE

The student should be able to arrange a set of two or more objects into arrays and to write an equation describing a given array for any of the multiplication facts up to 9 × 9.

MATERIALS

1 set of 20 objects (pennies, washers, bottle caps, poker chips or similar items) for each student
Envelope or box to contain each set of objects
3 worksheets for each student

TEACHER STRATEGY

An effective way to arrive at the total number of objects in a given set is to arrange the objects in orderly rows and multiply the number of rows by the number of columns. As each student arranges his objects into rectangular arrays he will discover the multiplication facts.

An array is an orderly arrangement of dots, pictures, or objects in even rows and columns. Each array is rectangular in shape. The number of rows times the number of columns gives the product, or total number of items in the array, as is illustrated in figure 1.

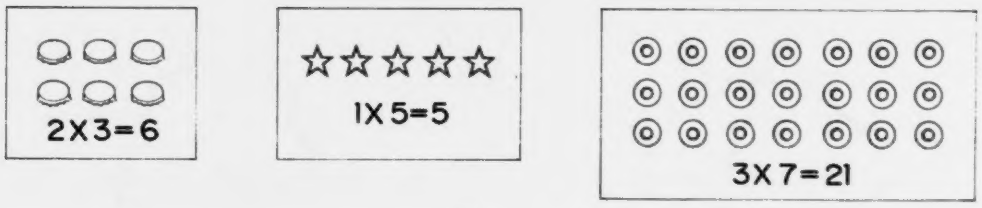


Fig. 1

One teacher gives the following account:

I introduced this experience by giving each student an envelope containing twenty objects such as those mentioned above. I asked the students to take twelve objects out and put them on their desks, leaving the rest of them in the envelopes.

Then I asked, "How can you arrange your objects into the shape of a rectangle?" I reminded the class to put the same number of objects in each row. As the students made different arrays, I asked them to go to the chalkboard and draw their arrays. I tried to get them to discover the six possible arrays, shown in figure 2. (Students may have some difficulty in seeing that a 12 × 1 or a 1 × 12 array is in the shape of a rectangle. A frame drawn around each array may help them see that it really is a rectangle.)

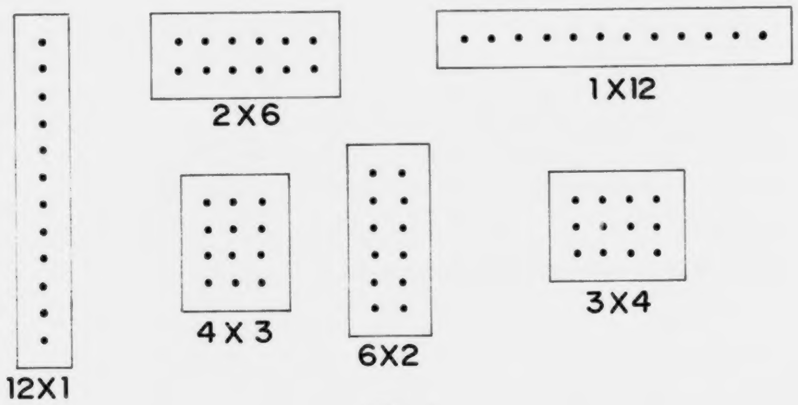
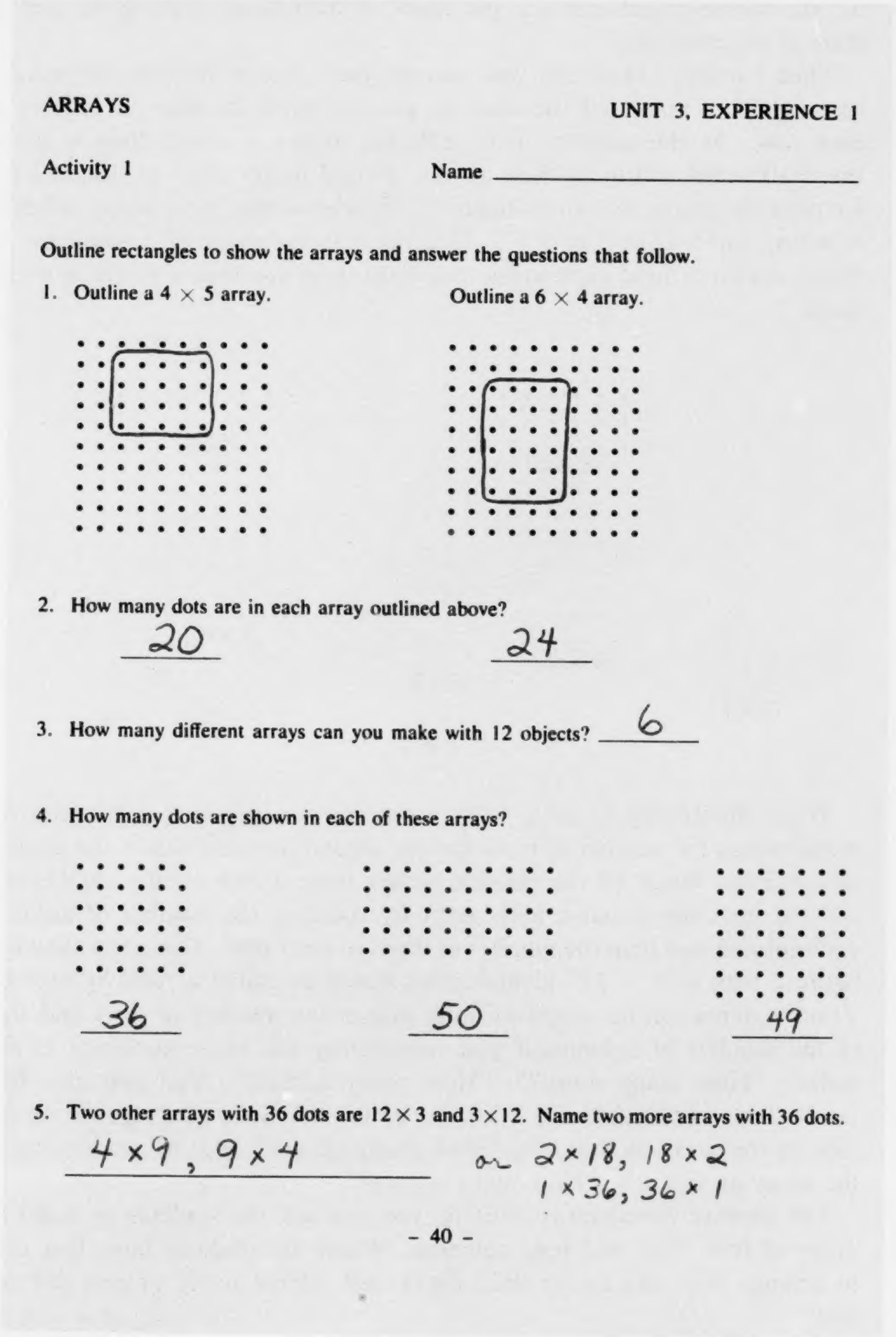


Fig. 2

When identifying an array with a product we agreed that the first numeral names the number of rows and the second numeral names the number of columns. When all the possible arrays were drawn on the chalkboard, I asked the class to name each array by counting the number of items in each column and then the number of items in each row. The array shown in figure 2 with a "1 × 12" identification would be called a "one *by* twelve." Your students can be taught to think first of the number of rows and then of the number of columns if you consistently ask these questions in this order: "How many down?" "How many across?" You can also help the students to visualize this sequence by running your finger down the left side of the array as you ask, "How many down?" and across the top of the array as you ask, "How many across?"

For another introductory activity, you can ask the students to make an array of four rows and four columns. When the students have had time to arrange their objects on their desks, ask, "How many objects did you use?"

[*Continued on page 75*]



ARRAYS **UNIT 3, EXPERIENCE 1**

Activity 1 Name ____________

Outline rectangles to show the arrays and answer the questions that follow.

1. Outline a 4 × 5 array. Outline a 6 × 4 array.

2. How many dots are in each array outlined above?
20 24

3. How many different arrays can you make with 12 objects? 6

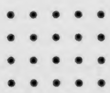
4. How many dots are shown in each of these arrays?
36 50 49

5. Two other arrays with 36 dots are 12 × 3 and 3 × 12. Name two more arrays with 36 dots.
4 × 9, 9 × 4 or 2 × 18, 18 × 2
1 × 36, 36 × 1

– 40 –

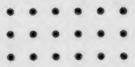
ARRAYS **UNIT 3, EXPERIENCE 1**

Activity 2 Name ______________________________

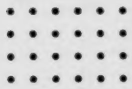


4 × 5 = 20.

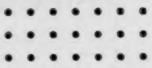
The number of dots in this array can be described with the number sentence given underneath it. Write a sentence for each array given below.

1.


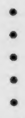
3 × 6 = 18

2.


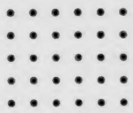
4 × 6 = 24

3.


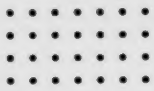
3 × 7 = 21

4.


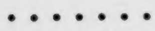
5 × 1 = 5

5.


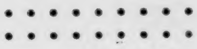
5 × 6 = 30

6.


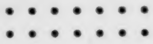
4 × 7 = 28

7.


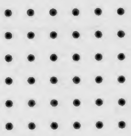
1 × 7 = 7

8.


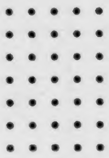
2 × 4 = 18

9.


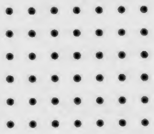
2 × 7 = 14

10.


6 × 6 = 36

11.


7 × 5 = 35

12.


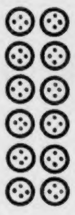
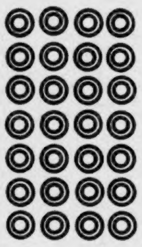
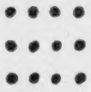
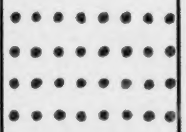
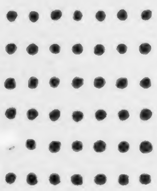
6 × 7 = 42

ARRAYS **UNIT 3, EXPERIENCE 1**

Activity 3 Name ____________________

Complete the tables.

Number of Rows	4	5	4	8	7	7	4
Number of Columns	3	4	6	3	6	8	7
Number of Objects	12	20	24	24	42	56	28

Picture of the Array					
Name of the Array	6×2	7 × 4	3×4	4 × 8	6×7
Number of Objects in the Array	12	28	12	32	42

To get the students to write sentences that describe their arrangement you might ask, "How could we write a number sentence that describes this array?" When each student has written his answer, have some student write the correct sentence, $4 \times 4 = 16$, on the chalkboard. Explain that this sentence is read "Four *times* four equals sixteen."

Repeat these two activities with different numbers of objects to give the students many opportunities for both making arrays and writing number sentences. You might have groups of students pool their objects to make arrays that represent the larger products. It is not necessary that every student make every possible array or write every sentence. It is more important that you keep the experience moving by providing activities that are fun.

After a careful introduction I handed out copies of the worksheet for Activity 1 and had the students look it over to see if they understood the directions. Then I told them to work individually. When most of them had finished, I called for answers. We found a number of different identifications named in answers to the last question.

At the conclusion of this activity, I handed out the remaining worksheets.

EVALUATION

The worksheets can be used to evaluate the students' ability to make arrays and to write the sentence that describes a given array. Encourage each student to use his set of objects to help him work the problems on the worksheet. If students fail to complete most of the worksheets correctly, provide more opportunities for making arrays and writing sentences.

EXPERIENCE 2

Solving Equations Using Arrays

OBJECTIVE

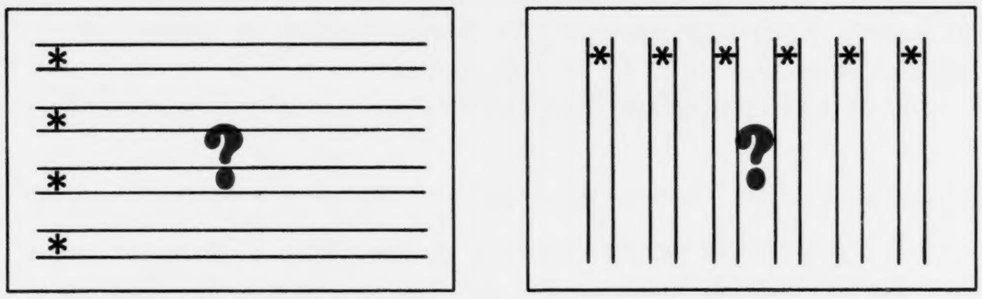
The student should be able to solve equations involving the basic multiplication facts when he is given one factor and the product.

MATERIALS

Set of objects for each student as described for Experience 1
2 worksheets for each student

TEACHER STRATEGY

If a product and one of the factors are given for a sentence involving multiplication, the missing factor may be determined by placing objects in an appropriate array. When the partial arrays shown in figure 3 are completed, the missing factors (5 and 3, respectively) are found easily. Experiences like these give your students insight into the meaning of multiplication and, also, assurance that they can help themselves when their memory fails about a particular multiplication fact.



$4 \times \square = 20$ $\square \times 6 = 18$

Fig. 3

I introduced this experience by asking each student to place twenty objects on his desk and to arrange them in an array to find the missing number in the sentence $4 \times \square = 20$. Through discussion I drew out the information that the "4" in the sentence indicates there should be four rows in the array.

Students may start the array by placing one object as the beginning of each row, continue by placing a second object in each row, and go on this way until all twenty objects are arranged in a rectangular array. The number of objects in each row (the number of columns) is the missing factor, 5.

When the students had solved several problems of this kind, I asked them to arrange eighteen objects on their desks to find the missing number in the sentence $\square \times 6 = 18$. I tried to get them to discover that, although the number of rows is missing, there should be six objects in each row.

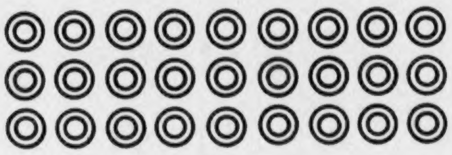
[*Continued on page 79*]

SOLVING EQUATIONS USING ARRAYS **UNIT 3, EXPERIENCE 2**

Activity 1 **Name** ______________________________

Write the name of the correct number in each box in the number sentences. If you are not sure of any result, use your set of objects to form the appropriate array.

$3 \times \boxed{9} = 27.$



$3 \times \boxed{7} = 21.$

$3 \times \boxed{5} = 15.$

$4 \times \boxed{2} = 8.$

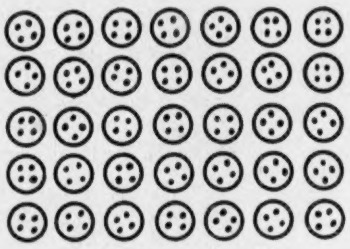
$4 \times \boxed{4} = 16.$

$4 \times \boxed{6} = 24.$

$4 \times \boxed{7} = 28.$

$5 \times \boxed{8} = 40.$

$5 \times \boxed{7} = 35.$



$5 \times \boxed{6} = 30.$

$\boxed{5} \times 7 = 35.$

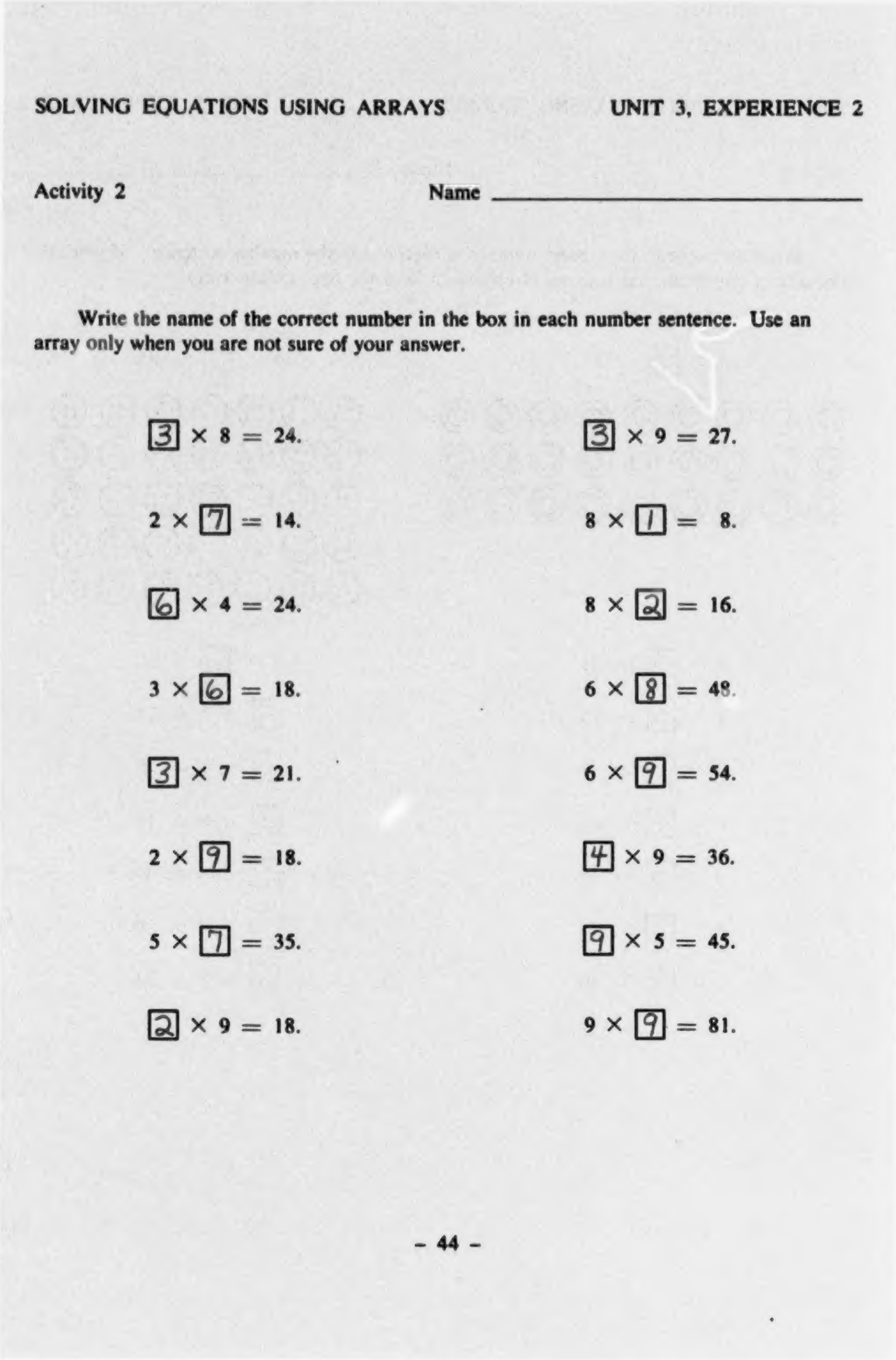
$\boxed{6} \times 6 = 36.$

$\boxed{3} \times 6 = 18.$

$\boxed{5} \times 6 = 30.$

$\boxed{1} \times 6 = 6.$

$\boxed{3} \times 7 = 21.$



SOLVING EQUATIONS USING ARRAYS **UNIT 3, EXPERIENCE 2**

Activity 2 **Name** ______________________________

Write the name of the correct number in the box in each number sentence. Use an array only when you are not sure of your answer.

$\boxed{3} \times 8 = 24.$

$2 \times \boxed{7} = 14.$

$\boxed{6} \times 4 = 24.$

$3 \times \boxed{6} = 18.$

$\boxed{3} \times 7 = 21.$

$2 \times \boxed{9} = 18.$

$5 \times \boxed{7} = 35.$

$\boxed{2} \times 9 = 18.$

$\boxed{3} \times 9 = 27.$

$8 \times \boxed{1} = 8.$

$8 \times \boxed{2} = 16.$

$6 \times \boxed{8} = 48.$

$6 \times \boxed{9} = 54.$

$\boxed{4} \times 9 = 36.$

$\boxed{9} \times 5 = 45.$

$9 \times \boxed{9} = 81.$

- 44 -

Then, by counting the rows in the completed array, they arrived at the missing factor.

After providing a few problems of this type I had groups of three or four students pool their objects and work together to arrange arrays that would help solve such number sentences as 6 × □ = 48 and □ × 6 = 42.

After this introduction I had the students work on the first worksheet for this experience, allowing them to work in pairs to pool their objects when necessary. Then I instructed them to complete the second worksheet individually.

EVALUATION

The worksheets can be used to evaluate each student's ability to solve number sentences by using arrays. Allow enough time so that the exercises do not become a test of the student's knowledge of memorized number facts. They are designed, rather, to test understanding of how the sentences can be solved.

EXPERIENCE 3
Multiplication Bingo

OBJECTIVE

The student should be able to match corresponding arrays of dots, pairs of factors, and products for multiplication facts up to 9 × 9.

MATERIALS

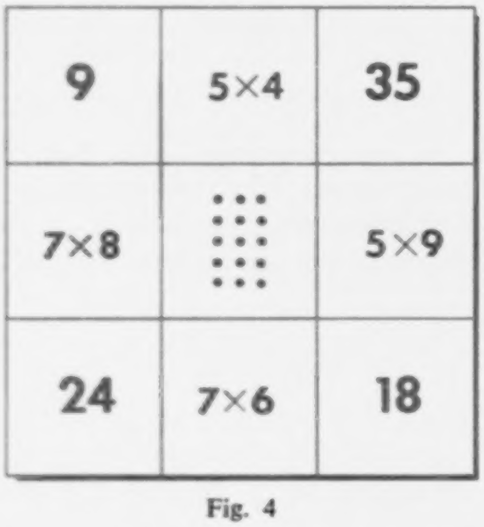
Large bingo card to demonstrate the rules of the game (from Teaching Package)
Multiplication-bingo announcements (also supplied in the Teaching Package)
1 bingo card for each student (from four sheets in the Teaching Package)
Buttons
1 worksheet for each student

TEACHER STRATEGY

In this experience your students get familiarity with the multiplication facts by matching corresponding arrays, pairs of factors, and products.

Multiplication Bingo

One teacher introduced the game by displaying a large bingo card (fig. 4, reproduced from the Teaching Package) and explaining, somewhat as follows, what is meant by a match:



9	5×4	35
7×8	• • • • • • • • • • • • • • •	5×9
24	7×6	18

Fig. 4

"If the caller announces 'Five times three,' we have a match with the center square because it is a 5 × 3 array of dots. If the caller announces 'Fifteen,' we again have a match with the center square. *When you have a match, cover the square with a button.* Similarly, if either 'Three times three' or 'Nine' is announced, we have a match with the upper left-hand square.

"How can we get a match for the top center square?"

Usually, correct answers will be given.

"Yes, if 'Five times four' is announced, or if 'Twenty' is announced, you should cover that square. But you should not cover it if 'Four times five' is called, since the factors are not written in that order. Neither should you cover the center square if 'Three times five' is called.

"Now let's see if you understand the game. I'll start announcing products or pairs of factors, and you raise your hand if you see a match on this large multiplication-bingo card."

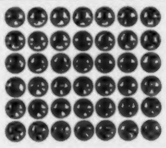
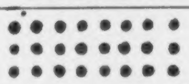
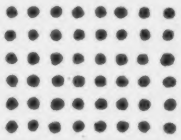
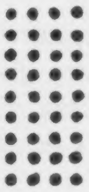
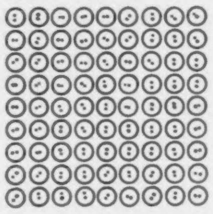
Then I announced slowly, "Fifty-four; nine times five; fifty-six"—and

[*Continued on page 82*]

MULTIPLICATION BINGO **UNIT 3, EXPERIENCE 3**

Activity 2 Name ____________________

Fill in each space with an array, a product, or missing factor(s).

Array	Factors	Product
	6 × 7	42
	3 × 8	24
	6 × 8	48
	9 × 4	36
	9 × 9	81

at this point several students immediately raised their hands. When I was sure that everyone understood the game, I distributed a card and about ten buttons to each student, explaining that they were to cover each matched square with a button and to shout "Bingo!" as soon as they had three buttons in a row, a column, or a diagonal.

We were able to play about a dozen games in one class period. It added interest to let the winner of a game be the caller for the next one. I saved the last ten minutes of the class period for individual work on the worksheet for Activity 2.

EVALUATION

The students' performance on the worksheet should indicate how well this experience helped the students meet the objective.

EXPERIENCE 4
Cross Products

OBJECTIVE

The student should be able to determine all of the pairs of objects which can be made by using one object from each of two given sets, and he should know that the number of pairs is the product of the number of elements in the first set and the number of elements in the second set.

MATERIALS

1 overhead-projector transparency
6 geoboards
Box of rubber bands of assorted sizes
Red nail polish
White nail polish
Envelopes
Construction paper (blue, brown, green, black, yellow, white, and orange)
Crayons or colored pencils in the same colors
3 worksheets for each student

Activity 1. Make geoboards as directed below and paint the posts with polish according to the patterns shown in figure 5, in which R represents red and W represents white. (The nail polish can be removed later for other geoboard lessons.) Label each geoboard, as shown, by attaching a strip of masking tape with a letter identification. Have the box of rubber bands available. Figure 5 pictures the geoboards with the loops added.

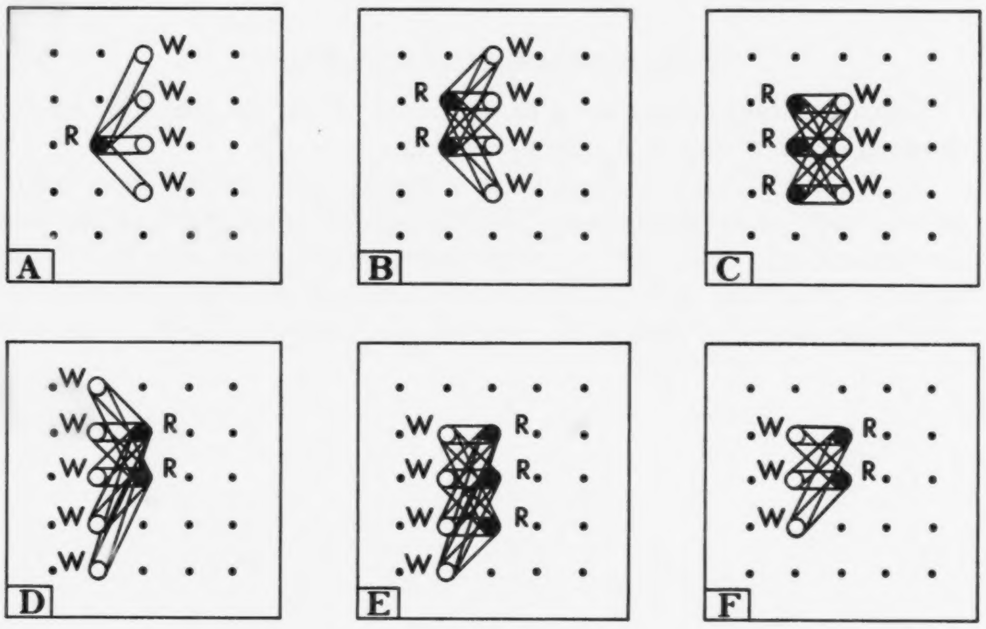


Fig. 5

Activity 2. For every two or three students prepare an envelope containing construction-paper triangles and construction-paper squares to be combined as shown in figure 6 (patterns are given in the Teaching Package). Each envelope for Activity 2 should contain the following:

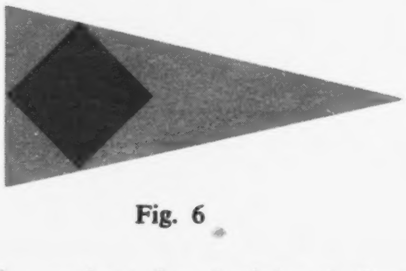


Fig. 6

1 black triangle
1 brown triangle
1 blue triangle
1 green triangle
1 orange square
1 yellow square
1 white square
1 crayon or colored pencil of each of the above colors

Activity 3. No materials other than worksheets are required for this activity.

Construction and Use of Geoboards

Geoboards may be purchased ready for use. However, they may be constructed easily at a cost of only about a quarter each.

The geoboards used in Experience 4 may be made from 9-inch squares of plywood (½ in. thick), with 25 roundheaded brass nails (¾ in. long) for each board. Mark off a ½-inch border, then make a grid of 2-inch squares. Drive the nails firmly into the board at the intersections of the lines, with about ¼ inch of the nail left protruding, as shown in figure 7.

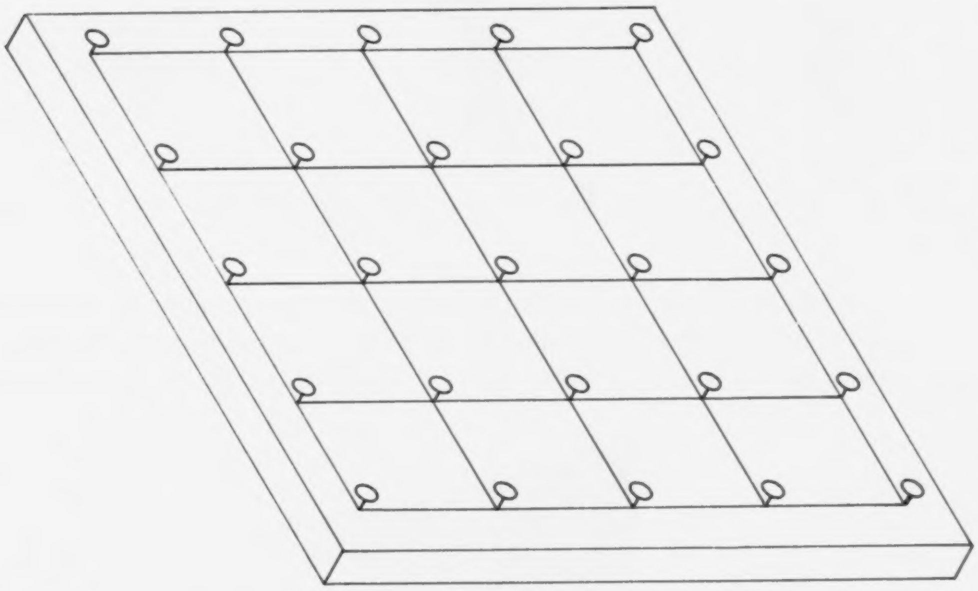


Fig. 7

The boards shown in figure 5 are being used, as previously explained, as models for finding a cross product. Geoboards may also be used as models for repeated addition and for arrays when the multiplication concepts are being taught. For example: If there are 5 nails in each row, how many nails are there in 2 rows? $5 + 5 = 10$; $2 \times 5 = 10$.

Other uses include finding the perimeter and area of plane figures. Either string or rubber bands may be used to construct the plane figures. For example, a student may be told that the perimeter of the enclosed region in figure 8 is 8 units and be asked to construct another region having a perimeter of 8 units (fig. 9).

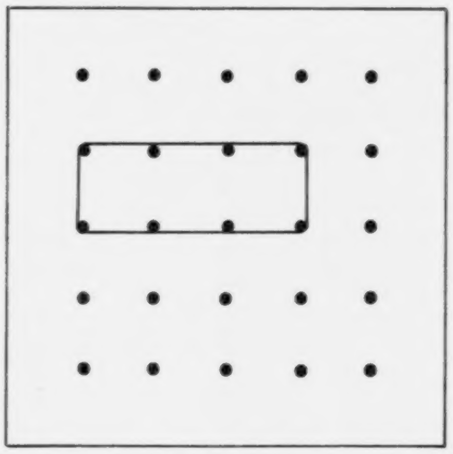


Fig. 8

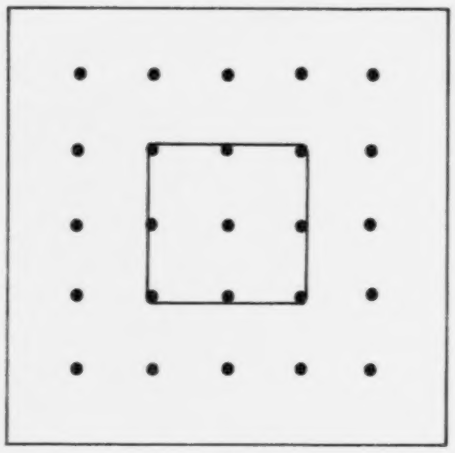


Fig. 9

Commercial materials are now available that provide teacher manuals and student problems using the geoboard to teach many other mathematical concepts.

TEACHER STRATEGY

The three activities of this experience will provide your students an opportunity to work with sets to determine a given product. The number of elements in a set multiplied by the number of elements in another set gives the number of ordered pairs that can be formed from the two sets.

I introduced this experience with a projectual from the Teaching Package (fig. 10) and asked the students how many different outfits could be formed using one of the shirts and one of the pairs of shorts. After students described the six outfits as "striped shirt and checked shorts," and so on, I tried to get from them the conclusion that from three choices of shirts and two choices of shorts there are 3×2 outfits possible:

How many shirts?	How many different outfits?
How many shorts?	$3 \times 2 = 6$.

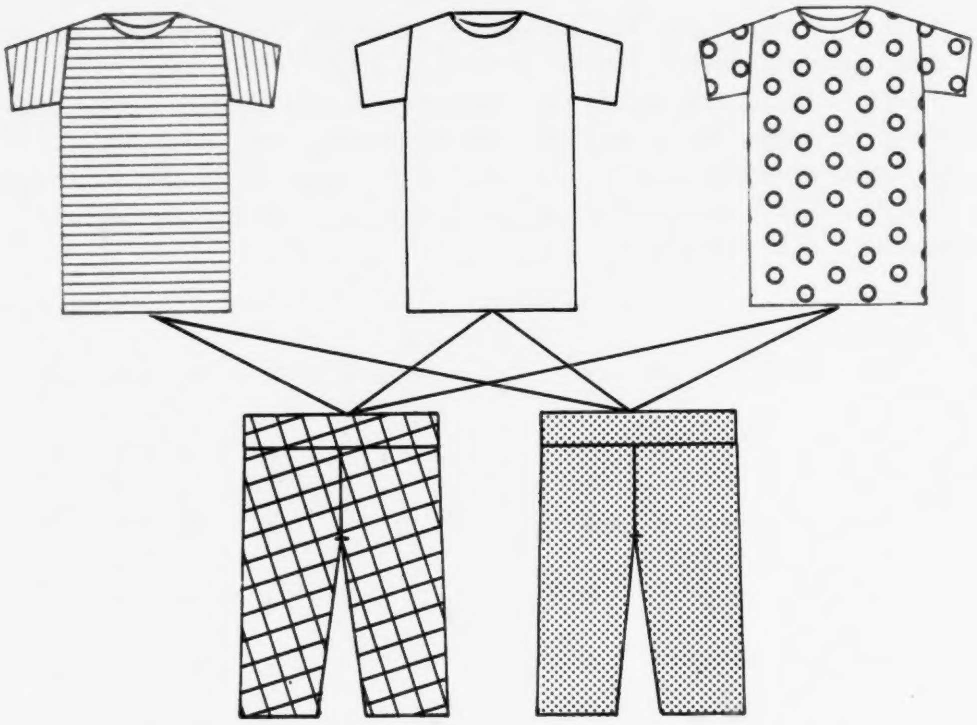


Fig. 10

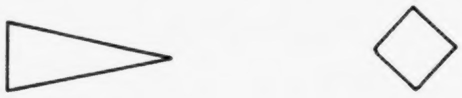
Then I divided the students into three groups and assigned each group to one of the three activities described below. (These activities may be performed in any order.)

Activity 1 should be conducted at a station or a work area. Give each of your students a copy of the appropriate worksheet. The six geoboards present six problems. The red posts of each board represent one set, the white posts another set. The student is directed to connect *each* red post with *each* white post by means of rubber bands. Each post is used as many times as it is needed. The total number of rubber bands used on each board is the product. The loops that are to be formed were illustrated in figure 5.

In Activity 2 the student constructs as many different pennants as he can from the set of construction-paper triangles and the set of construction-paper squares in his envelope. He makes different pennants by placing each of the various squares on each of the pennants. Then he records his answers on the worksheet for this activity by coloring its pictured pennants to correspond to those he has made. The number of pennants should equal the number of triangles times the number of squares.

More pennant outlines than are needed are purposely placed on the

Activity 2 worksheet. Some of the students may make duplicate pennants or devise "illegal" pennants of the wrong colors by reversing the triangle and square colors to use all the outlines. For example, a student may color an orange triangle with a black square even though there is no orange triangle or black square in the envelope. The student could be asked to find the corresponding triangle and square in the envelope to determine whether such a pennant is possible. When the evaluation session is held, the students should place a check by the correct pennants and cross out any pennants that are duplicates or illegal. On the chalkboard organize the possibilities as shown in figure 11 to give the students a logical method of determining when all the pennants have been named.



Black	Orange
Black	Yellow
Black	White
Brown	Orange
Brown	Yellow
Brown	White
Blue	Orange
Blue	Yellow
Blue	White
Green	Orange
Green	Yellow
Green	White

Fig. 11

The worksheets for Activity 3 require a student to draw lines representing sidewalks from all the doors of one building to all the doors of another building.

The student will also write number sentences showing that the number of doors in one building multiplied by the number of doors in another building is equal to the number of sidewalks.

EVALUATION

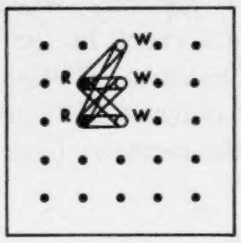
At the conclusion of the activities the students should assemble to talk about their findings. As each item is discussed ask one who has found an answer why he thinks the answer is true.

Students who have difficulty with Activity 3 of Experience 4 should be given an opportunity to make additional drawings to help them with their solutions.

CROSS PRODUCTS **UNIT 3, EXPERIENCE 4**

Activity 1 **Name** ______

Connect each red post with each white post by means of rubber bands. Record your answers, remove the bands, and repeat for the next geoboard.



Number of red posts 2
Number of white posts 3
Number of rubber bands 6
2 × 3 = 6.

A

Number of red posts 1
Number of white posts 4
Number of rubber bands 4
1 × 4 = 4

B

Number of red posts 2
Number of white posts 4
Number of rubber bands 8
2 × 4 **=** 8

C

Number of red posts 3
Number of white posts 3
Number of rubber bands 9
3 **× 3 =** 9

D

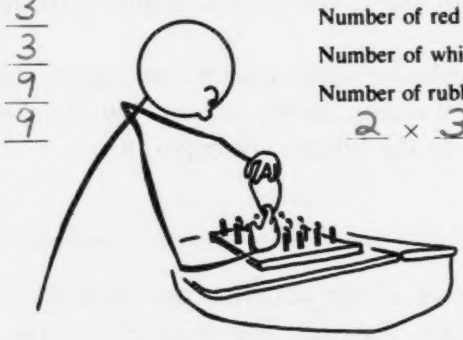
Number of red posts 2
Number of white posts 5
Number of rubber bands 10
2 **×** 5 **=** 10

E

Number of red posts 3
Number of white posts 4
Number of rubber bands 12
3 **×** 4 **=** 12

F

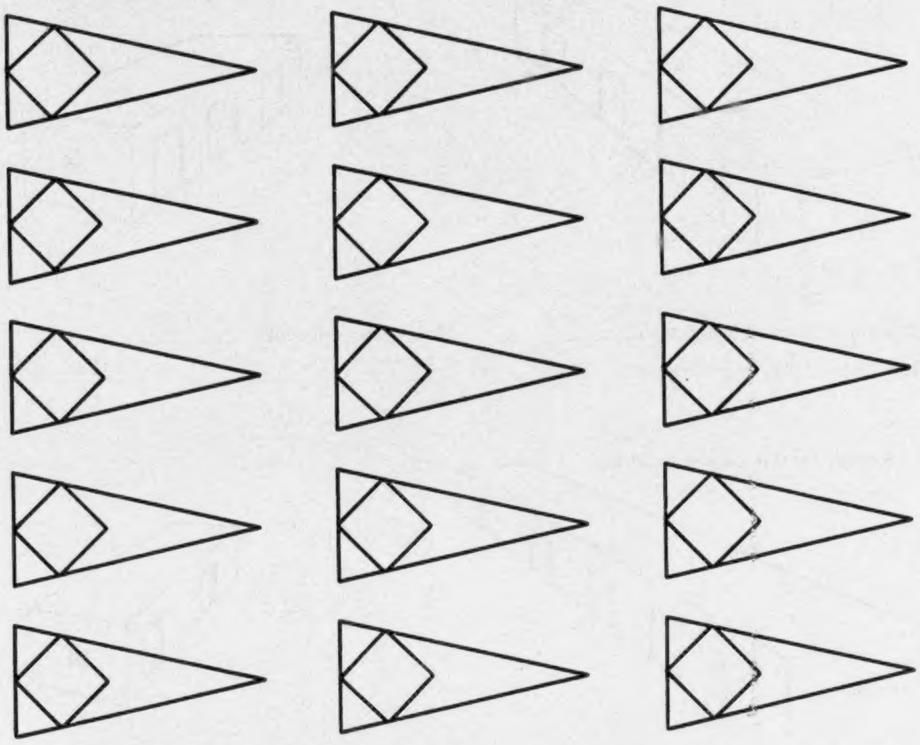
Number of red posts 2
Number of white posts 3
Number of rubber bands 6
2 **×** 3 **=** 6



CROSS PRODUCTS **UNIT 3, EXPERIENCE 4**

Activity 2 Name ______________________

Make as many different pennants as you can, using the colored triangles and squares in the envelope, and color the pennants below to match. Color only as many pennants as you can really make.



How many different shapes like this, ▷, do you have? 4

How many different shapes like this, ◇, do you have? 3

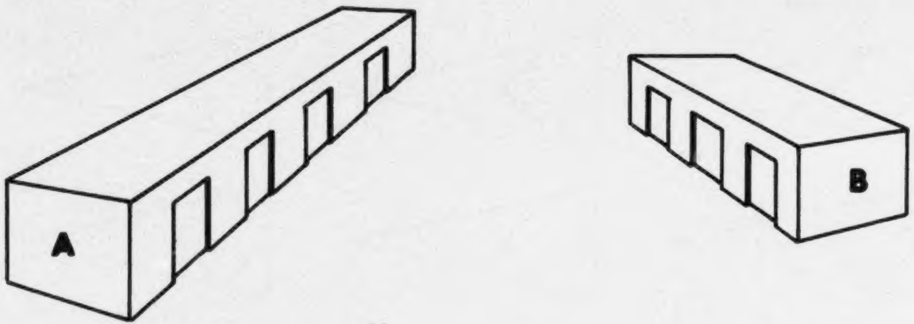
How many different pennants did you make? 12

4 × 3 = 12

CROSS PRODUCTS **UNIT 3, EXPERIENCE 4**

Activity 3 Name ______________________

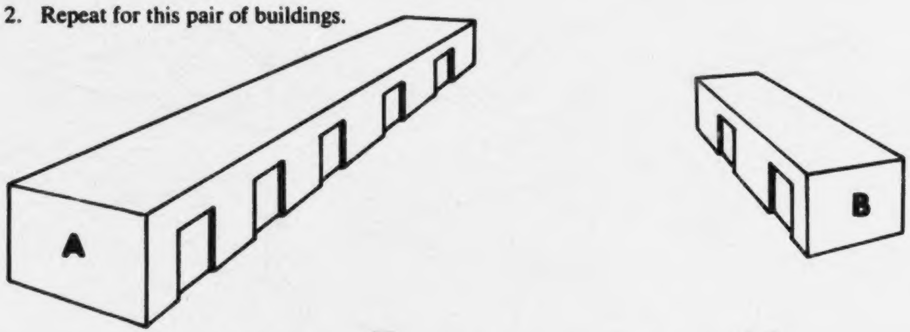
1. Draw a sidewalk from each door in Building A to each door in Building B, then answer the questions and write a number sentence.



How many doors in Building A? 4 How many sidewalks? 12

How many doors in Building B? 3 4 × 3 = 12.

2. Repeat for this pair of buildings.



How many doors in Building A? 5 How many sidewalks? 10

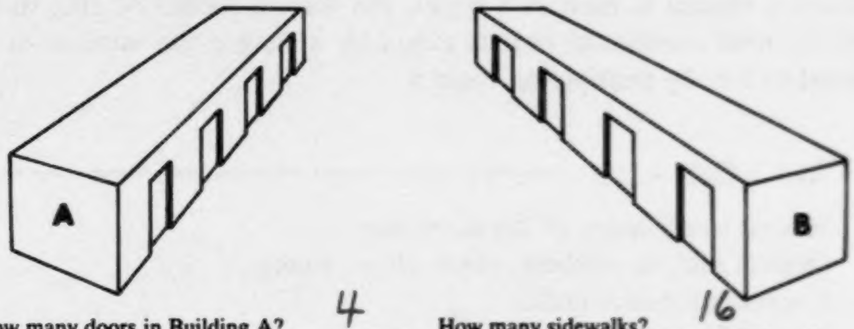
How many doors in Building B? 2 5 × 2 = 10.

[Continued]

CROSS PRODUCTS **UNIT 3, EXPERIENCE 4**

Activity 3–*Continued* Name ______________________

3. Repeat for this pair of buildings.



How many doors in Building A? 4 How many sidewalks? 16

How many doors in Building B? 4 4 × 4 = 16.

4. In the chart below different buildings labeled A and B have sidewalks connecting each door in one to each door in the other. Fill in all the blanks.

Doors in Building A	Doors in Building B	Total Sidewalks	Number Sentences
1	4	4	1 × 4 = 4
5	4	20	5 × 4 = 20
3	5	15	3 × 5 = 15
6	3	18	6 × 3 = 18
5	5	25	5 × 5 = 25
7	3	21	7 × 3 = 21
4	6	24	4 × 6 = 24

EXPERIENCE 5
Repeated Addition and Multiples

OBJECTIVE

Given *a* objects in each of *b* boxes, the student should be able to determine the total number of objects either by adding *a* the number of times denoted by *b* or by multiplying *a* and *b*.

MATERIALS

Several small boxes of the same size
Objects such as washers, poker chips, beans
1 spinner for each group
2 worksheets for each student

Construction of the Spinner

The completed spinner, shown in figure 12, is based on a face in the Teaching Package.

Paste a reproduction of this face on a piece of cardboard, then thumbtack a Popsicle stick to it so that the two ends are equidistant from the center. Finish by identifying the pointing end of the stick in some way; an easy method of doing this is to add poster paint or to color it with a crayon or a felt-tip marker.

TEACHER STRATEGY

I prepared six small boxes (milk cartons) with a different number of washers in each box.

When the class started I placed the six boxes on the table in the front of the room. I asked one of the students to choose a box and tell us what was in it. The student happened to choose one with three washers, whereupon I asked, "If there are the same number of washers in all the other boxes, how many washers do I have?"

The students responded on scratch paper.

I then asked, "If there were five washers in each box, how many washers would I have?"

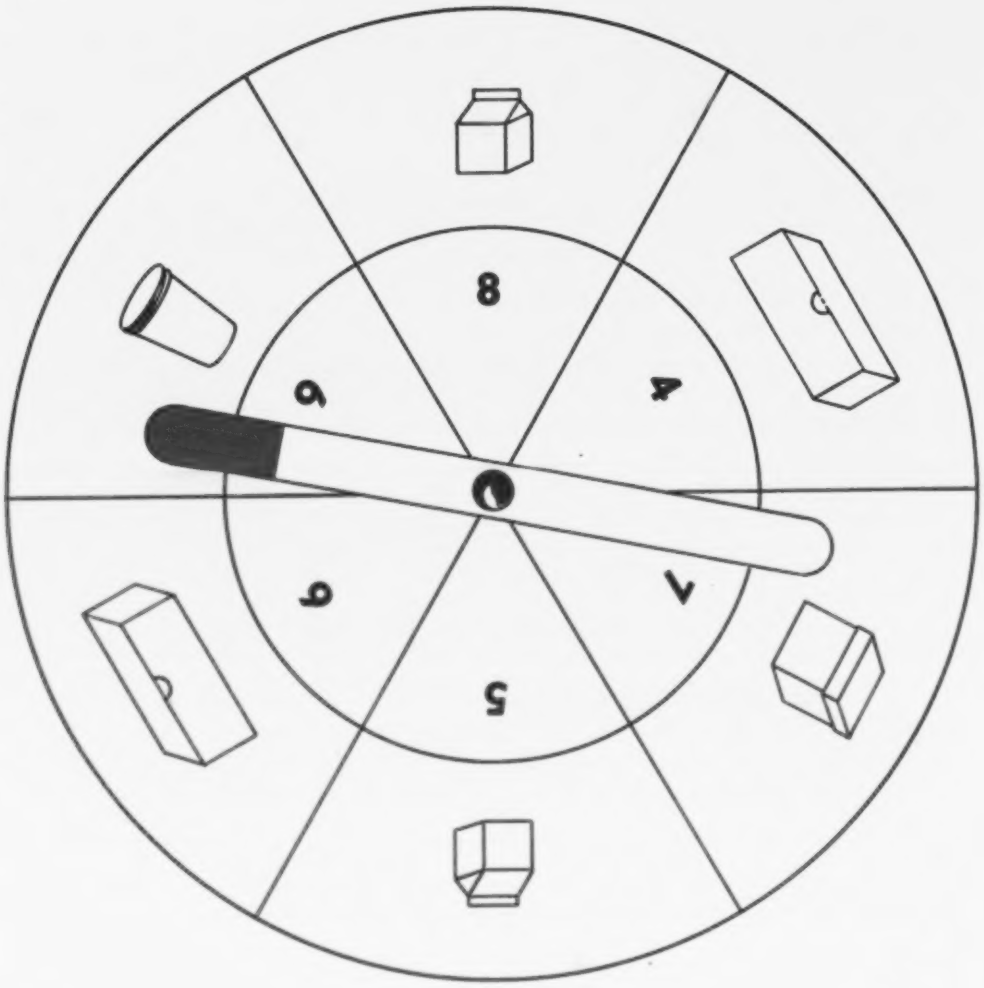


Fig. 12

Some of the students found the answer by adding five six times, and other students multiplied to find their answers.

After we had repeated this activity several times I divided the class into six groups to play the game called "Box the Circle," using the spinners described above.

Box the Circle

To play this game, students are divided into groups and given copies of the worksheets for Activity 1. On these every student is to record the scores of all the players in his group. Each player takes one turn, which consists of using the spinner twice—once to obtain the number of boxes

[*Continued on page 97*]

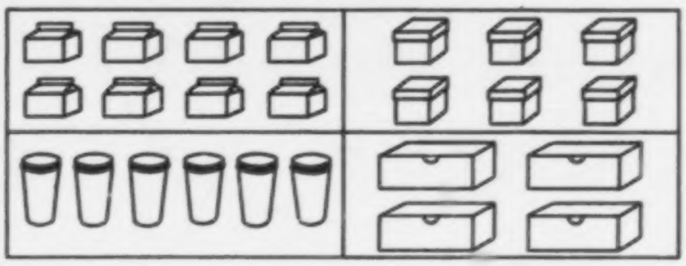
REPEATED ADDITION AND MULTIPLES **UNIT 3, EXPERIENCE 5**

Activity 1 Name ______________________________

BOX THE CIRCLE

All players are to keep score. The game is completed when each player has had one turn; the one with the highest score wins.

1. Spin the spinner once to select a box from the outer ring. Match this with the number of boxes, shown below, and enter that number on the scoresheet.
2. Spin the spinner a second time to select, from the inner ring, the number of objects in each box.
3. Enter the total number of objects in the last column.
4. At the end of the game, mark the winner's name with a star.



Game 1

Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

[Continued]

Name ________________________________

BOX THE CIRCLE—Continued

Game 2

Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

Game 3

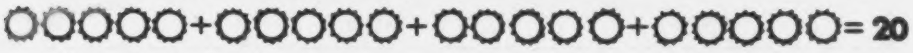
Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

REPEATED ADDITION AND MULTIPLES **UNIT 3, EXPERIENCE 5**

Activity 2 Name ______________________

Complete the number sentences below by writing the standard name for the answer.

Example: 5 + 5 + 5 + 5 = 20.



1. 6 + 6 + 6 + 6 = 24.
2. 8 + 8 + 8 + 8 = 32.
3. 7 + 7 + 7 + 7 = 28.
4. 9 + 9 + 9 + 9 = 36.

5. 3 + 3 + 3 + 3 + 3 + 3 = 18.
6. 5 + 5 + 5 + 5 + 5 + 5 = 20.
7. 4 + 4 + 4 + 4 + 4 + 4 = 24.
8. 7 + 7 + 7 + 7 + 7 + 7 = 42.

9. 6 + 6 + 6 = 18.
10. 8 + 8 + 8 = 24.
11. 7 + 7 + 7 = 21.
12. 9 + 9 + 9 = 27.

13. 2 + 2 + 2 + 2 + 2 + 2 + 2 = 14.
14. 9 + 9 + 9 + 9 + 9 = 45.
15. 3 + 3 + 3 + 3 + 3 + 3 + 3 + [illegible] = 24.
16. 9 + 9 = 18.

of a particular kind and once to obtain the number of washers in each of these boxes. Then all the players multiply these numbers to record the student's score. After everyone has had a turn, the student with the highest score is declared the winner.

I did *not* tell the students to look for shortcuts, but many of them did.

EVALUATION

If the game runs smoothly and students have little difficulty keeping score, the objective has probably been reached. To make sure individual students had reached it, during the last couple of minutes of class I asked my students to turn over their scoresheets and write answers to questions such as "If I had seven boxes with eight washers in each, how many washers would I have?" The worksheet for Activity 2 may be given to students who have not reached the objective.

EXPERIENCE 6
Problem Solving

OBJECTIVE

The student should be able to make sketches and write number sentences in solving word problems that involve basic multiplication facts.

MATERIALS

4 worksheets for each student

TEACHER STRATEGY

The problems on the student worksheets are similar to those found in most elementary textbooks. The worksheets for Activities 1 and 2 evaluate the student's skill in using arrays and in writing number sentences that describe the problem. The mathematics has been confined to the level of basic facts. Students who have mastered the basic multiplication facts can answer the problems easily, but it is important that they complete the sketch for each problem so that you can evaluate their ability to make an array and use it to solve problems. Students who have not mastered the basic multiplication facts can find the solutions to the problems by counting or using repeated addition. Worksheets for Activities 3 and 4 evaluate the student's ability to use cross products to solve problems.

One way to introduce this experience is by reviewing the problem used at the beginning of Experience 4, the one about the number of outfits that could be formed by different combinations of three shirts and two pairs of shorts.

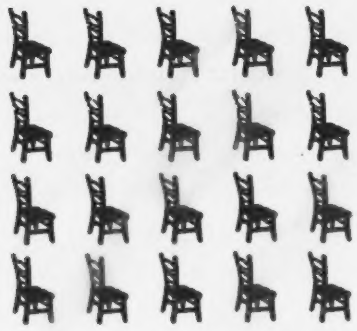
EVALUATION

The answers to the problems on the worksheets will indicate how well your students have mastered the ideas of the entire unit.

PROBLEM SOLVING **UNIT 3, EXPERIENCE 6**

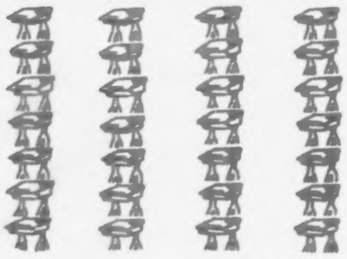
Activity 1 Name ______________________

1.

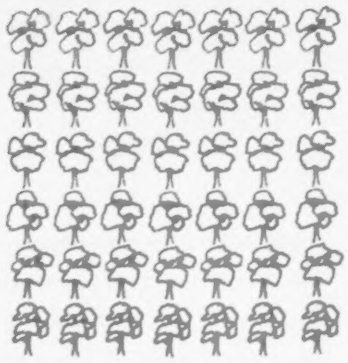


This classroom has 4 rows of chairs. There are 5 chairs in each row. How many chairs in all? 20
Write a sentence for this problem.
4 x 5 = 20

2. A classroom has 7 rows of desks. There are 4 desks in each row. How many desks in all? 28
Use the space at the right to draw a picture for this problem. Write a sentence for it.
7 x 4 = 28



3. An orchard has 42 trees. There are 6 rows of trees. How many trees in each row? 7 Use the space at the right to draw a picture for this problem. Write a sentence for it.
6 x 7 = 42

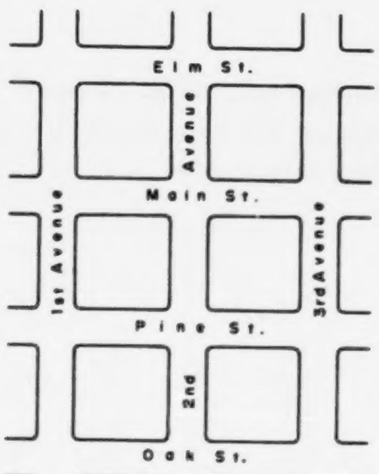


PROBLEM SOLVING **UNIT 3, EXPERIENCE 6**

Activity 2

Name ____________________

1.



This is a map of Tiny Town. Each place a street and avenue cross is called an intersection. Place an X on each intersection.

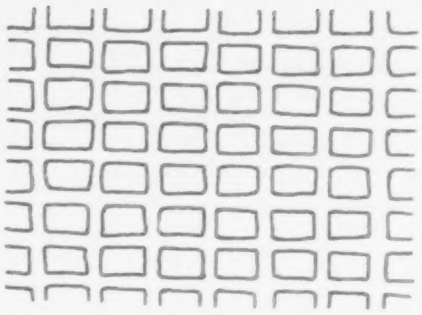
How many avenues? 3 How many streets? 4 How many intersections? 12

Write a sentence for this problem.

4 × 3 = 12

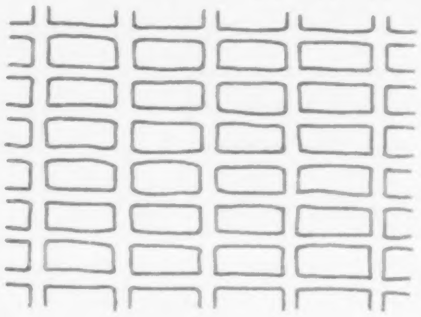
2. Big Town has 7 streets and 5 avenues. Use the space at the right to draw a map of Big Town. How many intersections in Big Town? 35 Write a sentence for this problem.

7 × 5 = 35



3. Little City has 49 intersections. It has 7 streets. Use the space at the right to draw a map of Little City. How many avenues in Little City? 7 Write a sentence for this problem.

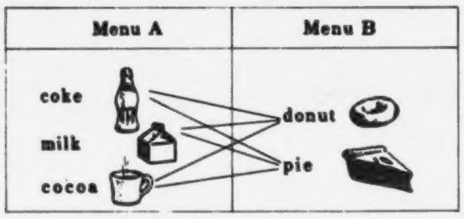
7 × 7 = 49



PROBLEM SOLVING **UNIT 3, EXPERIENCE 6**

Activity 3 Name ______________________

1.



coke, donut
coke, pie

milk, donut
milk, pie

cocoa, donut
cocoa, pie

In the space at the right, list all the possible pairs from Menu A and Menu B. You must choose one item from each menu for each pair you list. Write a sentence for this problem.

3 × 2 = 6

2. John, Mary
John, Betty
John, Susan

Jim, Mary
Jim, Betty
Jim, Susan

David, Mary
David, Betty
David, Susan

Bill, Mary
Bill, Betty
Bill, Susan

Don, Mary
Don, Betty
Don, Susan

Boys	Girls
John Jim David Bill Don	Mary Betty Susan

In the space at the left, list all the possible pairs from the boys and girls. You must use one boy's name and one girl's name for each pair you list. Write a sentence for this problem.

5 × 3 = 15

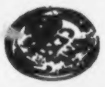
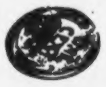
PROBLEM SOLVING

UNIT 3, EXPERIENCE 6

Activity 4

Name ____________________

1. Try tossing a penny with a friend.
   List all the ways the penny could land in two throws.

Penny 1	Penny 2
heads 1	**heads 2**
 **tails 1**	 **tails 2**

heads, heads
heads, tails
tails, heads
tails, tails

Write a sentence for this problem. 2 × 2 = 4

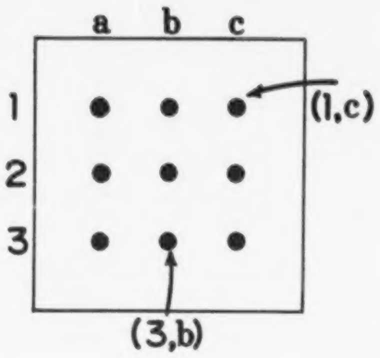
2. Each dot in the array is named with a number and a letter. List the names of all the dots in the array. Write a sentence for this problem.

3 × 3 = 9

(1, a)
(1, b)
(1, c)

(2, a)
(2, b)
(2, c)

(3, a)
(3, b)
(3, c)



4

EXPERIENCES WITH

Units of Measure

The experiences in this unit develop ideas common to all measurement situations. Whenever you measure, whether in units of length, area, volume, or weight, you are applying the basic principles of a standard unit. These principles are dealt with slowly but surely here.

Students, particularly those with a weak background in primary-grade mathematics, will profit from the emphasis on activity and manipulation in these experiences, an emphasis typically neglected because of lack of time and equipment. Without such experiences, however, the further development of measurement concepts is difficult, if not impossible, for these students.

Each experience introduces a primitive form of measurement to help the student develop the concept of a unit, then extends the concept to show the advantage of using more standard units in measurement situations.

Since most of the worksheets ask for answers in nonstandard units, no answers are given on the student worksheets reproduced in the text.

OVERVIEW

Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is one teacher's first-person account of a procedure that proved effective with his class.

The following brief summary will help you decide whether these experiences meet the needs of your own class and will also give you some indication of the amount of preparation involved.

It is assumed that duplicating facilities are available to you for making copies of the student worksheets and other "handout" materials in the Teaching Package.

Experience 1: Measuring Length

Students use lima beans to measure the length of line segments. Since so few of them get the same number for the length of the line segment, they discover that the lima bean is a very poor unit of length. In the second activity, the students use trading stamps to measure the line segments and find that their answers will agree if they all use the stamps the same way. Discussion brings out the characteristics of a uniform unit and the desirability of such a unit in the measurement process. The third activity gives students an opportunity to use measurement concepts in simple problem-solving situations.

Materials needed: lima beans, trading stamps, drawing, worksheets.

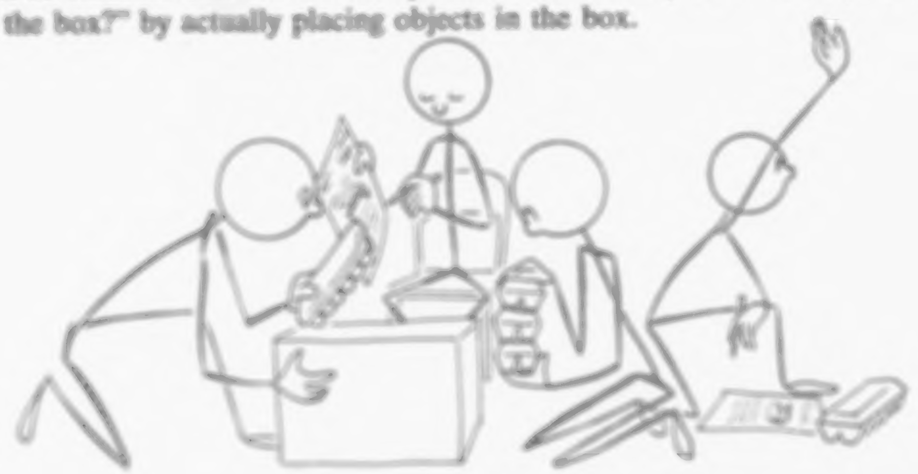
Experience 2: Measuring Area

The lima bean is used as an initial unit for measuring area, and again it proves unsatisfactory. Students find that the trading stamp is a good unit for rectangular areas but encounter difficulties in measuring triangular or irregular areas. Discussion brings out these problems and some possible solutions.

Materials needed: lima beans, trading stamps, drawing, worksheet.

Experience 3: Measuring Volume

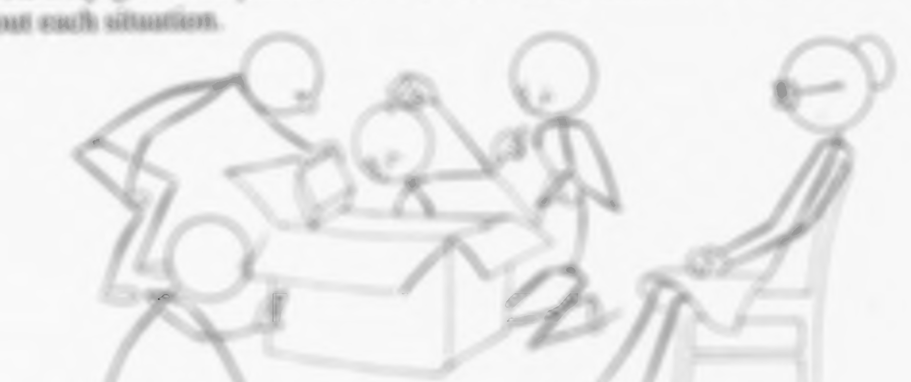
The students work on worksheets at the five stations to build the volume concept, using one-inch cubes and household containers. At each station there should be enough of the required material to make one layer in a box so students can answer the question "How many of these objects fit in the box?" by actually placing objects in the box.



Materials needed: blocks, egg cartons, cereal boxes, baby-food jars, frozen-juice cans, boxes, worksheets.

Experience 4: More about Measuring Volume

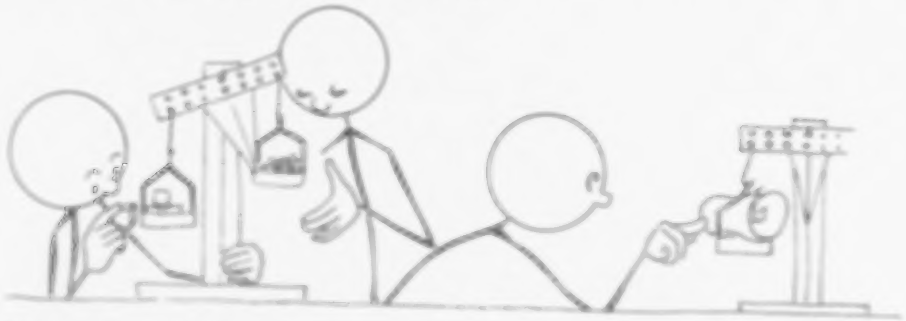
The volume concept is extended in the activities at the six stations. The student is given only two or three of the objects being used as the unit of measure. Since he doesn't have enough to make one layer in the box, he must develop for himself a technique for finding the volume. The worksheets help give a sequence to his activities and stimulate further thinking about each situation.



Materials needed: boxes, decks of cards, shoe box, baby-food jars, milk carton, gum erasers, cereal boxes, 1-in. cubes, worksheets.

Experience 5: Measuring Weight

The student develops the concept of a unit of weight by using a simple balance he can construct himself. He uses a paper clip as his unit of weight in resolving several problem situations.



Materials needed: paper clips and paper-clip boxes, 1-in. cubes, rubber erasers, typing paper, index cards, envelopes, nails, worksheets; coat hangers, wooden blocks, milk cartons, tagboard, pegboard, washers.

EXPERIENCE 1

Measuring Length

OBJECTIVE

The student should be able to recognize the need for a uniform unit for measuring length and to use such a unit in simple problem-solving situations.

MATERIALS

About 20 lima beans for each student
Trading stamps (several different sizes)
Copy of Drawing 1 (from the Teaching Package) for each student
3 worksheets for each student (see the Teaching Package)

TEACHER STRATEGY

I began the experience by giving each student a copy of Drawing 1, the first worksheet, and about twenty lima beans. After reading aloud the directions for Activity 1, I told the class to go ahead and work on it. In anticipation of the discussion, I walked around the room and noted the different ways the students were attacking the problem.

When most students had finished the activity, I asked several of them to give their answers for the length of $\overline{AB}$ and wrote their replies on the board. Few of them gave the same answer, although they agreed that they had all measured the same line segment. Some students claimed that if they all used the same lima beans they should have the same answer. The discussion became so heated that I had to interject questions like the following to make the discussion productive:

> If you drew a line segment eight lima beans long, would it be the same length as someone else's segment eight lima beans long? Why?
>
> If you measure two line segments and find that one is eleven lima beans long and the other is ten lima beans long, can you be sure that the first is actually longer than the second?
>
> Notice that in Drawing 1 line segment $\overline{AB}$ is the same length as segment $\overline{CD}$. Did you all assign the same number of lima beans to both line segments?
>
> Line segment $\overline{EF}$ is longer than the other two segments. Is the number assigned to it greater than the number assigned to the others?

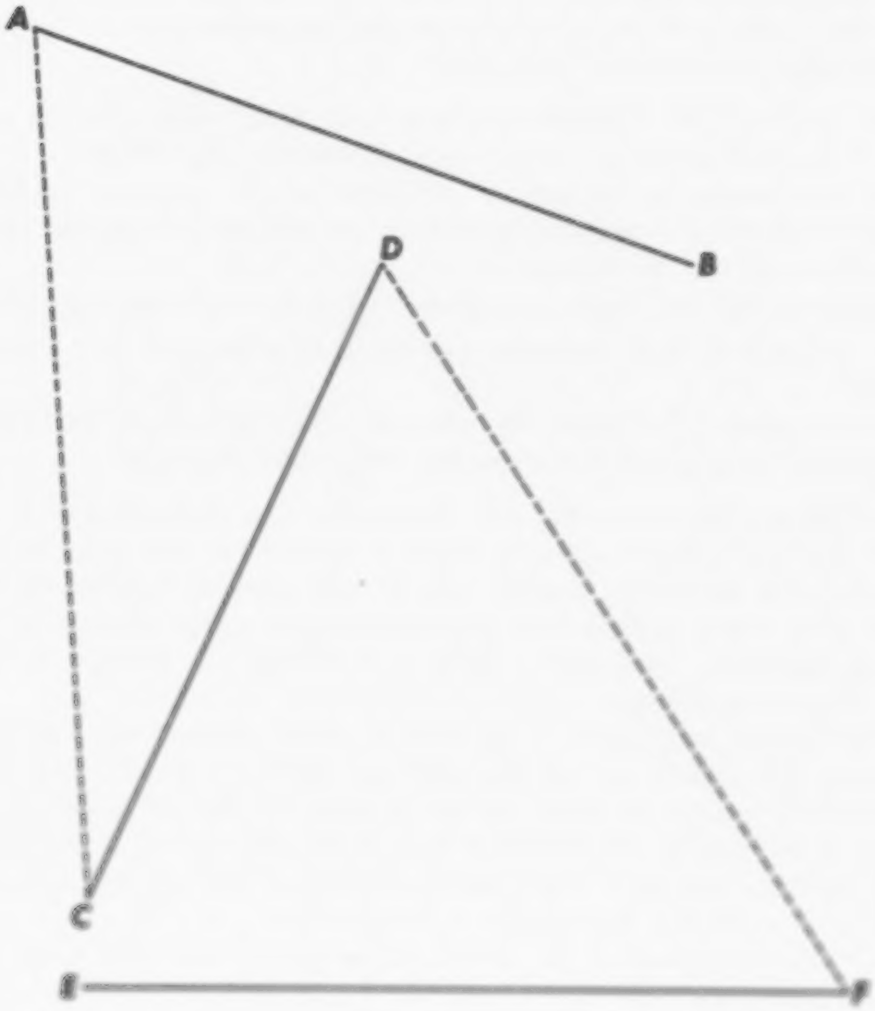
I tried to bring out the fact that, because the lima beans are not of uniform size, the numbers assigned to the lengths of the line segments may not represent the relative lengths. The obvious conclusion is that the lima bean is not a very useful unit for measuring length. Since I did not discuss all the questions, about five minutes of discussion was enough to bring out the necessary ideas.

For Activity 2 I passed out the trading stamps, making sure that those given to one student were all the same size and that each had enough to place them side by side along the line segments. I did not direct the students to use them in any particular way—some used the width and others the length as their unit. The primary function of this activity is to lead students to recognize the need for a uniform unit.

The brief discussion of the activity was similar to that after Activity 1. The desirability of agreement on whether length or width is used evolved, and I led the students to various conclusions about the need for a uniform unit. They noted the advantages of using the same unit over and over again and recognized that if their results were to be comparable to the results of others they must all use the same unit.

[Continued on page 112]

Drawing 1

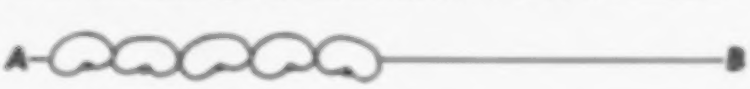


Name ______________________________

Activity 1–Using Lima Beans to Measure Length

Use your lima beans to measure the line segments in Drawing 1 to get the answers to the following questions.

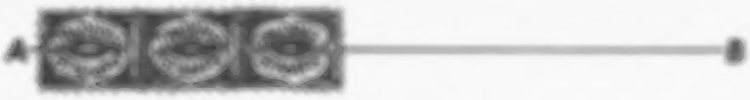
1. How many lima beans long is line segment $\overline{AB}$? ________ Segment $\overline{CD}$? ________
   Segment $\overline{EF}$? ________
2. Which line segments are longer than line segment $\overline{AB}$? ________
3. Which line segments are shorter than segment $\overline{AB}$? ________
4. In Drawing 1, how far is it from point A to point C (in lima beans)? ________
5. Will all the other students get the same answers to Questions 1–4? ________



Activity 2–Using Trading Stamps to Measure Length

Use your trading stamps and Drawing 1 to answer the following questions.

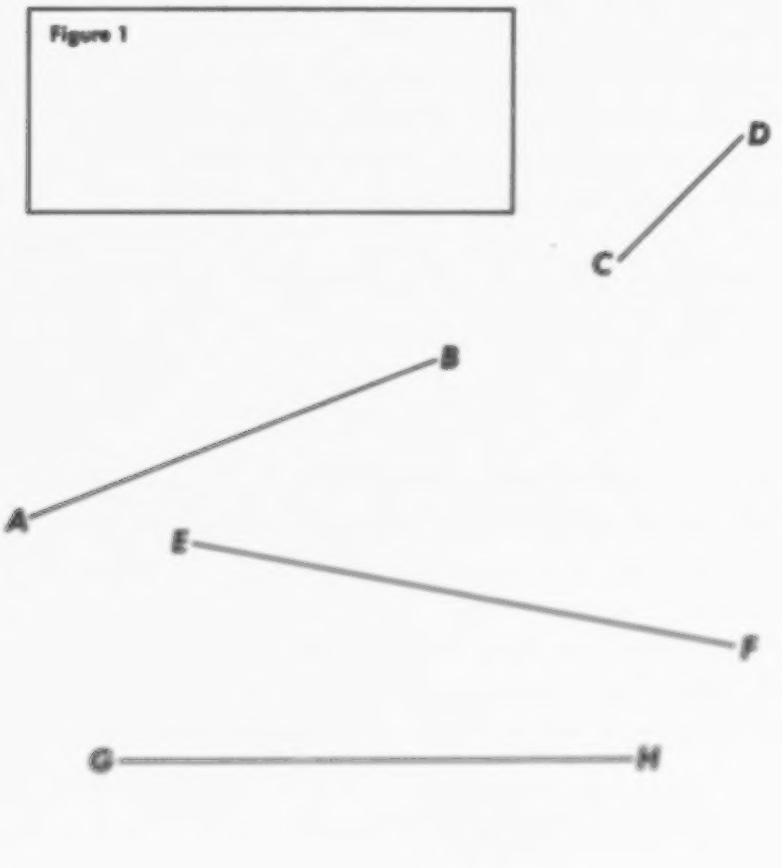
1. How many trading stamps long is line segment $\overline{AB}$? ________ Segment $\overline{CD}$? ________
   Segment $\overline{EF}$? ________
2. Which line segments are longer than segment $\overline{AB}$? ________
3. In Drawing 1, how far is it from point D to point F (in trading stamps)? ________
4. Will all the other students get the same answers to these questions? ________



MEASURING LENGTH UNIT 4, EXPERIENCE 1

Activity 3 Name ______________________________

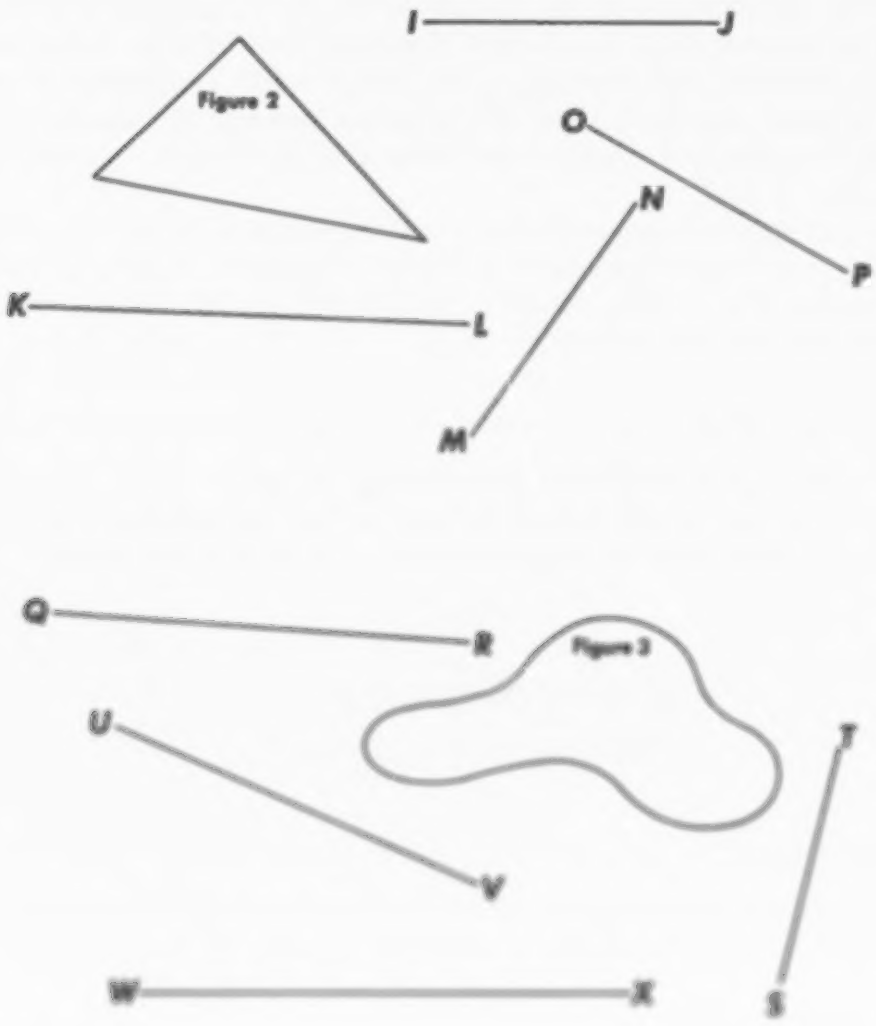
Each of the following three exercises shows one closed figure and several line segments. Draw a ring around those line segments that will fit inside the closed figure.



(Continued)

Activity 3—*Continued*

Name ________________________________


I
J
Figure 2
O
N
P
K
L
M
Q
R
Figure 3
U
T
V
W
X
S

In Activity 3, I encouraged students to apply the generalizations developed during the first two activities. In explaining that they were to determine which of a series of line segments could be placed within the given figure, I used somewhat loose mathematical terminology, but it communicated to the students. I thought that any attempt to use more precise mathematical language might defeat the purpose of this experience.

The students determined for themselves what measurement technique to use. Some simply eyeballed the segment to decide whether it would fit—and often reached faulty conclusions since they had to make positional changes mentally. For example, in the first problem the student could easily recognize that $\overline{EF}$ wouldn't fit within the rectangle horizontally, but it would be difficult for him to determine visually whether it would fit diagonally.

We concluded the experience with a brief discussion of the final activity. The next time I teach these activities I think I will take two class periods and add some skill-building exercises. I was a little pushed to get all three activities into one class period.

EVALUATION

This experience is successful if the students participate in it. The discussion at the end of each activity focuses on the generalizations and will help you evaluate the level of generalization developed in the students.

EXPERIENCE 2

Measuring Area

OBJECTIVE

The student should recognize the need of a uniform unit for measuring area and should see the advantage of having everyone use the same unit.

MATERIALS

About 50 lima beans for each student
Trading stamps (several different sizes)
Copy of Drawing 2 for each student
1 worksheet for each student

[Continued on page [illegible]]

Drawing 2

Figure 1

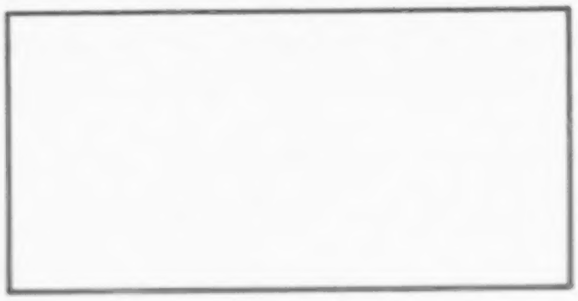
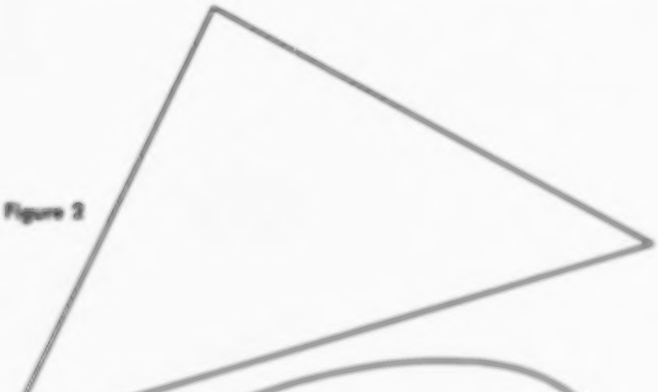


Figure 2



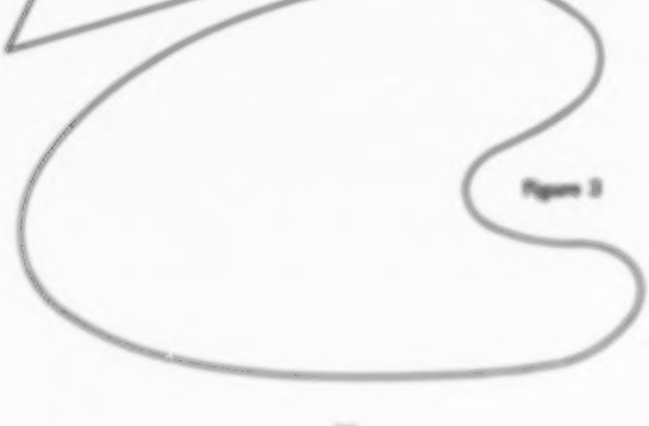
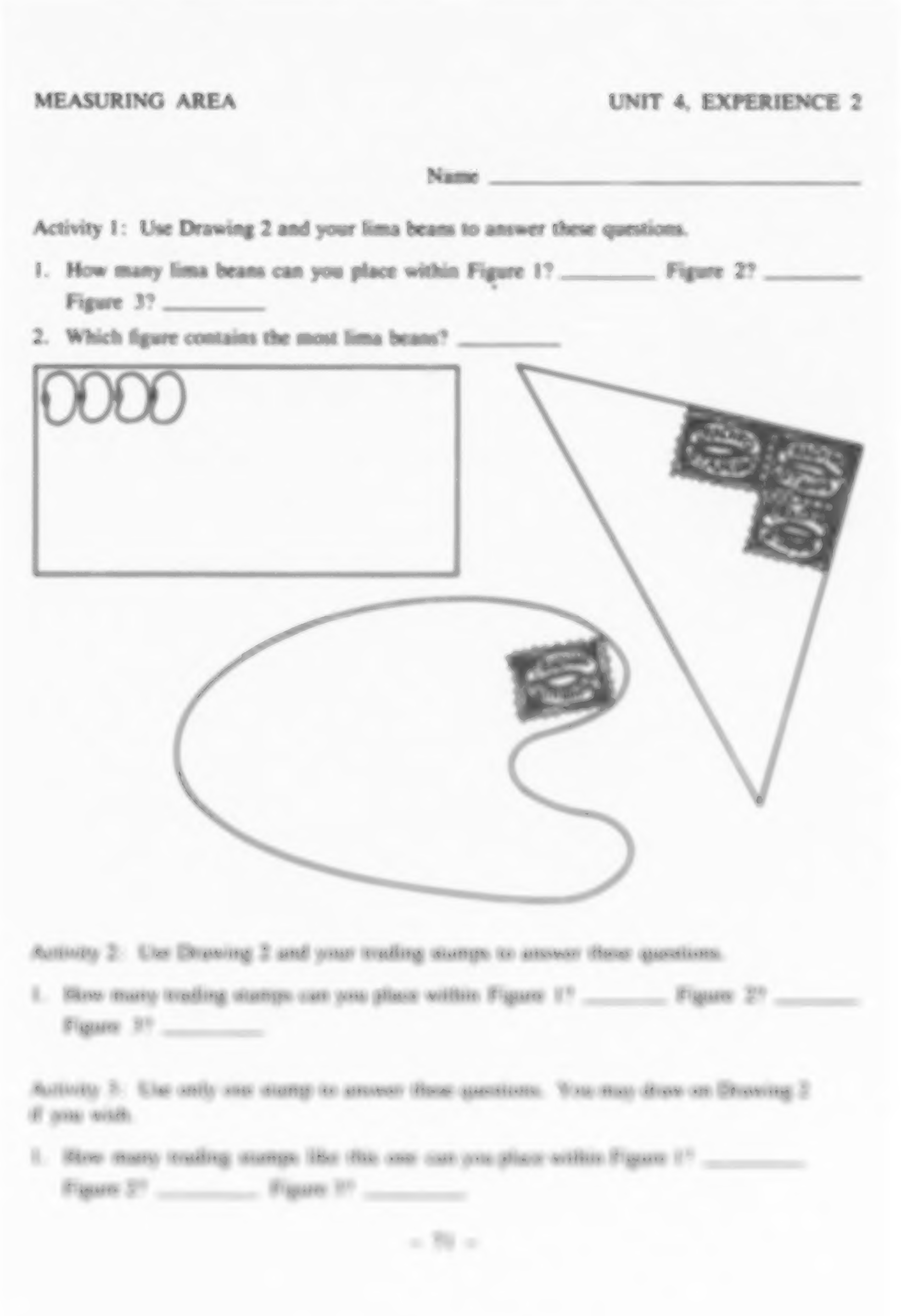


Figure 3



MEASURING AREA **UNIT 4, EXPERIENCE 2**

Name ______________________________

Activity 1: Use Drawing 2 and your lima beans to answer these questions.

1. How many lima beans can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________
2. Which figure contains the most lima beans? ________

Activity 2: Use Drawing 2 and your trading stamps to answer these questions.

1. How many trading stamps can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________

Activity 3: Use only one stamp to answer these questions. You may draw on Drawing 2 if you wish.

1. How many trading stamps like this one can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________

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TEACHER STRATEGY

I gave each student a copy of Drawing 2, the worksheet, and about fifty lima beans. While all worked on Activity 1, I circulated, asking questions and observing their techniques. As each student completed it, I gave him the trading stamps for Activity 2. I had three different sizes available and gave individual students stamps of a uniform size. I suggested that they measure to the nearest whole stamp, but some decided to cut stamps to get a more precise measure of the area. When all students had completed these two activities, we compared answers and discussed the results. Meanwhile, I had given each individual student one stamp of a size different from that of his original stamps so he could begin work on Activity 3.

EVALUATION

These activities took about thirty minutes of class time. The discussion assured me that many of the students understood the basic nature of a unit of area and realized that to measure area we had to cover the region being measured. Several students pointed out that the measures were only approximate no matter how they cut up the stamps to arrive at the answer.

EXPERIENCE 3

Measuring Volume

OBJECTIVE

The student should be able to get a rough measure of the volume of a rectangular box by packing it with one layer of smaller objects.

MATERIALS

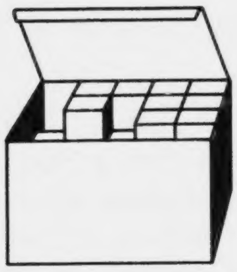
25 blocks
Egg cartons
15–20 single-serving cereal boxes
20–30 baby-food jars
25–35 frozen-juice cans
5 cardboard boxes
5 worksheets for each student

[Continued on page [illegible]]

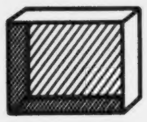
MEASURING VOLUME **UNIT 4, EXPERIENCE 3**

Station A **Name** ____________________

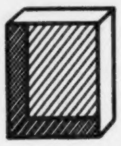
At this station are some blocks and Box A. Use them to answer these questions.



1. **What is the largest number of blocks like these that you can fit into the box?** ________
2. **How many blocks can you fit into one layer?** ________
3. **How many layers can you get into the box?** ________
4. **If you put the box on its long side, how many blocks can you put in one layer?** ________



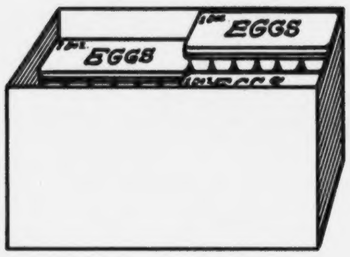
5. **If you stand the box on end, how many blocks can you put in one layer?** ________



MEASURING VOLUME **UNIT 4, EXPERIENCE 3**

Station B **Name** ______________________________

At this station you find a large cardboard box and some egg cartons. Use them to answer these questions.



1. How many egg cartons like this would it take to fill the box? __________

2. How many eggs does each carton hold? __________

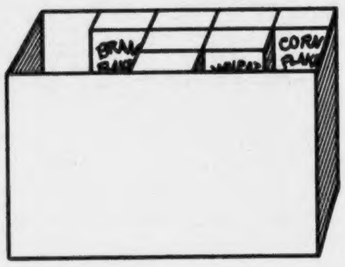
3. If the box were filled with egg cartons and the egg cartons were filled with eggs, how many eggs would the box hold? __________

4. How many dozens of eggs are in the box when it is filled with egg cartons? __________

MEASURING VOLUME **UNIT 4, EXPERIENCE 3**

Station C **Name** ____________________________

At this station there are small cereal boxes and a large cardboard box. Use these materials to answer the following questions.



1. **What is the *largest* number of cereal boxes that you can place in one layer on the bottom of the box? __________**
2. **What is the *smallest* number of cereal boxes that you can place in one layer and still cover the bottom of the box? __________**
3. **How many cereal boxes would it take to fill the large box? __________**

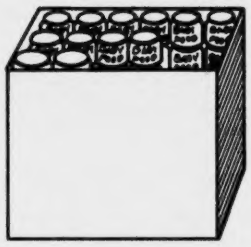
MEASURING VOLUME

UNIT 4, EXPERIENCE 3

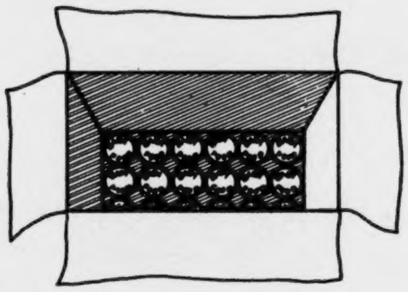
Station D

Name ____________________

Baby-food jars and a cardboard box are at this station. Use them to answer the following questions.



1. How many baby-food jars high is the box? __________
2. How many of the jars can you put in one layer? __________



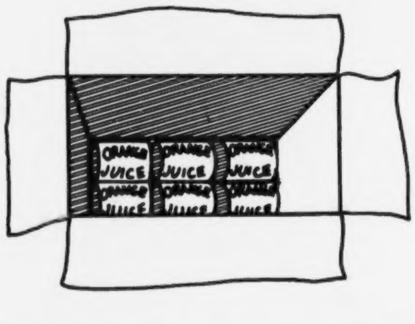
3. How many baby-food jars will the box hold? __________
4. If a jar of baby food weighs 4 ounces, how many pounds of baby food in jars will the box hold? __________

MEASURING VOLUME **UNIT 4, EXPERIENCE 3**

Station E **Name** ______________________________

At this station there are some frozen-juice cans and a box. Use them to answer the questions.

1. **How many cans does it take to cover the bottom of the box?** __________


ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE

2. **How many cans will cover the bottom of the box if you lay the cans on their sides?**

   ____

3. **How many frozen-juice cans can you pack in the box?** __________

4. **Can you pack in more cans by laying them down or by standing them up?**

   ____________________________ **How many more?** __________

Stations. Stations A, B, C, D, and E should be set up around the room. At each there is a box labeled with the station letter and enough of the objects being used to fill one layer in the box. For example, if one-inch cubes are used for the blocks at Station A, the box should be about four-by-five-by-three inches. At Station B, the size of Box B depends on the size and number of the egg cartons you use. The cereal boxes at Station C may be the kind that contain a single serving. At Stations D and E there are baby-food jars and frozen-juice cans, respectively, and boxes of the appropriate sizes.

TEACHER STRATEGY

For my class I set up several duplicates of each station. I did not demand that students complete the stations in order or that they complete them all. Each student moved from one station to another at his own pace.

The students "measured" the box at each station with the given objects. I provided more than enough objects to cover the bottoms of the boxes. The layer of objects should fit the dimensions of the box as nearly as possible.

I planned the period so that we had time in a short discussion to focus on generalizations and to reinforce correct answers.

EVALUATION

A few questions, such as "If a box contains twenty objects in one layer and has five layers, how many objects are in the box?" should serve both to evaluate the success of the experience and to focus on the generalizations developed.

EXPERIENCE 4

More about Measuring Volume

OBJECTIVE

The student should be able to use only two or three small objects to get a rough measure of a rectangular box.

MATERIALS

2 or 3 decks of cards (each deck in its own box)
Shoe box
2 or 3 baby-food jars
1-gal. milk carton (or box of a similar size)
2 or 3 gum erasers
2 cornflakes boxes
4–6 1-in. cubes
Sealed cereal box
Boxes: 2 in. by 3 in. by 5 in.
    12 in. by 18 in. by 24 in.
    2 ft. by 2 ft. by 3 ft.
    6 in. by 5 in. by 3 in.
6 worksheets for each student

Stations. Stations A, B, C, D, E, and F are set up around the room. At each there is a box labeled with the station letter and no more than two or three of the objects being used. The decks of cards and the shoe box are at Station A. Station B has the baby-food jars and the milk carton. At Station C are the erasers and the first two of the four boxes listed above. At Station D are the cornflakes boxes and the two-by-two-by-three-foot box. At Station E are two or three one-inch cubes and a six-by-five-by-three-inch box. At Station F are two or three one-inch cubes and a sealed cereal box.

TEACHER STRATEGY

The basic difference between this experience and the preceding one is the number of objects provided: here I placed, at most, three of the objects being used as units at each station. Any of the several strategies open to the students required more careful planning and technique than in Experience 3, and I was careful not to tell them how they were to go about measuring. However, some students, after struggling with the first worksheet themselves, required a hint as to how to get started. This might focus on a relatively primitive strategy; for example, I asked "Could you use a pencil to mark around the base of your unit?"

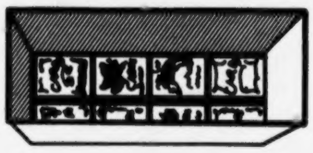
While the students worked at the stations, I circulated among them to give encouragement and minimal guidance. The second question on each worksheet tested their understanding of the principle involved.

[*Continued on page 129*]

MORE ABOUT MEASURING VOLUME UNIT 4, EXPERIENCE 4

Station A Name ____________________

You will find decks of cards and a shoe box at this station. Use them to answer the first question, then go on to the second question.



1. What is the largest number of decks of cards that you could put in the box? ________

   How many would you put in each layer? ________

   How many layers would you have to make? ________

2. A packing case has 4 layers and 18 boxes in each layer. How many boxes are there in the packing case? ________

MORE ABOUT MEASURING VOLUME **UNIT 4, EXPERIENCE 4**

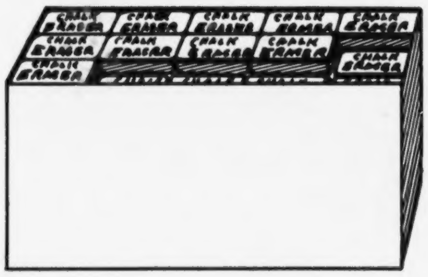
Station B **Name ______________________________**

There are baby-food jars and a box at this station. Use them to help you answer the first question, then answer the second.

1. What is the largest number of jars you could put in the box? __________

   How many would you put in one layer? __________

   How many layers would you make? __________

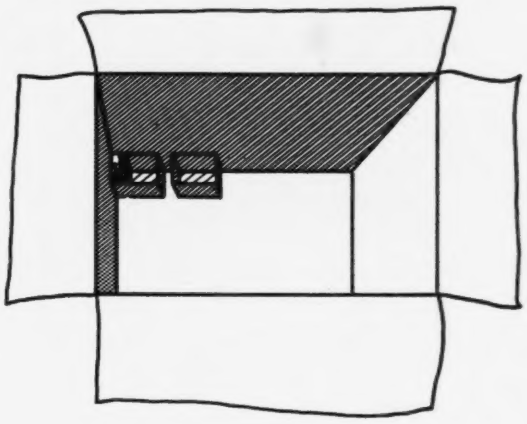


2. Jim packed 100 erasers in a box. He started by putting 15 in each layer. How many layers did he make? __________

MORE ABOUT MEASURING VOLUME **UNIT 4, EXPERIENCE 4**

Station C **Name** ______________________________

There are erasers and two boxes at this station. Use them to help you answer the first question, but do not stack the small boxes inside each other! Then go on to the second question.



1. What is the largest number of erasers that you could put in the small box? __________

   What is the largest number of small boxes that you could put in the large box? ________

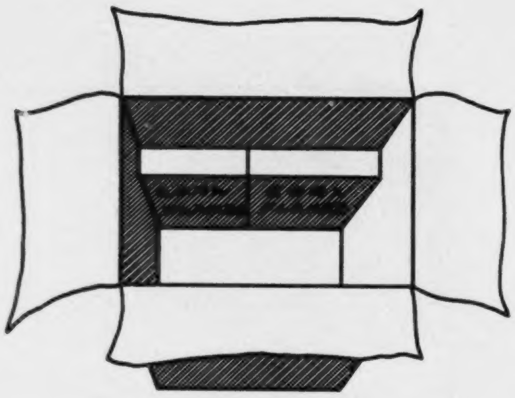
   What is the largest number of erasers that you could put in the large box? __________

2. A carton of playing cards holds 10 decks with 52 cards in each deck. How many cards are there in the carton? __________

MORE ABOUT MEASURING VOLUME **UNIT 4, EXPERIENCE 4**

Station D **Name ____________________**

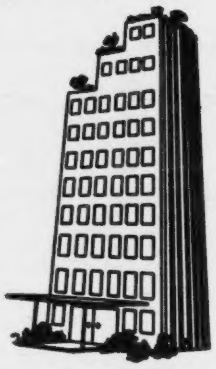
At this station there are two cereal boxes and one large box. Use them to answer the first question, then go on to the next one.



1. What is the largest number of cereal boxes that you could put in the large box? ________.

   How many boxes would you put in each layer? ____________

   How many layers would you have? ____________



2. This apartment building has 25 apartments on each floor.

   If it is an eight-story building, how many apartments does the building have? ____________

   If all of them are three-room apartments, how many rooms does the building have? ____________

MORE ABOUT MEASURING VOLUME **UNIT 4, EXPERIENCE 4**

Station E **Name** ______________________________

At this station there are cubes and a box. Use them to help you answer the first question, then answer the second.

1. What is the largest number of cubes you could put in the box? __________

How many cubes long is the box? __________

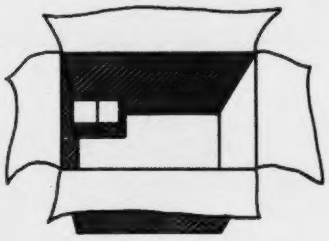
How many cubes wide is the box? __________

How many cubes high is the box? __________

How many cubes would you put in each layer?

____

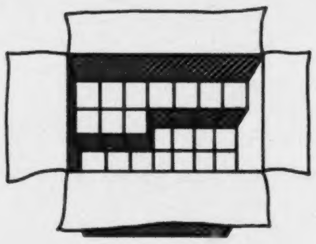
How many layers would you have? __________



2. Dick filled two boxes with blocks. Box A had 6 layers with 28 blocks in each layer.

Box B had 8 layers with 21 blocks in each layer. Which box held more blocks?

____



MORE ABOUT MEASURING VOLUME **UNIT 4, EXPERIENCE 4**

Station F Name ______________________

Special–for experts only!

Use the cubes and the sealed cereal box to help you answer the first question, then go on to the second.

1. What is the largest number of cubes that you could put in the cereal box? __________

2. Tom visited his dad's warehouse. One stack of cases was 10 cases long and 15 cases wide. Tom figured out that there were 1,800 cases in the stack. How many cases high was the stack? __________

Near the end of the period I called the class together and asked some of them to explain how they went about solving the problems in the various situations. Several students gave good explanations of volume and suggested strategies I hadn't even thought of.

EVALUATION

These activities took two class periods. I evaluated the success of the experience by giving the students a problem similar to those on the worksheet and the information necessary to solve it. I asked them to write out their solutions so I could see their method of attacking the problem. Only about half of my class were successful, but perhaps I made the question too difficult.

EXPERIENCE 5

Measuring Weight

OBJECTIVE

The student should be able to use a simple balance to weigh various objects in terms of a nonstandard unit, such as the paper clip.

MATERIALS

30–40 paper clips (all the same size) for each student
Balance for each student (see directions for construction below)
1-in. cube for each student
Rubber eraser for each student
Sheet of typing paper for each student
5-by-7-in. index card for each student
Envelope for each student
10 sealed envelopes, each containing a different number of paper clips
Empty paper-clip box for each student
6 sealed paper-clip boxes (or pill boxes), each containing a different number of nails
1 nail for each student (all nails should be the same size)
2 worksheets for each student

Construction of Student Balance

A simple balance can be constructed in ten or fifteen minutes from materials that are inexpensive and readily available.[1] The total cost of materials for an entire class is less than five dollars. The balance weighs to the nearest fifth of a paper clip (about .1 gram) and is useful in both mathematics and science classes. The following materials are needed for each balance:

Coat hanger
Small wooden block (about 7 in. by 2½ in.)
Nails
2 milk cartons
Tagboard
Piece of pegboard (10½ in. by 2½ in.)
3 paper clips
Small washer

To construct the balance, cut a twenty-inch piece of wire from the hanger and straighten it. Make a loop at one end of the wire and a small hook at the other end, and fasten the loop securely to the wooden block with nails (see fig. 1).

Now cut a two-by-eight-inch strip of tagboard, draw a center line on it from top to bottom, punch a hole at the center top, and tape a two-inch piece of hanger horizontally to the bottom. Hang this background strip on the hook as shown in figure 2.

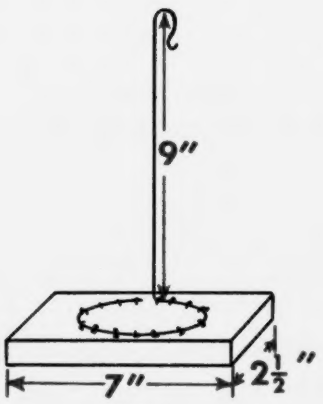


Fig. 1

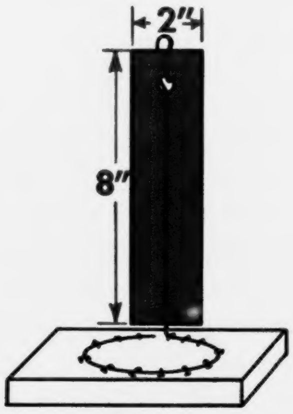


Fig. 2

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[1] The method of constructing a student balance given here was suggested by Louis Finsand, professor of (science) teaching, Malcolm Price Laboratory School, University of Northern Iowa, Cedar Falls, Iowa.

Next, cut a tagboard pointer, mark it with a center line, and tape it to the 10½-by-2½-inch piece of pegboard (see fig. 3). Cut off the tops of the two milk cartons as shown in figure 4 and punch holes near the ends of the strips. Place the small washer on the hook as a spacer and hang the pegboard on the hook over it. Attach the milk-carton trays to the pegboard with partially straightened paper clips, and use the other paper clip wherever it is needed to balance the balance—that is, to line up the center line on the pointer and the center line on the background strip. The finished balance is shown in figure 5.

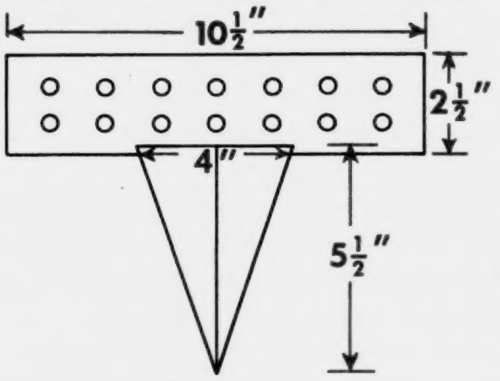


Fig. 3

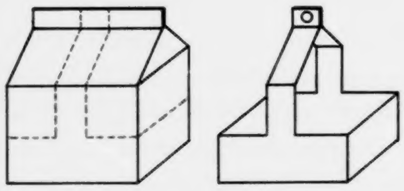


Fig. 4

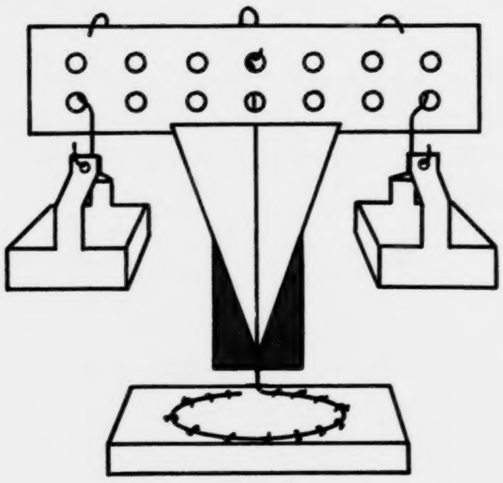


Fig. 5

TEACHER STRATEGY

I gave each student a balance, a supply of paper clips, and the two worksheets. The balances can be constructed by the students themselves if you give them step-by-step instructions, and such an activity might make it easier for them to understand the principles involved.

In Activity 1, the student uses the paper clip as his unit of measure to weigh a sheet of paper, the eraser, an index card, and the cube. He uses this information to answer the questions on the worksheet.

The student uses his paper clips in Activity 2 to determine the number of paper clips in small sealed envelopes. I labeled the envelopes A–J and placed them at the front of the room.

In Activity 3, the student determines the number of nails in small sealed boxes by using his balance and paper clips. I labeled the boxes A–F and put from five to fifteen nails in each. I made them available at a table and provided each student with an empty box and a single nail.

In a short discussion we noted the advantages and disadvantages of using the paper clip as a unit and the balance as a means of measuring weight. We compared the results of various students, and I tried to reinforce correct answers.

EVALUATION

The success of the experience was obvious in my class—all of the students were involved in it. Many of them had to prove to themselves that the heavier side of the balance would go down before they could begin the activities.

As a final evaluation, I gave the students a problem similar to the ones in Activity 3 and asked them to discuss the method of solution. Then two students did the weighing, and all of them computed the solution and turned in their results to me.

MEASURING WEIGHT

UNIT 4, EXPERIENCE 5

Name ______________________________

Use your balance and paper clips to help you answer the questions for these two activities.

Activity 1

1. If you use the paper clip as your unit of weight, what is the weight of the wooden cube? ______________ The eraser? ______________ 5 sheets of typing paper? ______________ 50 index cards? ______________
2. How many paper clips would it take to balance 2 cubes and 3 erasers? ______________
3. What is the total weight, in paper clips, of a ream (500 sheets) of typing paper? ______________

Activity 2

1. Find out how many paper clips are in any 5 of the sealed envelopes. DO NOT OPEN THEM!

Envelope A ________
Envelope B ________
Envelope C ________
Envelope D ________
Envelope E ________
Envelope F ________
Envelope G ________
Envelope H ________
Envelope I ________
Envelope J ________

2. Suppose you had as many of each of these 5 envelopes as you needed. Describe the envelopes you would need to have exactly 100 paper clips. ____
3. Describe the envelopes you would need to have exactly 51 paper clips. ____

MEASURING WEIGHT **UNIT 4, EXPERIENCE 5**

Activity 3 Name ______________________________

Use your balance and paper clips to help you answer these questions.

1. Find out how many nails are in Box A and in Box B. Then find the number of nails in as many of the other boxes as you have time for. DO NOT OPEN THE BOXES!

Box A ________ Box D ________
Box B ________ Box E ________
Box C ________ Box F ________

2. Suppose you had as many boxes of each kind as you needed. Could you make a stack of Box A's and a stack of Box B's so that both stacks would have the same number of nails? ___________ If so, how would you do it? ________________________
____

5

EXPERIENCES WITH

Physical Models for Fractions

The activities in this unit provide students with learning experiences that can help them grasp the fraction concept. Each experience is designed to bring the student into a personal encounter with fractions as they are represented by physical models or referents. You can use the experiences periodically with an entire class in conjunction with a more comprehensive unit on fractions. Or you may wish to use them with selected groups of students who appear to need additional practice with concrete representations in order to become familiar with basic concepts. Although under some circumstances these experiences may constitute a five-day sequence, it is more likely that with slow learners the activities will extend over a longer period of time.

The experiences are laboratory-oriented. The student is given the opportunity to work independently, to explore, to guess, and to see whether his guesses are correct. For Experiences 1 and 3 stations are set up within the classroom, and the student moves from station to station involving himself in a wide variety of experiences having to do with the fraction concept. The very nature of these experiences implies that a certain level of "productive noise" can be expected. Many students, undoubtedly, will have to learn a great deal about working productively in this laboratory-like situation. In Experiences 2, 4, and 5 students work individually on the worksheets; but these experiences are introduced by group discussions, which give pupils the opportunity to pool their resources and to meet with more success than they would have if they simply worked alone.

The activities require equipment and materials that are relatively easy to obtain and safe to use with a minimum of classroom supervision. In most cases, substitutions may be made where necessary without detracting from the essential point of the activity.

OVERVIEW

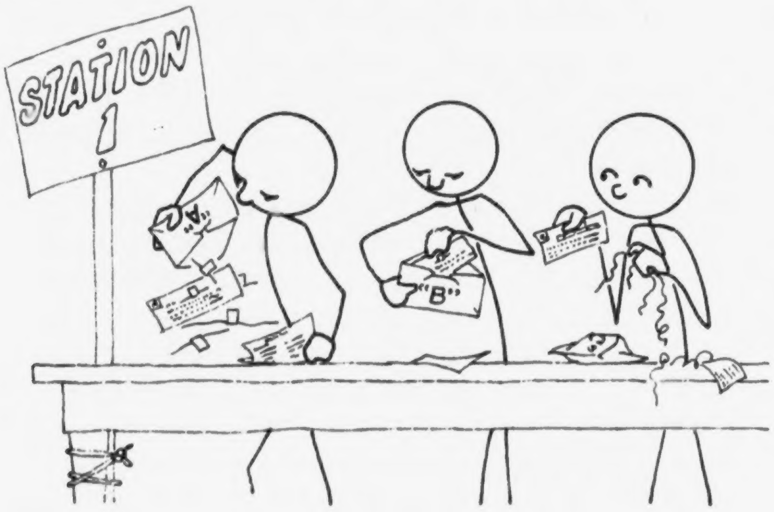
Each of the five experiences in this unit includes a detailed section called "Teacher Strategy," which is an account of a procedure that proved successful in actual classroom use.

The following brief summary will help you decide whether these experiences meet the needs of your particular class and will also give you some indication of the amount of preparation involved.

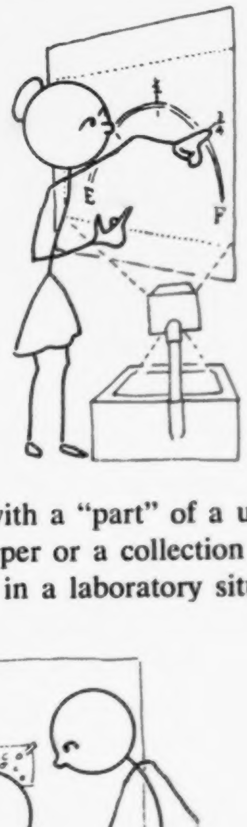
Sample student worksheets and teacher materials are provided in the Teaching Package. It is assumed that duplicating facilities are available to you.

Experience 1: The Hidden "One"

Four stations are established in which activities require the student to focus on the relation between the unit fraction and the unit of reference. For example, the student is told: "Here is a piece of wire that is one-fourth as long as one of the pieces of string in this envelope. Find that piece of string in the envelope."



Materials needed: envelopes, bell wire, toothpick, ribbon, drinking straw, string, graph paper, containers for water, cartons, supply of water, index cards, lima beans, labels, record cards.



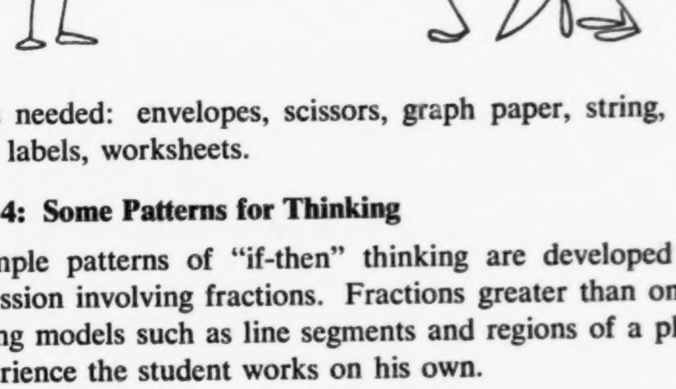
Experience 2: Fractions around Us

Situations from the real world, in which fractions are represented by points on gauges, are the subject of group discussion. Basic properties of fractions are applied in elementary problems.

Materials needed: overhead projector and transparencies (or chalkboard drawings), worksheets.

Experience 3: Focus on "One"

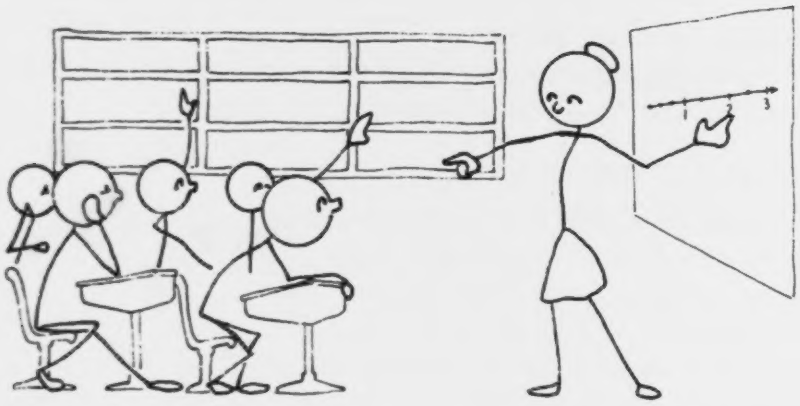
Students are required to identify a fraction with a "part" of a unit that is a physical model such as a piece of graph paper or a collection of lima beans. As in Experience 1, the students work in a laboratory situation.



Materials needed: envelopes, scissors, graph paper, string, plastic jar, lima beans, labels, worksheets.

Experience 4: Some Patterns for Thinking

Some simple patterns of "if-then" thinking are developed through a group discussion involving fractions. Fractions greater than one are dealt with by using models such as line segments and regions of a plane. Later in this experience the student works on his own.

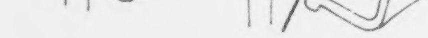


Materials needed: overhead projector and transparency (or chalkboard drawing), worksheets.

Experience 5: Reading and Using Tenths

Decimal fractions are introduced so simply in a group situation that this experience should serve as a confidence builder. The number-line map used as a model presents interesting real-life problems that can be solved by a variety of student strategies.

Materials needed: counter (such as a bicycle odometer or a supermarket Add-a-matic), overhead projector and transparency (or chalkboard drawing), worksheets.



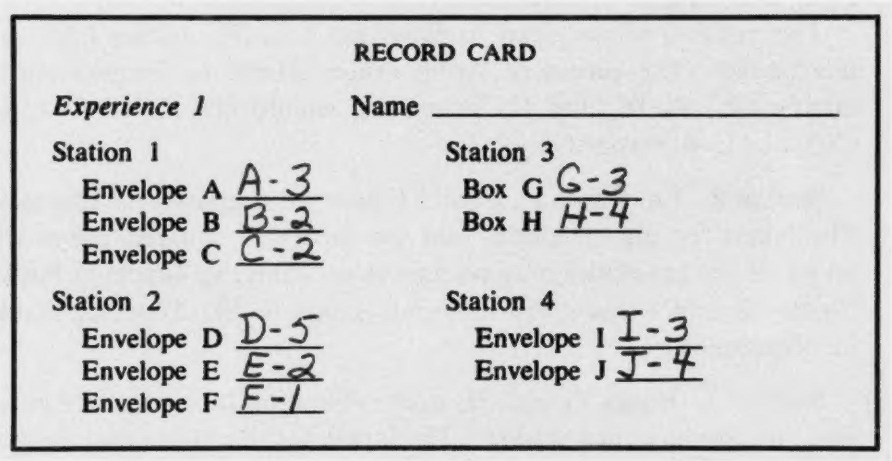
EXPERIENCE 1
The Hidden "One"

OBJECTIVE

When a student is given a piece of string identified by $\frac{1}{3}$ and four other pieces of string which are three, four, five, and six times as long, respectively, as the first piece of string, he should be able to respond correctly to these instructions: This piece of string (identified by 1/5) is one-fifth as long as one of the other pieces. Find that other piece.

MATERIALS

8 envelopes (6 small and 2 large)
28 in. of bell wire
Toothpick
31 in. of ribbon
Drinking straw
59 in. of string
Graph paper (or patterns of squared paper reproduced from the Teaching Package)
4 containers, with capacities of 3, 8, 12, and 32 fl. oz.
6 containers with unspecified capacities
2 cartons, each to hold 4 of the above containers
1 gal. of water, in 2 containers
8 index cards
81 lima beans
10 labels (made from masters in the Teaching Package)
1 record card for each student (from the Teaching Package; see fig. 1)



RECORD CARD

Experience 1 Name ____________

Station 1
Envelope A A-3
Envelope B B-2
Envelope C C-2

Station 3
Box G G-3
Box H H-4

Station 2
Envelope D D-5
Envelope E E-2
Envelope F F-1

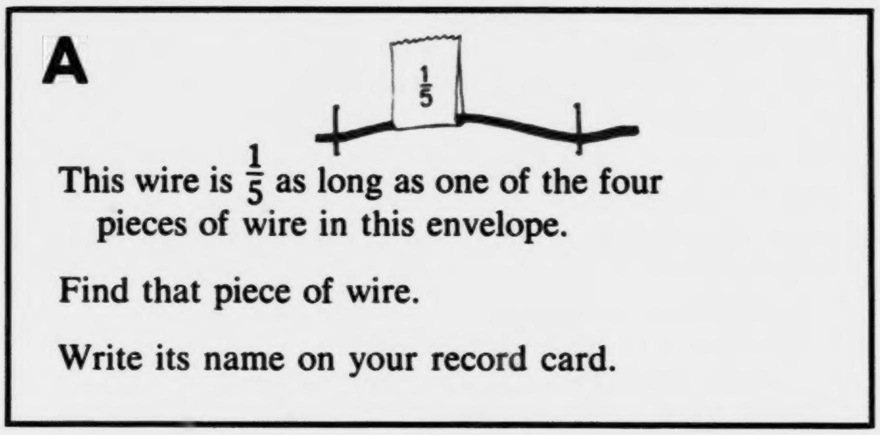
Station 4
Envelope I I-3
Envelope J J-4

Fig. 1

Station 1. Prepare Envelopes A, B, and C by gluing to them the labels reproduced from the Teaching Package.

The wire attached to Envelope A is 1½ inches long and is labeled

⅕ with masking tape, as shown in figure 2. There are four pieces of wire inside the envelope. They have lengths of 5 inches, 6 inches, 7½ inches, and 9 inches, and they are labeled A–1, A–2, A–3, and A–4, respectively.



A

This wire is $\frac{1}{5}$ as long as one of the four pieces of wire in this envelope.

Find that piece of wire.

Write its name on your record card.

Fig. 2

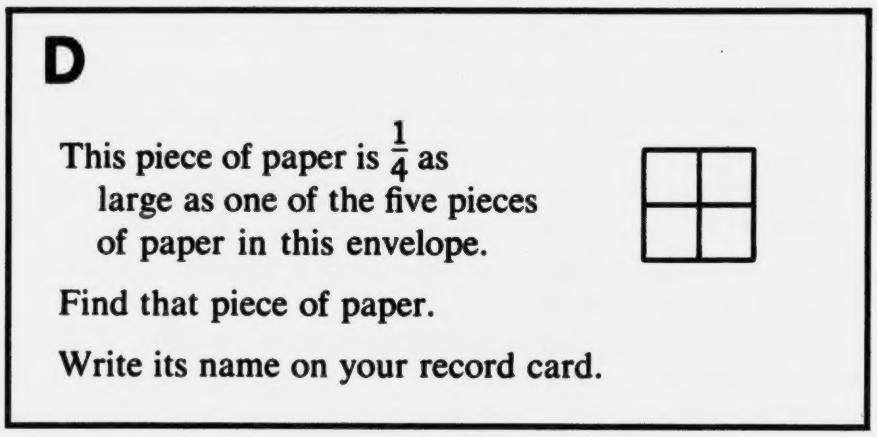
The toothpick stapled to Envelope B is 2¼ inches long and is labeled ⅓ with masking tape. Inside are four pieces of ribbon of lengths approximating 5, 6¾, 9, and 10 inches and labeled B–1, B–2, B–3, and B–4, respectively.

The piece of straw glued to Envelope C is 3½ inches long and is labeled ¼. The pieces of string inside should be lengths approximating 12, 14, 16, and 17 inches and should be labeled C–1, C–2, C–3, and C–4, respectively.

Station 2. Envelopes D, E, and F must be prepared for this station. The labels for the envelopes and the pieces of squared paper which go inside the envelopes may be reproducd from the Teaching Package. Figures 3 and 4 show the materials given in the Teaching Package for Envelope D.

Station 3. Boxes G and H, each with four labeled containers inside, are found at this station. The labels for the boxes may be reproduced from the masters in the Teaching Package.

Outside, but near, Box G is a container labeled G–1 which has a capacity of 3 fluid ounces. Inside the box is a container labeled G–3 with a capacity of 12 fluid ounces. Containers G–2, G–4, and G–5 should have capacities considerably different from 12 ounces—4, 18, and 40 fluid ounces, for example.



D

This piece of paper is $\frac{1}{4}$ as large as one of the five pieces of paper in this envelope.

Find that piece of paper.

Write its name on your record card.

Fig. 3

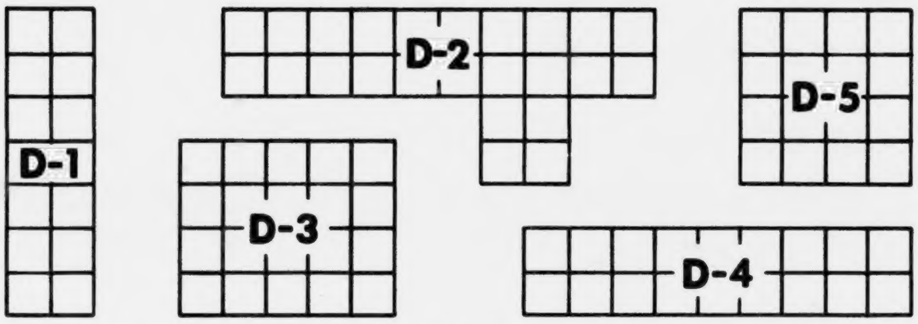


Fig. 4

Box H should be prepared in the same way. H–1, outside the box, has a capacity of 8 fluid ounces; and H–4, inside the box, has a 32-ounce capacity. H–2, H–3, and H–5 should have capacities considerably different from 32 ounces.

A supply of water should be available near each box to encourage students to experiment with volume.

Station 4. At this station are Envelopes I and J with labeled index cards inside. The labels for the envelopes may be reproduced from the Teaching Package.

Envelope I has two lima beans taped or glued securely to the label. Inside are cards labeled I–1, I–2, I–3, and I–4. Attached to the cards are 6, 7, 8, and 9 lima beans, respectively.

Envelope J has three lima beans on the label. The cards are labeled J–1, J–2, J–3, and J–4 and have 7, 10, 12, and 15 lima beans, respectively.

TEACHER STRATEGY

It is essential to the success of this experience that each student become involved in the station activities. He should be encouraged to make his own decisions and to record them on his record card.

This activity approach to learning makes safety precautions especially important. For example, the wire in Envelope A at Station 1 should be pliable, insulated bell wire. At Station 3 you could use paper or plastic cups instead of glass containers.

It is not necessary that students work at the stations in any particular order or that the activities at any one station be performed in a given sequence. When a student finishes the activities at one station, he should move on to any station that is not crowded.

For groups of more than eight students, you should make duplicate sets of materials for each activity so that more than one student can work on the activity at the same time.

EVALUATION

If most of your students respond correctly to seven of the ten questions in this experience, you should be satisfied that the objective has been accomplished.

EXPERIENCE 2

Fractions around Us

OBJECTIVE

The student should be able to identify positions which correspond to $\frac{3}{5}$ and $\frac{1}{4}$ on a gasoline gauge and other calibrated scales.

MATERIALS

Overhead projector and 3 transparencies (made from the master in the Teaching Package)

2-page worksheet for each student

TEACHER STRATEGY

It is intended that this experience be introduced by a group discussion of the ways fractions are used in the "real world." You might ask your students to consider in particular the role fractions play in the use of gauges. Each student should be encouraged to give examples of gauges he has used or knows about and to tell how fractions are used with them.

The two situations described on the following pages provide examples of simple settings in which fractions are commonly used. In presenting them to your students, encourage them to refer to the illustrations as they search for answers. For maximum effectiveness, figures 5, 6, and 7 should be put on overhead-projector transparencies by duplicating the sheet in the Teaching Package. However, you can put all three on a single transparency and mask out the figures not being discussed at the moment, or you can simply draw the figures on the chalkboard.

You may be tempted to develop this experience into a treatment of basic operations on fractions, such as addition or multiplication. Resist this temptation. The emphasis here is on providing many different physical models, or referents, for the fraction concept.

"A Trip to Canada"

For their vacation last summer, the Johnsons decided to drive to Canada for a fishing trip. They started out with a full tank of gasoline. [Place fig. 5 on the overhead projector and have someone show where the needle on the gauge is when the tank is full.] After they had traveled 280 miles, the gasoline gauge showed that the tank was $\frac{1}{4}$ full. [Have someone show where the needle on the gauge would be now.]

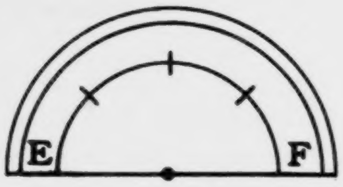


Fig. 5

When they pulled up to a service station, Mr. Johnson told the attendant to put in four dollars' worth of gasoline. The gauge now showed that the tank was $\frac{3}{4}$ full. [Show this.]

By the time they reached the fishing lake in Canada, the gauge showed that the tank was $\frac{1}{3}$ full. [Show this.]

The Johnsons stayed in a cabin by the lake. The cabin's stove burned fuel oil. Peter Johnson checked the gauge on the fuel-oil tank [place fig. 6 on the overhead projector] and found that it showed $\frac{1}{2}$ full. [Show this.] Peter noticed that this gauge also showed how many gallons were in the tank. How many gallons are in the tank when it is full? [Have someone

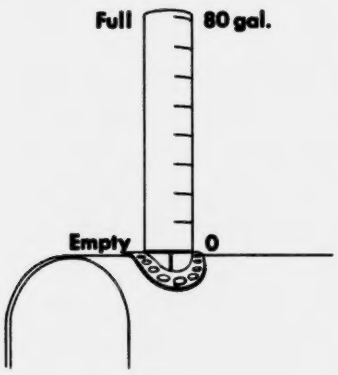


Fig. 6

point to "80."] How many gallons are in the tank when it is ½ full?

Before the week was over, Mrs. Johnson noticed that the gauge was down to ⅛. [Show this.] She asked Peter, "How many gallons of oil do we have left?" Mr. Johnson decided to have the tank filled. [Show this.]

When it was time to leave for home the gauge was down to ⅝. How many gallons were still in the tank?

Using his observations in reading the gauge, Peter made up the partial table shown below. Can you complete it? [This table might be copied on the chalkboard.]

Fraction	Gallons
⅛	10
2/8	?
?	50
¼	?
6/8	?
?	70

"Raising Money for the School Band"

The Band Mothers at City High had a fund drive to help send the band to the state fair. Terry's mother was chairman of the fund drive. She made a large chart on which she could show how well the drive was progressing. She called this chart her "success gauge." [Place fig. 7 on the overhead projector, or put it on the bulletin board, using strips of colored paper to show the various degrees of success.]

After the first day, Terry reported to the band director that the drive was off to a "good start."

[*Continued on page 147*]

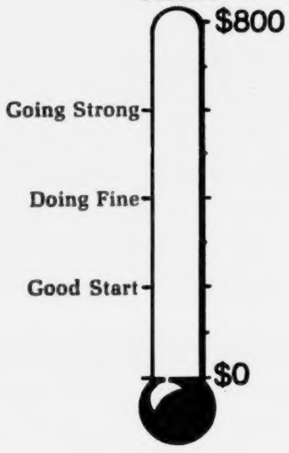


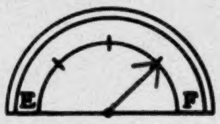
Fig. 7

FRACTIONS AROUND US UNIT 5, EXPERIENCE 2

Name ______________________________

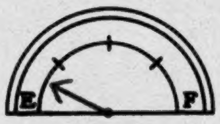
Draw an arrow on each gauge to show the situation described below the gauge.

1.



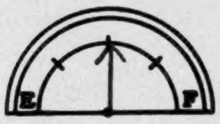
$\frac{3}{4}$ full

2.



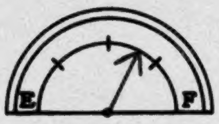
$\frac{1}{8}$ full

3.



$\frac{4}{8}$ full

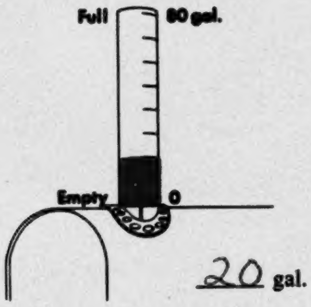
4.



$\frac{5}{8}$ full

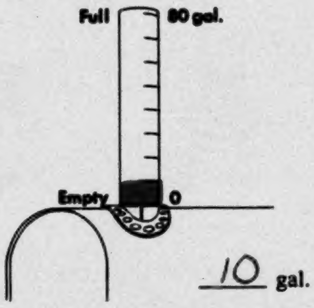
Shade each gauge to show the situation described at the left of it, then record the number of gallons in each case.

5. $\frac{1}{4}$ full


Full
80 gal.
Empty
0

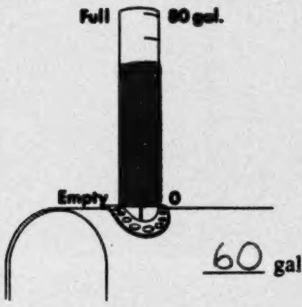
20 gal.

6. $\frac{1}{8}$ full


Full
80 gal.
Empty
0

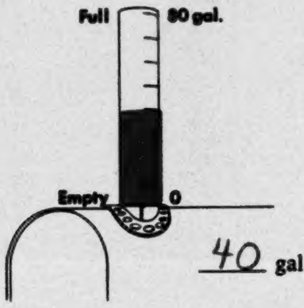
10 gal.

7. $\frac{3}{4}$ full


Full
80 gal.
Empty
0

60 gal.

8. $\frac{3}{6}$ full


Full
80 gal.
Empty
0

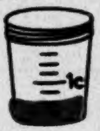
40 gal.

FRACTIONS AROUND US **UNIT 5, EXPERIENCE 2**

Name ______________________

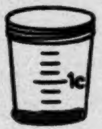
Shade in the indicated level in each of these peanut butter jars to show the part filled with peanut butter.

9.

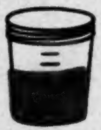


$\frac{1}{2}$ cup

10.



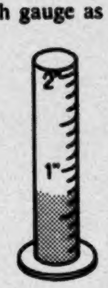
$\frac{1}{4}$ cup



$1\frac{1}{4}$ cup

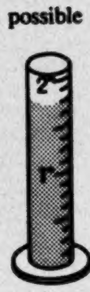
Shade each rain gauge pictured below to show the amount of rainfall indicated directly below it.

12.



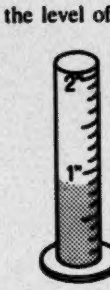
Mon. $\frac{1}{2}$"

13.



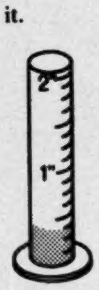
Tues. $\frac{5}{8}$"

14.



Wed. $1\frac{1}{2}$"

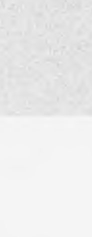
15.



Thurs. $1\frac{3}{8}$"

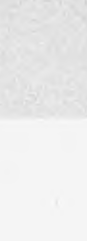
Read each gauge as carefully as possible and write the level of water below it.

16.



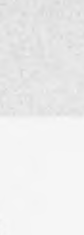
$\frac{6}{8}$" or $\frac{3}{4}$"

17.



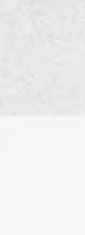
$1\frac{6}{8}$" or $1\frac{3}{4}$"

18.



$\frac{7}{8}$"

19.



$\frac{3}{8}$"

The director replied, "Good! That means we are ⅕ of the way to our goal of $800."

Was the director correct? What fraction would have been the correct response?

The students at City High had a car wash and bake sale to raise money for the drive. The next day Terry noticed that the success gauge was at the halfway point. Would Terry have said they were "off to a good start," "doing fine," or "going strong"? [Have someone show this on the success gauge.] How much money had they collected?

What fraction of the way to the goal was the fund drive when $600 had been collected?

Two weeks later they had reached ⅞ of the goal. How much money had they collected then?

After this discussion hand out the two-page worksheet for Experience 2.

EVALUATION

The individual work on the worksheet will give you a measure of how well your students can identify points that represent fractions on various calibrated lines.

EXPERIENCE 3

Focus on "One"

OBJECTIVE

Given a one-by-three-inch piece of graph paper, the student should be able to cut off a piece one-third the size of the original piece.

MATERIALS

1 large envelope and 3 regular envelopes for each student
6 regular envelopes for use at the stations
Scissors (several pairs)
Graph paper
String (about 3 ft.)

2-qt. plastic jar
24 lima beans for each student
9 labels (made from masters in the Teaching Package)
1 worksheet for each student

Station 1. Envelopes A, B, and C are found at this station. Each envelope should contain a piece of graph paper, of the size shown in figure 8, for each student. Labels for the envelopes can be reproduced from the Teaching Package.

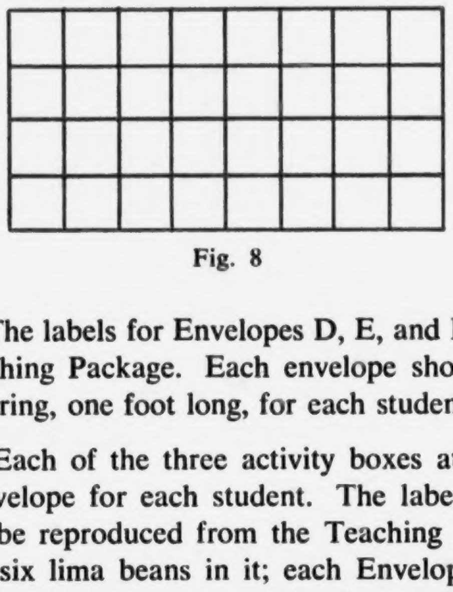


Fig. 8

Station 2. The labels for Envelopes D, E, and F can be reproduced from the Teaching Package. Each envelope should contain at least one piece of string, one foot long, for each student.

Station 3. Each of the three activity boxes at this station should contain an envelope for each student. The labels for Envelopes G, H, and I can be reproduced from the Teaching Package. Each Envelope G has six lima beans in it; each Envelope H has eight lima beans, and each Envelope I has ten lima beans. The envelopes should be closed with paper clips so they can be opened and reclosed without tearing.

TEACHER STRATEGY

Students should be encouraged to work individually on the worksheet and at each of the three stations in this experience, but you will need to help those who have difficulty reading and following the instructions. It will take much ingenuity and effort on your part to reduce the anxieties of those meeting difficulties. Praise the student for trying, attempt to clarify the instructions for him, and encourage him to try again. Plenty of materials should be available at each station so that several trials and errors can be accommodated.

[*Continued on page 150*]

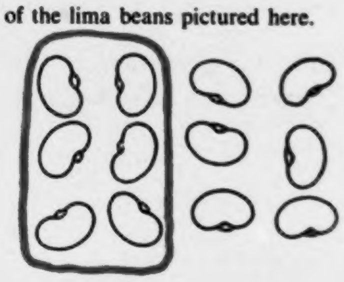
FOCUS ON "ONE"

UNIT 5, EXPERIENCE 3

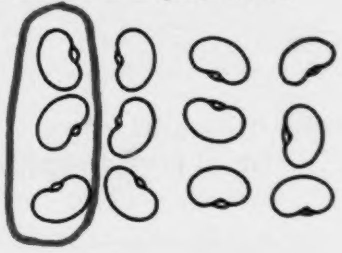
Name ______________________________

Put this sheet in your envelope when you have finished it.

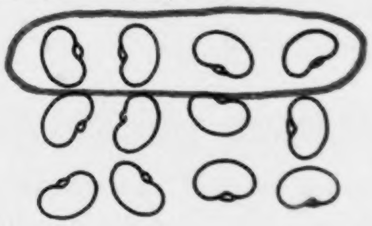
1. Draw a ring around $\frac{1}{2}$ of the lima beans pictured here.



2. Draw a ring around $\frac{1}{4}$ of the lima beans pictured here.



3. Draw a ring around $\frac{1}{3}$ of the lima beans pictured here.



The activities need not be done in any particular order. The large envelope is to be carried by the student throughout the entire experience. He should put his name on it and collect objects in it as he moves from station to station. He should also put the worksheet for this experience in it.

Watch to make sure that the directions are followed carefully—as a student leaves a station it should be ready for others. If more than ten students are participating in the activities, you will need to make duplicates of some or all of the envelopes.

EVALUATION

Check the contents of students' envelopes at the end of this experience to determine how well they have been able to carry out the directions.

EXPERIENCE 4

Some Patterns for Thinking

OBJECTIVE

Given an example in which a physical referent for one is marked into thirds, the student should be able to identify points such as $\frac{1}{3}$ and $2\frac{1}{3}$.

MATERIALS

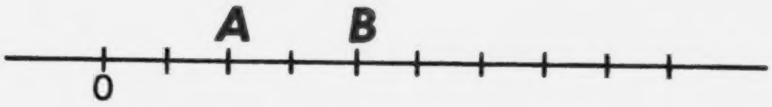
Overhead projector and 1 transparency (made from the master in the Teaching Package)

8 worksheets for each student

TEACHER STRATEGY

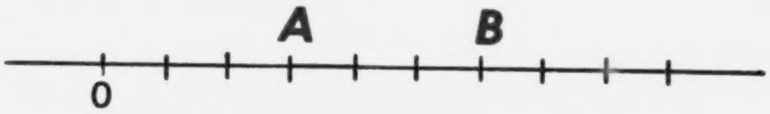
This experience establishes "if-then" patterns of thinking by group discussion of various examples. This type of thought process is then used in the activities to focus on the fraction concept.

You can introduce the experience by using first a number line, then a lattice of points, as the physical referent. In giving the number line as an example, indicate the points without using the letter identifications needed here.



Draw a number line on the board and, indicating first the point at *A* and then the point at *B*, say: "If I name this point '1,' what would I name this point?" The answer, of course, is "2." Then, indicating the points for 3, ½, and so on, ask "What would I name this point?"

Leave the same number line on the board but erase the numerals you have written on it except for the zero.

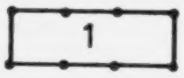


Indicating *A* and *B* respectively, ask: "If I name this point '1,' what would I name this point?" Again the answer is "2." Now, indicating ⅓, 2⅔, and so on, you can ask questions such as "What would I name this point?" and "Where would 2⅔ be?"

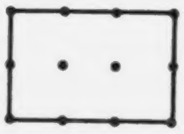
In this manner you can continue calling different points "1" and asking students to assign names of fractions to various other points. You may make this phase of the experience quite short or expand it into a full class period, depending on the capability of your students and the amount of previous work they have had with the number line as the referent for fractions.

Next, using the lattice of points as the referent for one, lead a discussion something like the following. (You may prefer to substitute a chalkboard drawing for the transparency made from the modified version of this discussion in the Teaching Package.)

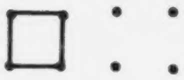
If we agreed to name this figure "1,"



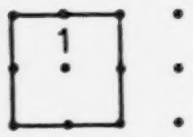
what name would we give to this?



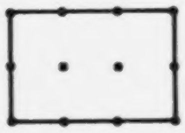
To this?



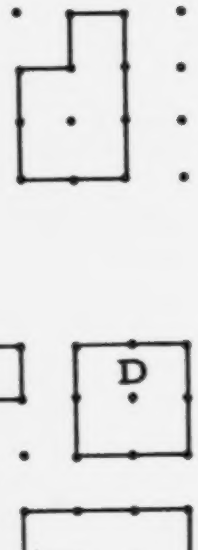
Now suppose we name *this* figure "1."



What would we name this?



And this?

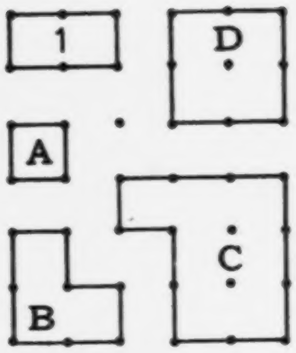


Everyone take out a sheet of scratch paper.

Suppose we name *this* figure "1."

What names would we give each of these others?

Write them down.



Keep in mind that this experience does not involve precise measurement, although many students may wish to measure the objects supplied in the activities. Keep the focus on the fraction concept by using only those examples that are simple enough to be handled by counting.

Encourage your students to feel at ease about assigning the number "one" to the many different sorts of models serving as the unit—the length of string, the piece of graph paper, the set of lima beans, and so on. It may take some students several days to grasp this basic concept of unit.

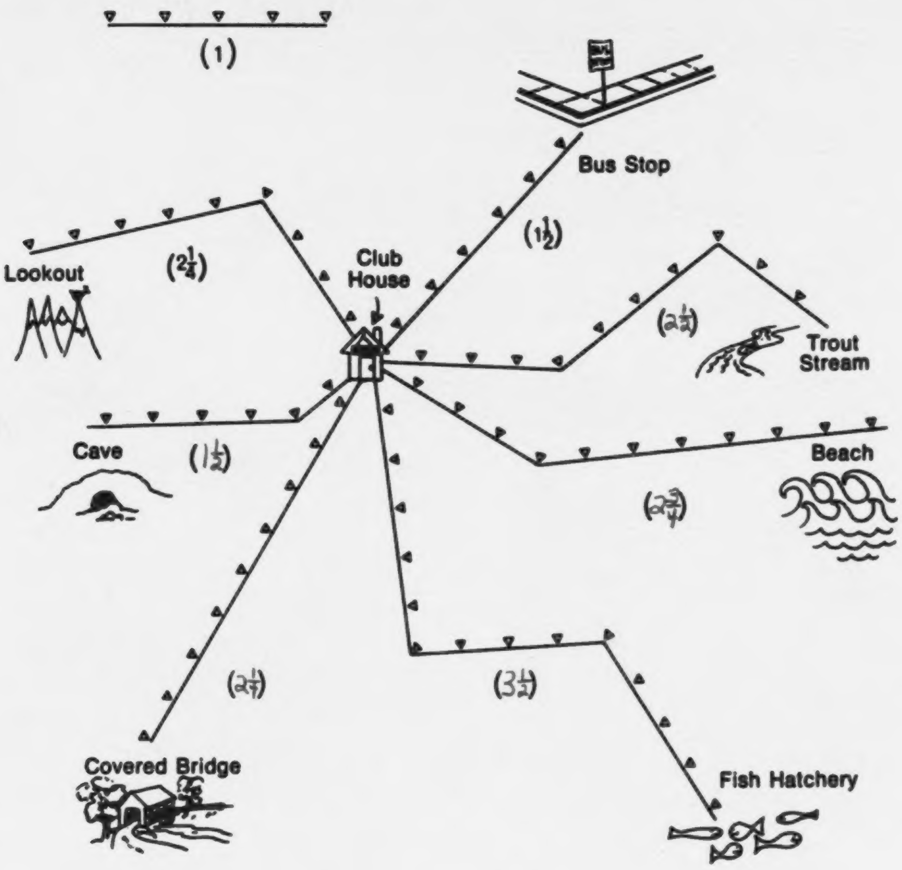
The rest of this experience involves your students in individual work

[*Continued on page 161*]

SOME PATTERNS FOR THINKING UNIT 5, EXPERIENCE 4

Activity 1 Name ________________________________

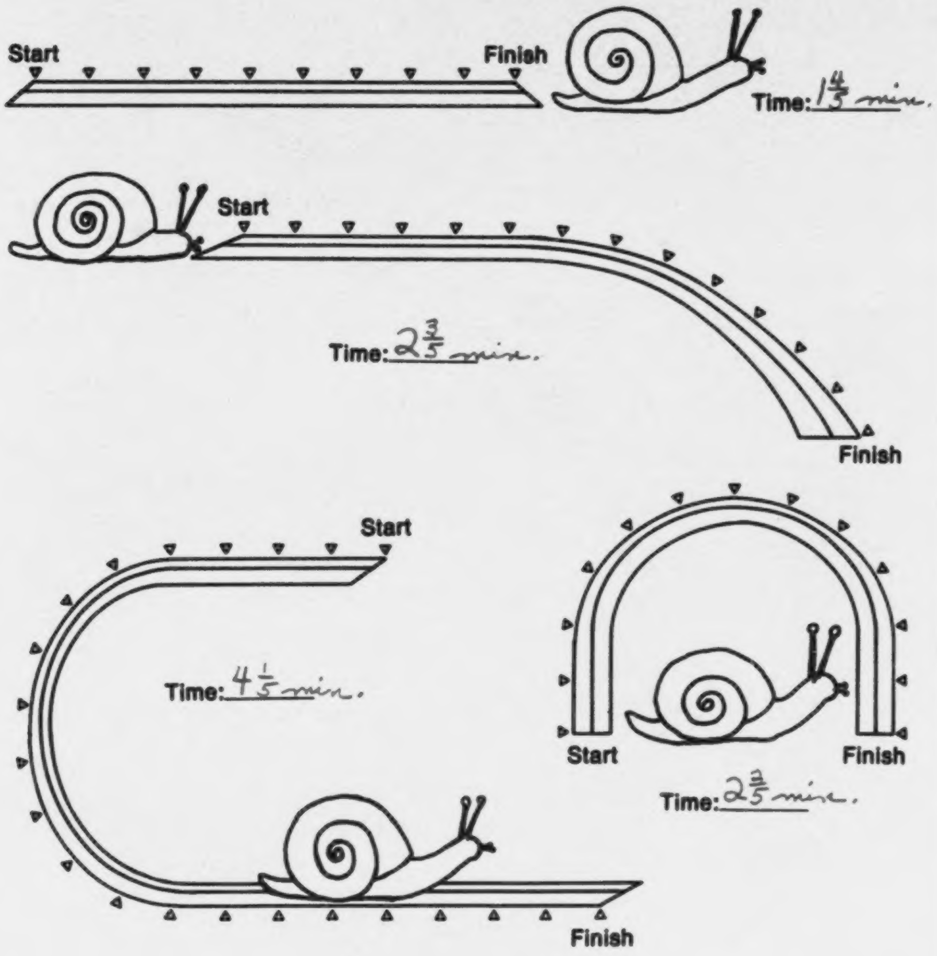
The Paul Bunyan Hiking Club names its hikes with numbers that indicate how many hours they are expected to take. A 4-mile hike is named "1" because it takes about 1 hour. This system is used for all hikes of 20 miles or less. The map below shows popular hikes with the mileposts marked ▽. Two of the hikes are already named. Name the others.


(1)
Bus Stop
(1½)
Lookout
(2¼)
Club House
(2½)
Trout Stream
Cave
(1½)
Beach
(2¾)
(2¼)
(3½)
Covered Bridge
Fish Hatchery

SOME PATTERNS FOR THINKING UNIT 5, EXPERIENCE 4

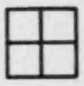
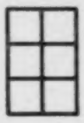
Activity 2 Name ______________________________

Samson, the trained racing snail, crawled at the rate of 5 centimeters in 1 minute. Four of the racetracks he crawled on are shown below. Centimeters are marked ▽. Write his time for each track on the blank provided.

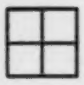

Start
Finish
Time: $1\frac{4}{5}$ min.
Start
Time: $2\frac{3}{5}$ min.
Finish
Start
Time: $4\frac{1}{5}$ min.
Finish
Start
Finish
Time: $2\frac{3}{5}$ min.

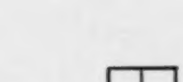
SOME PATTERNS FOR THINKING UNIT 5, EXPERIENCE 4

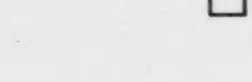
Activity 3 Name ______________________

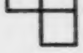
If  is 1, then  is $1\frac{1}{2}$.

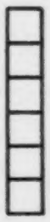
Complete:

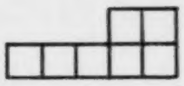
If  is 1, then

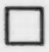
 is $2\frac{1}{2}$

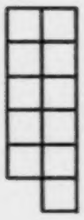
 is $3\frac{1}{4}$

 is $\frac{3}{4}$

 is $1\frac{1}{2}$

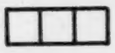
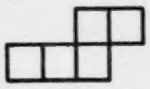
 is $1\frac{3}{4}$

 is $\frac{1}{4}$

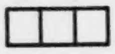
 is $2\frac{3}{4}$

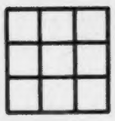
SOME PATTERNS FOR THINKING **UNIT 5, EXPERIENCE 4**

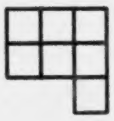
Activity 4 Name ______________________

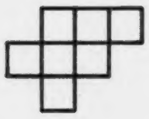
If  is 1, then  is $1\frac{2}{3}$.

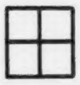
Complete:

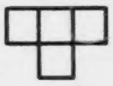
If  is 1, then

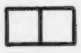

is 3


is $2\frac{1}{3}$


is $2\frac{1}{3}$

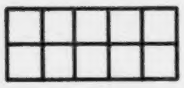
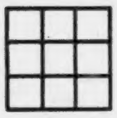

is $1\frac{1}{3}$


is $1\frac{1}{3}$

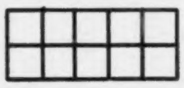

is $\frac{2}{3}$

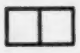
SOME PATTERNS FOR THINKING UNIT 5, EXPERIENCE 4

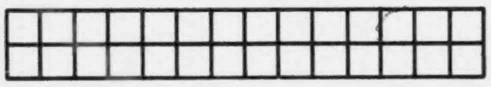
Activity 5 Name ______________________________

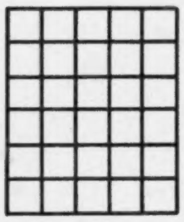
If  is 1, then  is $\frac{9}{10}$.

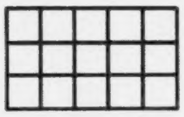
Complete:

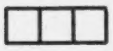
If  is 1, then

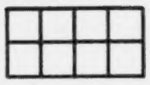
 is $\frac{2}{10}$

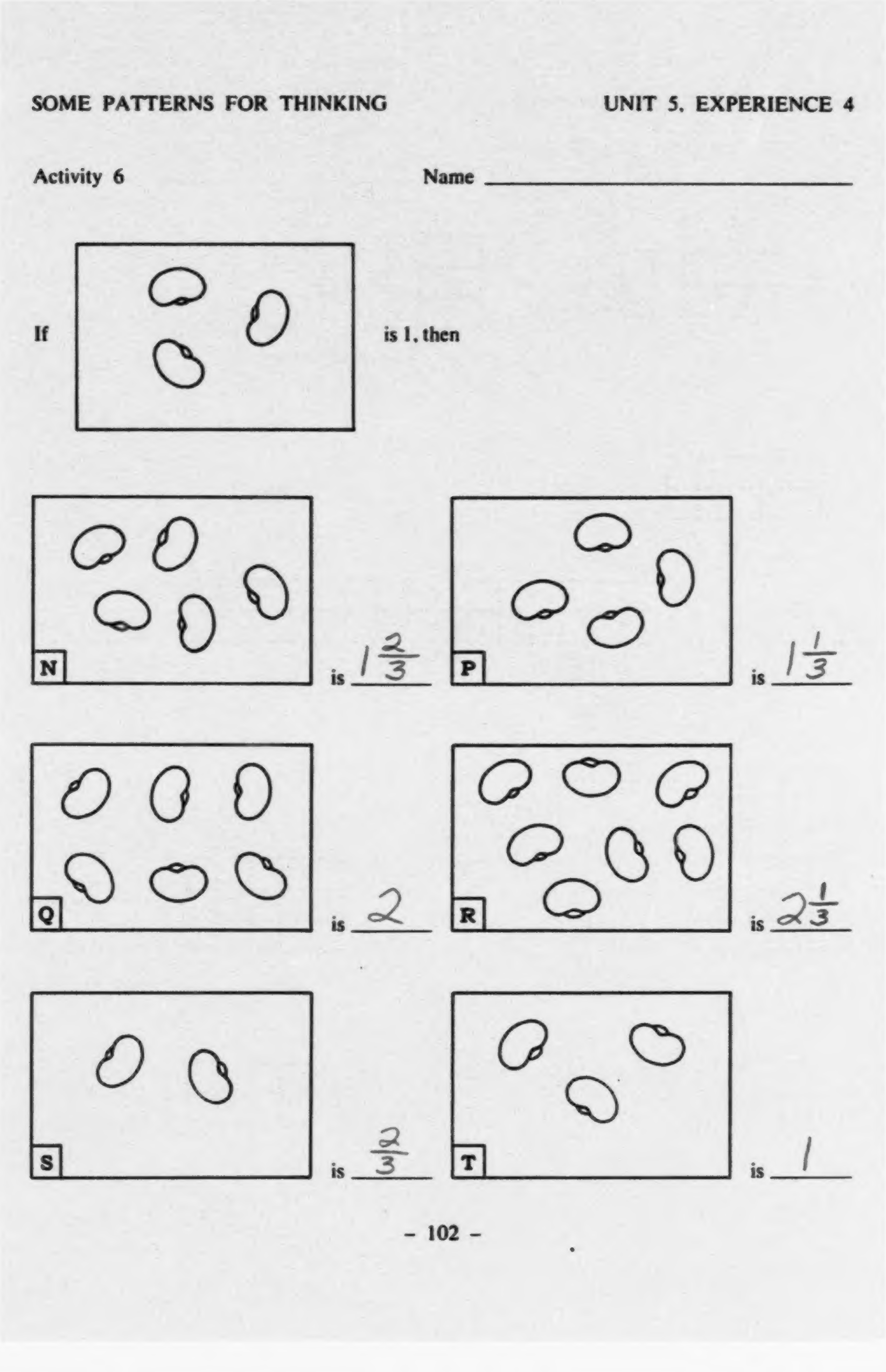
 is $2\frac{8}{10}$

 is 3

 is $1\frac{5}{10}$

 is $\frac{3}{10}$

 is $\frac{8}{10}$



SOME PATTERNS FOR THINKING UNIT 5. EXPERIENCE 4

Activity 6 Name ________________

If is 1, then

N is $1\frac{2}{3}$

P is $1\frac{1}{3}$

Q is 2

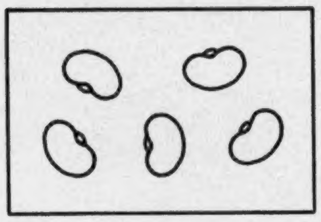
R is $2\frac{1}{3}$

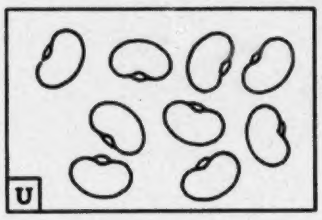
S is $\frac{2}{3}$

T is 1

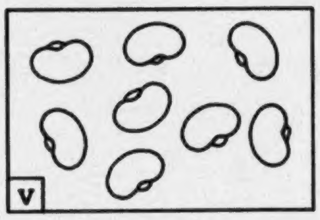
Activity 7

Name ______________________

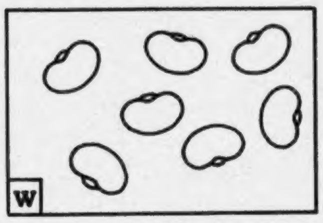
If  is 1, then



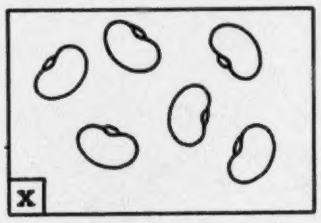
is $1\frac{4}{5}$,



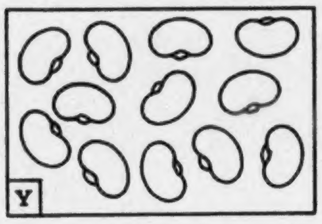
is $1\frac{3}{5}$



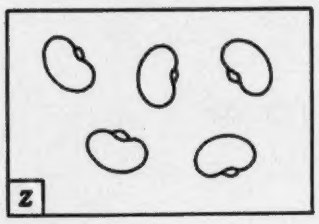
is $1\frac{2}{5}$



is $1\frac{1}{5}$



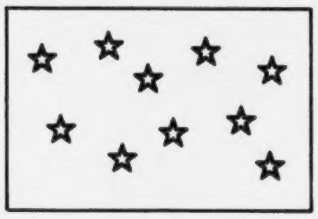
is $2\frac{2}{5}$

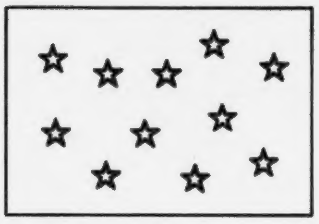


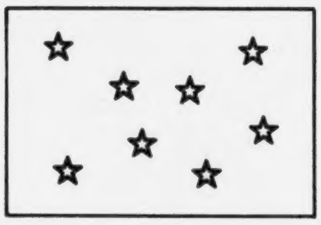
is 1

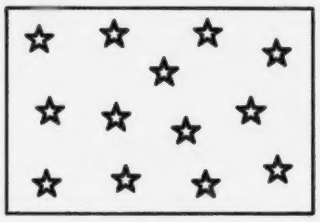
SOME PATTERNS FOR THINKING UNIT 5, EXPERIENCE 4

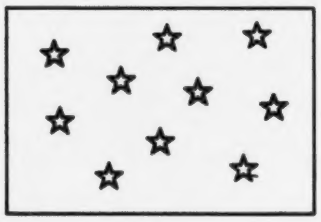
Activity 8 Name ______________________

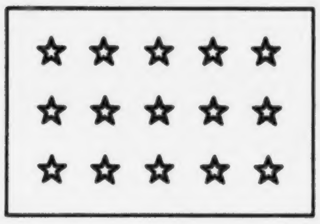
If  is 1, then

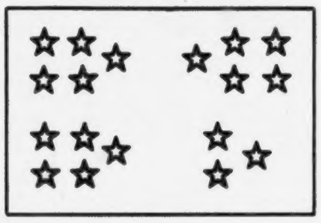

is $1\frac{1}{10}$


is $\frac{8}{10}$


is $1\frac{3}{10}$


is 1


is $1\frac{5}{10}$


is $1\frac{8}{10}$

on eight activity sheets. You may want to place copies of the worksheets, with answers inserted, around the room for students to use as keys to check their work.

EVALUATION

Your students should be able to complete at least the first five worksheets in one class period.

EXPERIENCE 5

Reading and Using Tenths

OBJECTIVE

Given a number line calibrated in tenths, with zero and one designated, the student should be able to identify correctly any point within the given segment of the number line.

MATERIALS

Counter (such as a bicycle odometer or a supermarket Add-a-matic) or a tagboard model of an odometer

Overhead projector and 1 transparency (made from the master in the Teaching Package)

4 worksheets for each student

TEACHER STRATEGY

Since this experience is intended to be a confidence builder, try to be as nondirective as possible and thereby encourage the student to decide for himself what techniques will provide him with easy ways to succeed.

Identification of the decimal fraction with the physical referents—the counter and the number-line map—is basic to the student's ability to relate decimal fractions to the world in which he lives. The transition from the physical counter to the number line is first carried out in a relatively simple situation and then solidified by the four activities, which utilize the number line in analyzing map situations.

The group discussion introducing the experience should be very uncomplicated. Some type of decimal counter would provide a valuable visual aid. Possibilities include a bicycle mileage gauge, a "shopper's counter," and a hand counter such as the kind often used in school cafeterias to tally the number served. If none of these are available, a tagboard model of an odometer would be a satisfactory substitute. The use of a counter as a referent or model provides a link between the physical situation to which the decimal fraction relates and the relatively abstract symbol that is imposed on the number line.

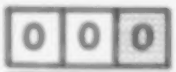
Each activity sheet should be worked on independently, if possible, by the students and then discussed in detail before the class proceeds to the next worksheet.

Activity 2 is included here to avoid the inference that the decimal numeration system is the system used in map interpretation. Activity 3 can be expanded easily by identifying other points of interest and generating questions related to them. There are many interesting subtleties connected to Activity 4, and it is critical that you be as nondirective as possible. Students will evidence a variety of strategies in attacking these problems.

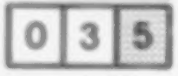
You may introduce the experience with discussions like the following. (You could either make a transparency from the modified version of this discussion in the Teaching Package or use chalkboard sketches of the illustrations to accompany your commentary.)

Tod got a new bicycle with a mileage gauge, an odometer, on it. He decided to find out how far it was from his home to the city park.

He set his gauge to look like this when he started:

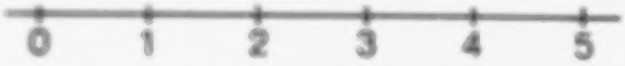


When he got to the park his gauge looked like this:



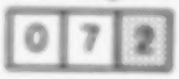
How many miles had he gone? [Seek 3.5, $3\frac{5}{10}$, and $3\frac{1}{2}$ as correct responses.]

Where would we find that number on this number line:

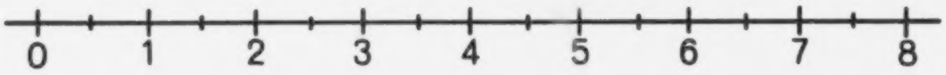


[Have someone mark the point and name it.]

Tod took a longer route home. When he got there his gauge looked like this:



How many miles had he gone? Where would the number be on this number line?



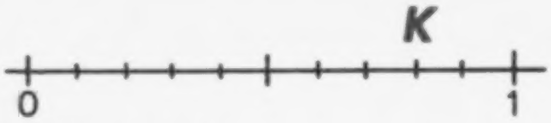
[Have someone mark the point and name it—precision is relatively unimportant.]

Bob and Jack have delivery routes after school. They argued about whose route was the longer. To find out, each set his odometer at zero when he started his route and wrote down the reading when he finished.

Bob's gauge read 6.2 when he finished, and Jack's read 5.9.

Find each of these numbers on the number line [above]. Which boy's route was longer? Jack said the difference was less than half a mile. Was he correct? How do you know?

Kim said she checked the distance from her home to school. She showed the class the distance by marking *K* on this number line:



How far did she live from school?

The students should now be ready to work individually on the four worksheets.

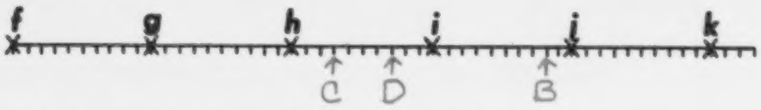
EVALUATION

All students should be able to complete the first three activity sheets in one class period, and the very best students should make some progress in Activity 4.

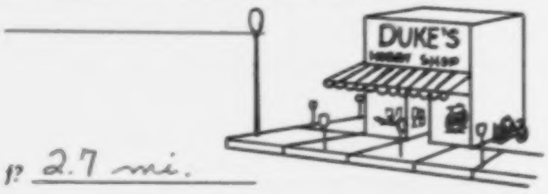
READING AND USING TENTHS **UNIT 5, EXPERIENCE 5**

Activity 1 Name ____________________

In Central City 10 blocks equal 1 mile. The "map" below shows 5.3 miles of Main Street. The mileposts are marked with an × and given a letter name. Each block is shown by the marks between the mileposts.

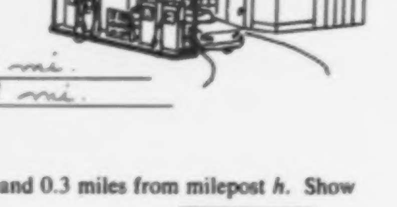


1. Duke's Hobby Shop is 2.3 miles from milepost *k*. Show where his shop is by putting a D on the map.



How far is Duke's from milepost *f*? 2.7 mi.

2. Bob's service station is 3.8 miles from milepost *f*. Show where it is by putting a B on the map.



How far is Bob's station from milepost *k*? 1.2 mi.
How far is Bob's station from Duke's shop? 1.1 mi.

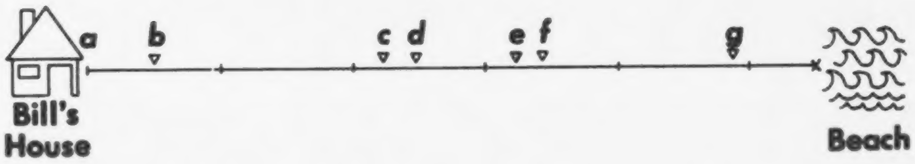
3. Cora's Beauty Shop is 1.7 miles from milepost *j* and 0.3 miles from milepost *h*. Show where her shop is by putting a C on the map.

BEAUTY SHOP

How far is Cora's shop from Duke's? .4 mi.
How far is Cora's shop from Bob's station? 1.5 mi.

READING AND USING TENTHS UNIT 5, EXPERIENCE 5

Activity 2 Name ______________________________



The "map" shows that the beach is more than 5 blocks from Bill's house, which is marked *a*. Jack's house is marked *b*. Jack's house is $\frac{1}{2}$ block from Bill's. Tom's house is between Bill's and the beach. It is $2\frac{1}{2}$ blocks from Bill's.

Which point locates Tom's house? d

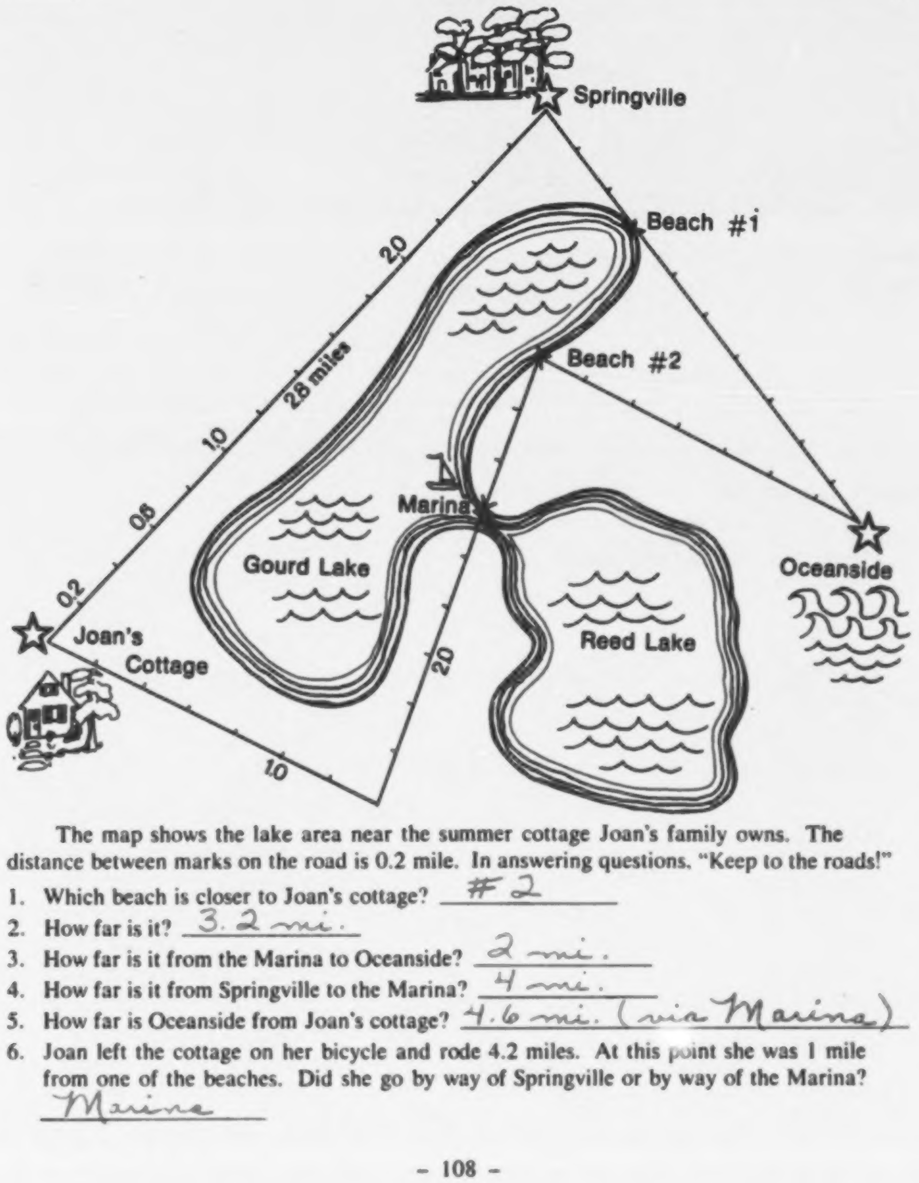
The Candy Kitchen, a favorite hangout, is $3\frac{1}{4}$ blocks from Bill's (on the way to the beach).

Which of the points shown on the map is nearest the Candy Kitchen? e

Which point shown on the map is 3 blocks from Jack's house? f

READING AND USING TENTHS **UNIT 5, EXPERIENCE 5**

Activity 3 Name ______________________________


Springville
Beach #1
Beach #2
Oceanside
Marina
Gourd Lake
Reed Lake
Joan's Cottage
2.0
2.8 miles
1.0
0.6
0.2
2.0
1.0

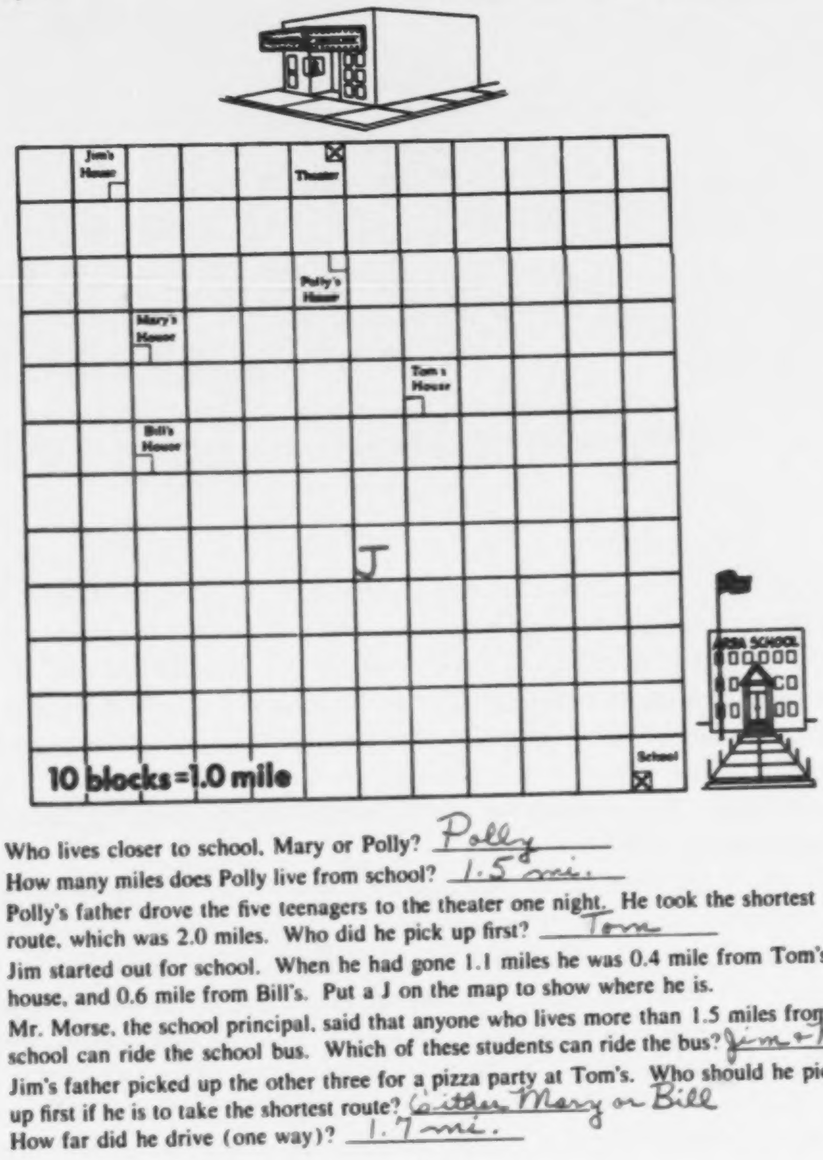
The map shows the lake area near the summer cottage Joan's family owns. The distance between marks on the road is 0.2 mile. In answering questions, "Keep to the roads!"

1. Which beach is closer to Joan's cottage? #2
2. How far is it? 3.2 mi.
3. How far is it from the Marina to Oceanside? 2 mi.
4. How far is it from Springville to the Marina? 4 mi.
5. How far is Oceanside from Joan's cottage? 4.6 mi. (via Marina)
6. Joan left the cottage on her bicycle and rode 4.2 miles. At this point she was 1 mile from one of the beaches. Did she go by way of Springville or by way of the Marina?
Marina

READING AND USING TENTHS UNIT 5, EXPERIENCE 5

Activity 4

Name ______________________


Jim's House
Theater
Polly's House
Mary's House
Tom's House
Bill's House
J
AREA SCHOOL
School
10 blocks = 1.0 mile

1. Who lives closer to school, Mary or Polly? Polly
2. How many miles does Polly live from school? 1.5 mi.
3. Polly's father drove the five teenagers to the theater one night. He took the shortest route, which was 2.0 miles. Who did he pick up first? Tom
4. Jim started out for school. When he had gone 1.1 miles he was 0.4 mile from Tom's house, and 0.6 mile from Bill's. Put a J on the map to show where he is.
5. Mr. Morse, the school principal, said that anyone who lives more than 1.5 miles from school can ride the school bus. Which of these students can ride the bus? Jim + Mary
6. Jim's father picked up the other three for a pizza party at Tom's. Who should he pick up first if he is to take the shortest route? Either Mary or Bill
How far did he drive (one way)? 1.7 mi.

6

EXPERIENCES WITH

Physical Models for Decimals

The four experiences included in this unit involve the student in a wide variety of laboratory-oriented activities that focus on the unique properties of those fractions called decimal fractions. It is assumed that the student has had some introduction to the decimal symbol but that he lacks a basic understanding of the number idea represented by the symbol—that he is a member of the subset of the class which was unsuccessful with the regular curriculum. The unit can be used profitably with an entire class, however, to reinforce previous learning.

Each of the first three experiences begins with a very basic activity at the student's desk. After he finishes this he proceeds to one of the various stations. (The order in which they are visited is unimportant.) The final activity for each of these experiences is also done at the student's desk; however, if less able students do not have time to work on it, they should still be able to succeed in subsequent experiences. This organization allows you to give individual help to students who need it as they work on Activity 1, and it also provides valuable, stimulating, independent learning experiences for the more able of the group.

The fourth experience is entirely laboratory-oriented, with a focus on fractional parts of a set of one hundred.

Some of these experiences may require more than one class period.

Your judgment on the feasibility and effectiveness of postexperience discussions will be a factor in determining how long the unit actually takes.

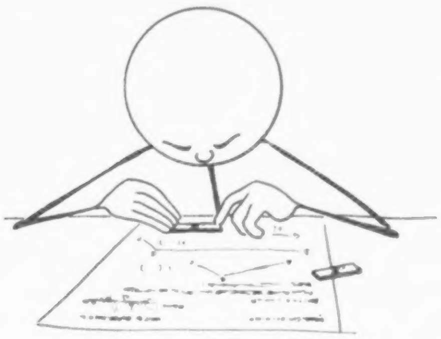
OVERVIEW

Each of the four experiences in this unit includes a detailed section called "Teacher Strategy," which is based on classroom-tested procedures. The following brief summary of each experience will help you decide whether or not that experience satisfies the needs of your group. The necessary materials for each experience are listed. Much of the software for the students is to be made by reproducing pages provided in the Teaching Package. It is assumed that duplicating facilities are available.

Experience 1: The Concept and the Numeral

The first experience utilizes the student's familiarity with elementary measurement and the number line to associate decimal numerals with the concept of tenths. The last activity introduces hundredths.

Materials needed: balsa strips (or plastic straws), beakers or cylindrical glass jars, tagboard, rulers, envelope, worksheets.



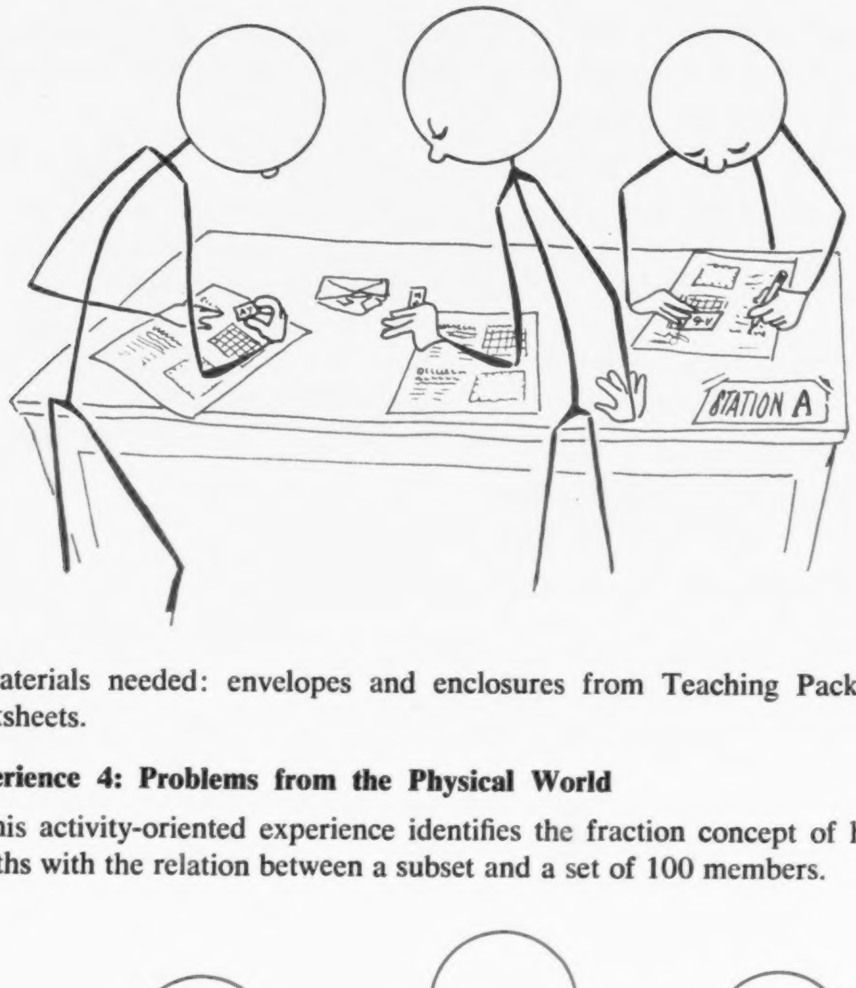
Experience 2: Problem Solving

The student's concept of the decimal fraction is enlarged by physical models. These models use the additive characteristics of the decimal system to focus on the fraction concept. Though hundredths are used in the culminating activity, the primary focus is on tenths.

Materials needed: balsa strips, envelopes, crayons, worksheets.

Experience 3: Area Model for Hundredths

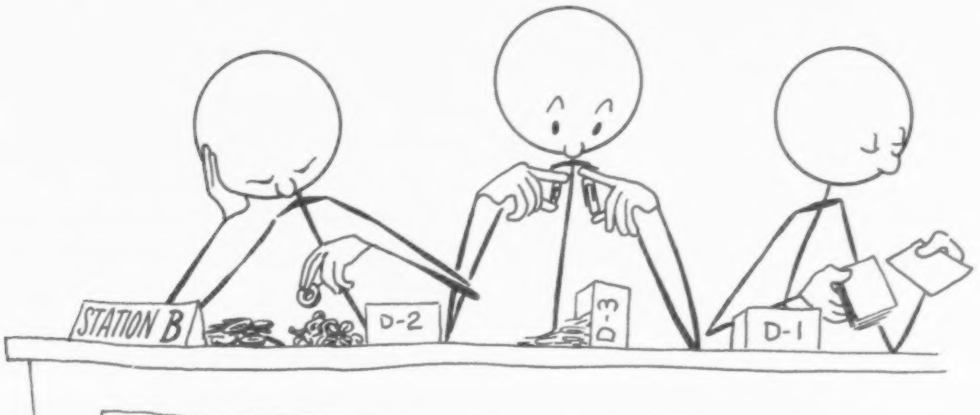
The concept of hundredths is the major focus of this experience. Several activities use the multiplication model as it relates to area.



Materials needed: envelopes and enclosures from Teaching Package, worksheets.

Experience 4: Problems from the Physical World

This activity-oriented experience identifies the fraction concept of hundredths with the relation between a subset and a set of 100 members.



Materials needed: books, newspapers, telephone books, index cards, metal washers, paper clips, small boxes, worksheets.

EXPERIENCE 1
The Concept and the Numeral

OBJECTIVE

Given a scale marked in tenths of a unit, the student should be able to measure an object and use a decimal symbol to record its length; for example, he would record the length of an object one and three-tenths units long as 1.3.

MATERIALS

Balsa strip or plastic drinking straws (about 18 in. for each 5 students in the class)
Envelopes (1 for each 5 students)
5 400-mm beakers or substitutes (duplicate this set if there are more than 20 students in the class)
Tagboard
Rulers (patterns from 3 sheets in Teaching Package)
4 worksheets for each student

Station A. In an envelope place 5 balsa strips cut to lengths of 2.4, 4.6, 3.7, 5.2, and 0.8 inches and labeled A, B, C, D, and E, respectively. To cut these strips to the exact length, place the wood on the patterns given in the Teaching Package and cut it with a single-edge razor blade.

Station B. If five 400-mm beakers are available from the science department of your school, label them from A through E. Make scales from the patterns given in the Teaching Package for tenths of a cup and cups. Using one of these as a gauge, pour colored liquid into the beakers as follows: A, 13 tenths of a cup; B, 11 tenths of a cup; C, 15 tenths of a cup; D, 6 tenths of a cup; and E, 10 tenths of a cup.

If beakers are not available, you may substitute 5 identical cylindrical jars with straight vertical sides, such as olive jars. Scales for these jars can be made and calibrated in cups, then in tenths of a cup.

Making and Calibrating Scales

Put one jar on a level surface and pour into it exactly one cup of liquid. Place a blank strip of tagboard as shown in figure 1 and mark the strip to indicate the 1-cup level.

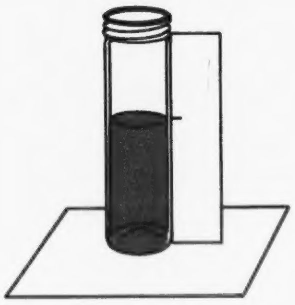


Fig. 1

Position the tagboard strip on the set of parallel lines in figure 2 so that the 0 mark is on the 0 line and the 1-cup mark is on the 10 line. Mark the tagboard strip at each point where the parallel lines intersect the strip.

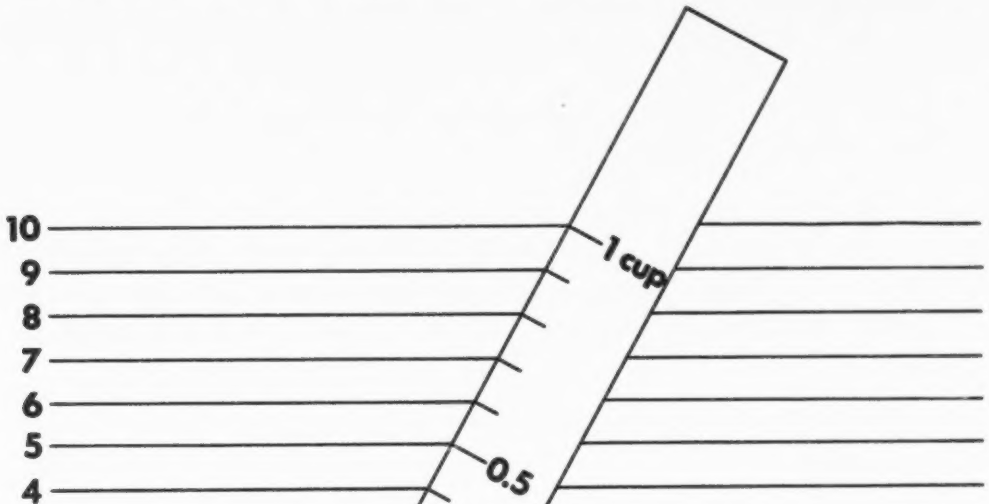


Fig. 2

Extend this scale to 1.5 cups, then make copies of it for use with the other four jars. Using the same measurements, make five scales marked in tenths of a cup. These are to be used with the jars in the same way the scales shown in the Teaching Package are to be used with the 400-millimeter beakers.

TEACHER STRATEGY

This experience builds upon your students' previous work with measurement to help them to identify decimal numerals with the length or height of physical objects. The act of matching the measurement line to the object and reading the scale is critical to understanding the relative order of the fractions involved. The relation between centimeters and decimeters is incidental—important only in that it helps focus on the fraction concept of tenths.

In all activities you should encourage but not require independence on the part of the learner. The two parts of Activity 1 are to be done in order, as seatwork. Each student is given copies of the worksheets for the activity. The focus is on recognizing that 37 centimeters and 3.7 decimeters name segments of equal length. Spot-check each student's work before he goes on to Station A or B, and don't hesitate to help those who need it. Care must be taken to assure a high degree of pupil success with this initial activity.

Activities 2 and 3, performed at the two stations located in the room, need not be taken in that order, but it is essential in each case that the *first* part be done before the *second* part. Activity 2 requires the student to find lengths in tenth-inch units and then in inches and tenths of an inch. Objects must be matched to the appropriate scale on the worksheet. Activity 3 imposes the decimal concept on capacity measure.

There should be no more than five students working at Station B. If necessary, supply duplicate materials at this station.

Activity 4 is seatwork that introduces the decimal symbol for hundredths. The choice of hundredths of a foot is based on the assumption that most children will have had considerable experience with the foot as a unit. Future lessons will expand the student's referents for hundredths. The less able student may not have time to do Activity 4, but this will not seriously jeopardize his chances for success in subsequent experiences.

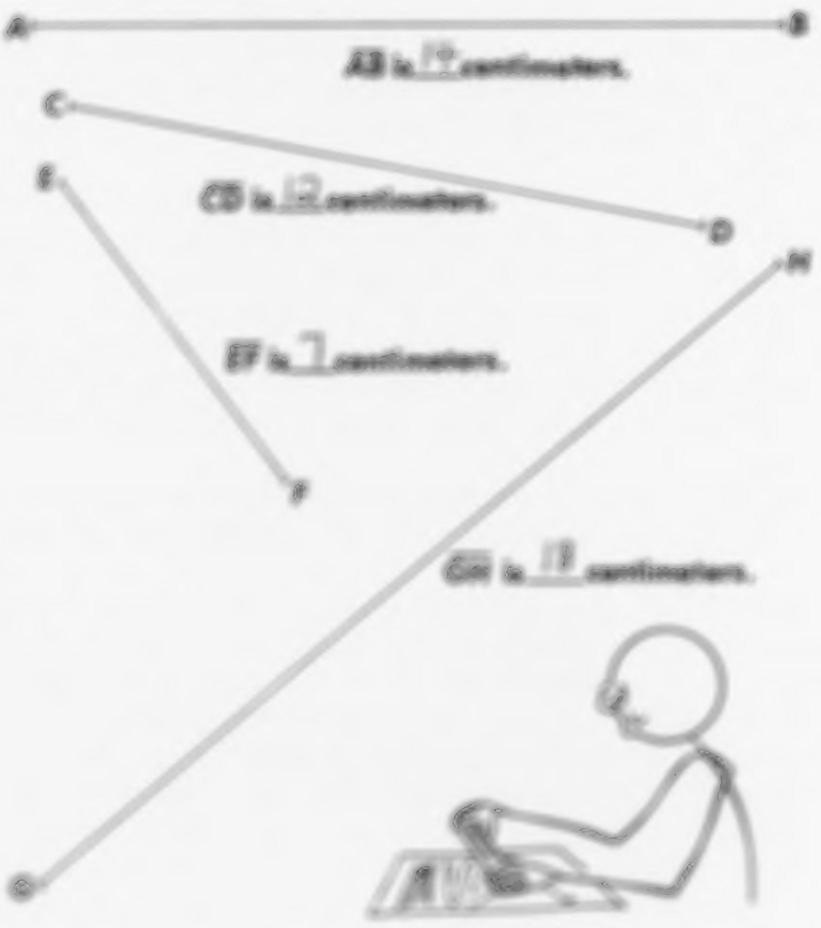
A game called "Complete the Square" is played as one of the beginning activities of Experience 2. It might be convenient to introduce this game

[*Continued on page 179*]

THE CONCEPT AND THE NUMERAL UNIT 6, EXPERIENCE 1

Activity 1, Part 1 Name ______________________

The ruler for this activity is marked off in centimeters. Use it to measure the length of each of these line segments. Write the lengths in the blanks.



$\overline{AB}$ is 19 centimeters.

$\overline{CD}$ is 12 centimeters.

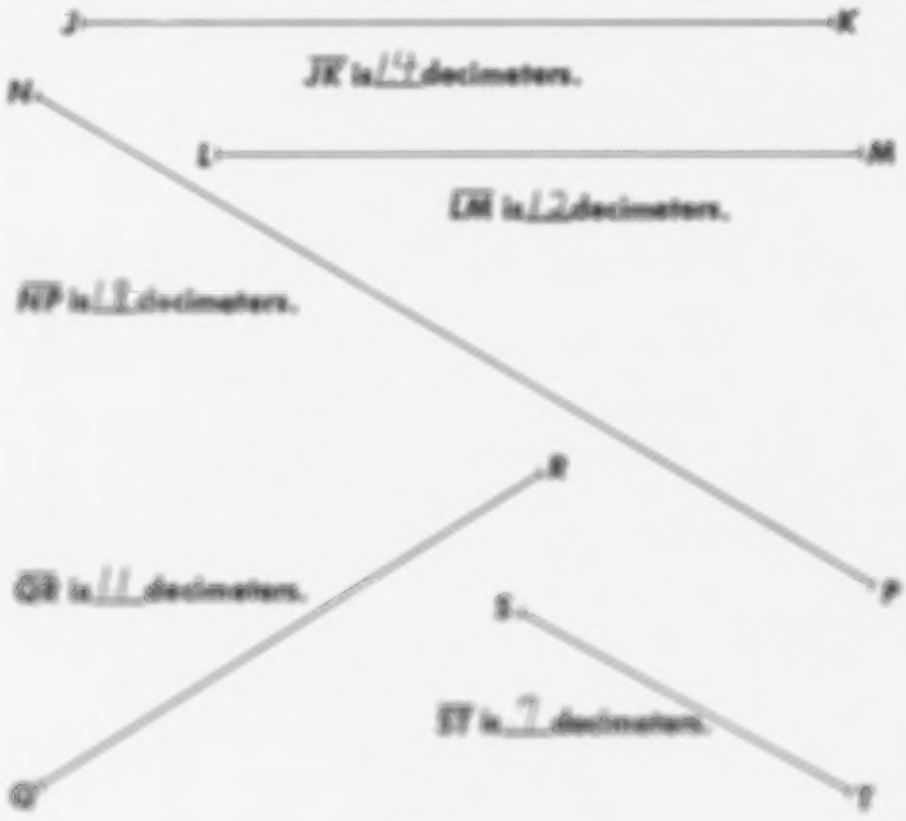
$\overline{EF}$ is 7 centimeters.

$\overline{GH}$ is 19 centimeters.

THE CONCEPT AND THE NUMERAL UNIT 6, EXPERIENCE 1

Activity 1, Part 2 Name ______________________

The ruler for this activity is marked off in decimeters. Use it to measure these line segments.



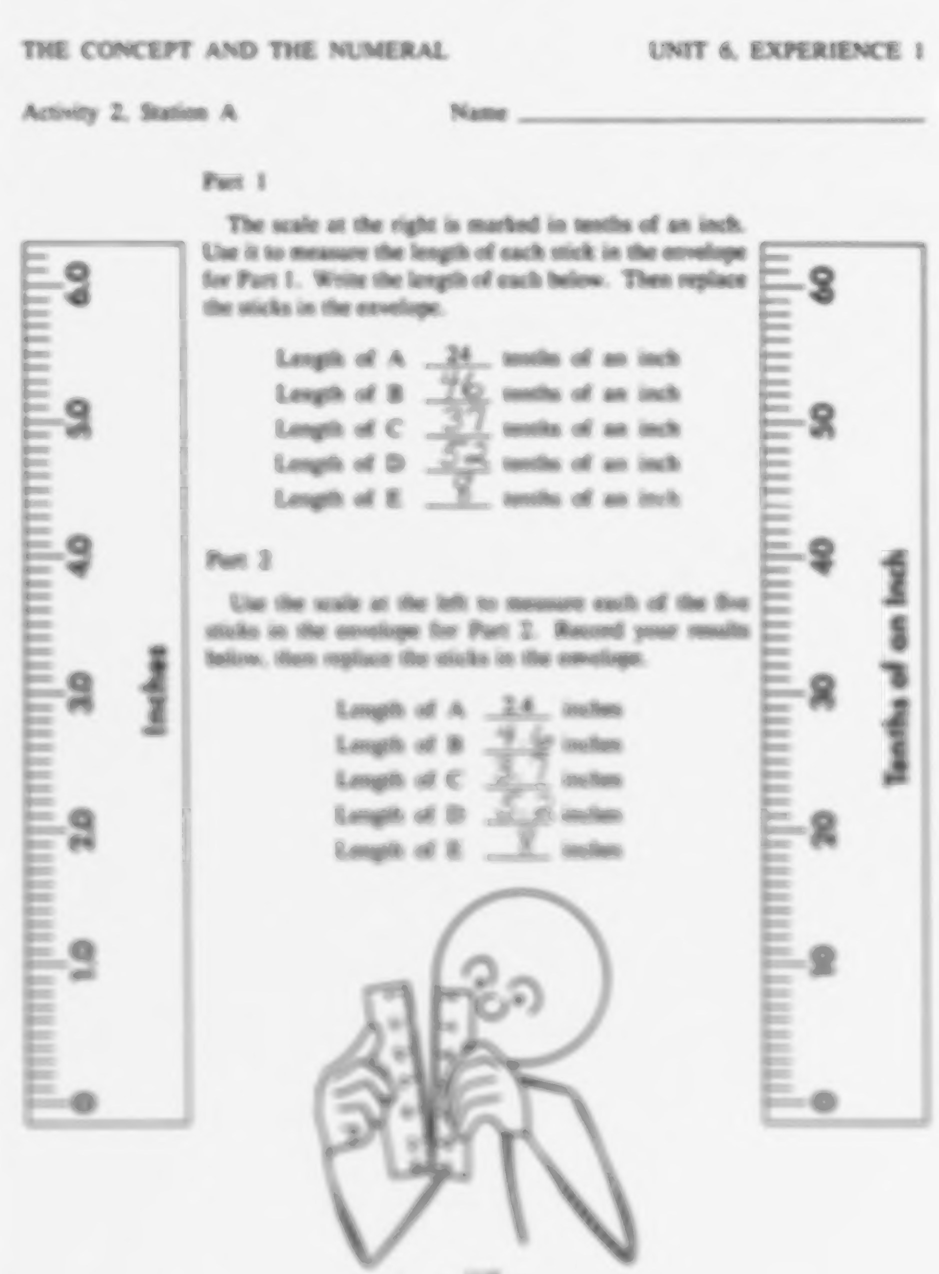
$\overline{JK}$ is 14 decimeters.

$\overline{LM}$ is 12 decimeters.

$\overline{NP}$ is 13 decimeters.

$\overline{QR}$ is 11 decimeters.

$\overline{ST}$ is 7 decimeters.



THE CONCEPT AND THE NUMERAL UNIT 6, EXPERIENCE 1

Activity 2, Station A Name ______________________

Part 1

The scale at the right is marked in tenths of an inch. Use it to measure the length of each stick in the envelope for Part 1. Write the length of each below. Then replace the sticks in the envelope.

Length of A 24 tenths of an inch
Length of B 46 tenths of an inch
Length of C 37 tenths of an inch
Length of D 25 tenths of an inch
Length of E 9 tenths of an inch

Part 2

Use the scale at the left to measure each of the five sticks in the envelope for Part 2. Record your results below, then replace the sticks in the envelope.

Length of A 2.4 inches
Length of B 4.6 inches
Length of C 3.7 inches
Length of D 2.5 inches
Length of E .9 inches

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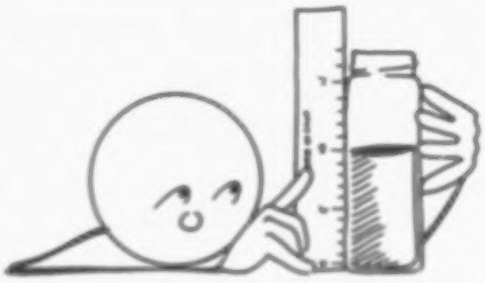
THE CONCEPT AND THE NUMERAL UNIT 6, EXPERIENCE 1

Activity 3, Station B Name ______________________

Part 1

Use the ruler for tenths of a cup to determine the number of tenth-cups in each jar at this station. Write your results below.

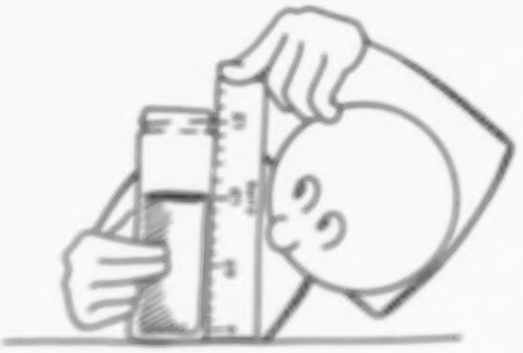
Jar A 13 tenth-cups
Jar B 11 tenth-cups
Jar C 15 tenth-cups
Jar D 6 tenth-cups
Jar E 10 tenth-cups



Part 2

Use the scale for cups to determine the number of cups in each jar at this station. Write your results below.

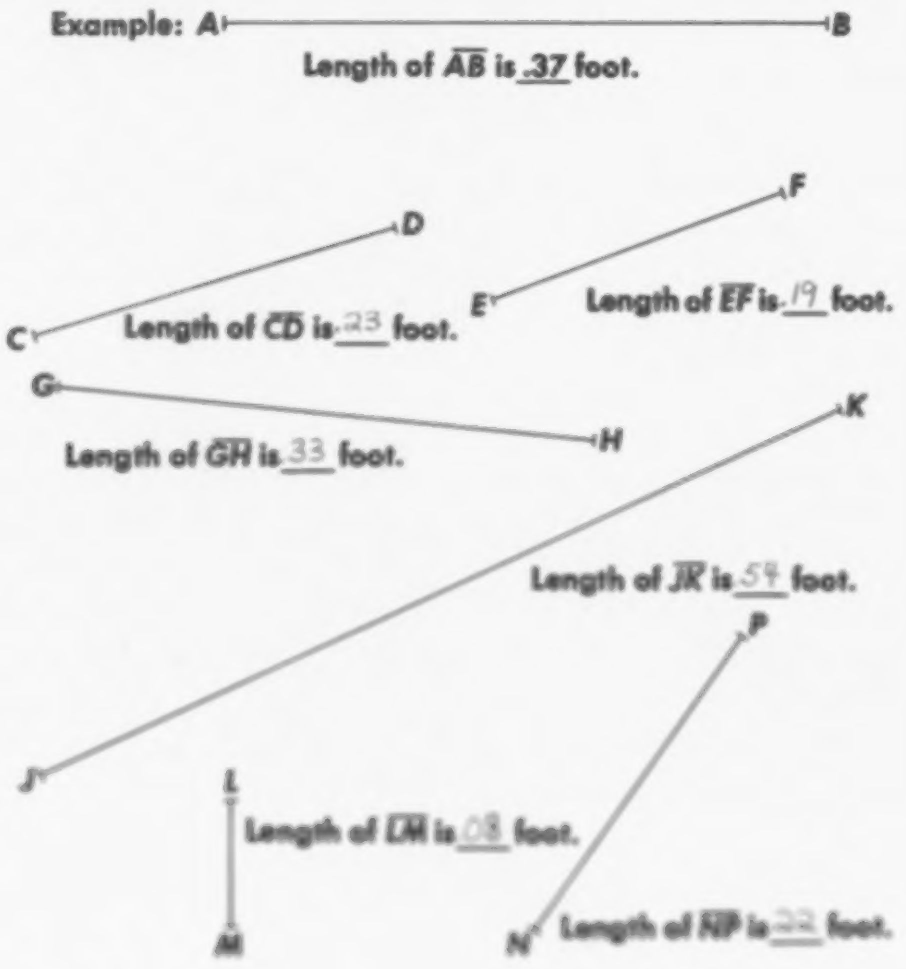
Jar A 1.3 cups
Jar B 1.1 cups
Jar C 1.5 cups
Jar D .6 cups
Jar E 1.0 cups



THE CONCEPT AND THE NUMERAL UNIT 6, EXPERIENCE 1

Activity 4 Name ____________________

The ruler is marked in hundredths of a foot. Use it to measure these line segments, following the example below, which measures 37 hundredths of a foot.



Example:

Length of $\overline{AB}$ is .37 foot.

Length of $\overline{CD}$ is .23 foot.

Length of $\overline{EF}$ is .19 foot.

Length of $\overline{GH}$ is 33 foot.

Length of $\overline{JK}$ is 54 foot.

Length of $\overline{LM}$ is 08 foot.

Length of $\overline{NP}$ is 22 foot.

now to some students while others are completing Activity 3 or Activity 4. Directions are given under Experience 2.

EVALUATION

If each student is able to name correctly at least three of the five measurements in Part 2 of either Activity 2 or Activity 3, the experience is a success. It is hoped, of course, that most students do considerably better.

EXPERIENCE 2

Problem Solving

OBJECTIVE

Given a stick marked .4 and another stick marked .7, the student should be able to indicate correctly a line segment that would be marked .3 and a different segment that would be marked 1.1.

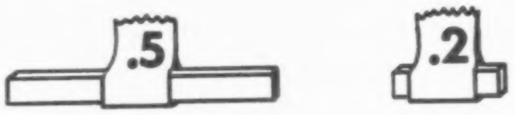
MATERIALS

Balsa strip (15 in. long), or plastic drinking straws
4 envelopes for each 6 students or fractional part of 6
Green and red crayons
6 worksheets for each student

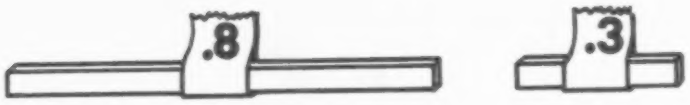
Stations. At each station there is an envelope marked with the same letter as the station. Each envelope contains 2 strips of balsa labeled with masking tape as shown in figure 3 and as drawn to scale in the patterns in the Teaching Package. Five inches represents one unit of measurement for the first three stations; 2½ inches represents one unit for the fourth.

TEACHER STRATEGY

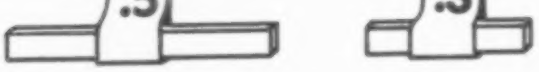
This experience is a necessary stage in the extension of the student's concept of tenths. It also gives him a second brief exposure to the hundredths concept. Physical models of decimals are manipulated by the student to solve problems that focus on the additive nature of the decimal



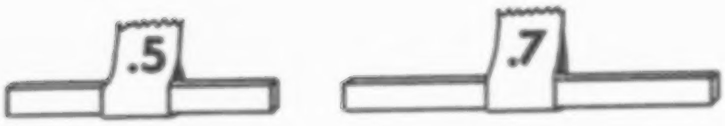
Envelope A



Envelope B



Envelope C



Envelope D

Fig. 3

numeration system. These activities give him a meaningful referent for the order, or relative size, of the decimal fractions.

In all activities you should encourage but not require independence on the part of the student.

If the students have not played "Complete the Square," they should be taught the game now. They should be given an opportunity to play it themselves before they begin Activity 1.

Complete the Square

To demonstrate how the game is played, draw on the chalkboard the array of dots shown in figure 4. Explain that the game is for two players and that the goal is to complete as many squares as possible. A turn consists of drawing a horizontal or vertical line segment; but if a player is able to complete a small square with his segment, he writes his initial in the square and takes another turn. The winner of the game is the player who completes the most squares.

In giving the students an example, point to the dots and add the line segments as illustrated, but without using the letter identifications for points and segments which are needed here.

Suppose that the game has begun and the lattice now looks as it does in figure 5. It's B's turn. If he draws the line segment $\overline{QR}$, player A will

Fig. 4

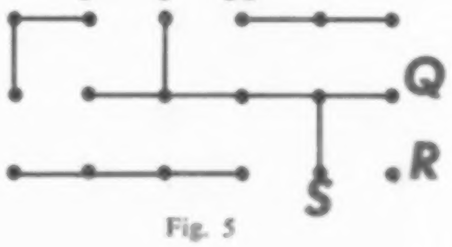


Fig. 5

probably draw $\overline{RS}$. Since A must draw one more segment, he would wisely draw $\overline{VT}$. Now the game looks like figure 6.

B will undoubtedly complete squares 2 and 1 and then draw $\overline{TW}$. This makes it possible for A to complete squares 3, 4, and 5 but forces him to draw a segment that will allow B to complete squares 6, 7, 8, and 9. The final lattice looks like figure 7. B wins, 6 to 4.

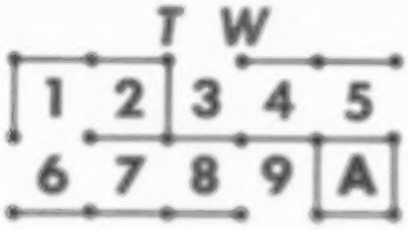


Fig. 6

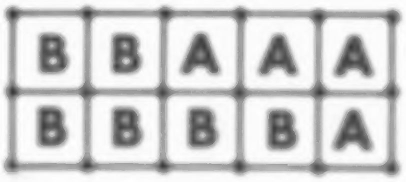


Fig. 7

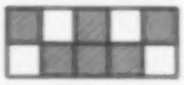
Activity 1, of which Complete the Square is a part, is designed to fix the tenths concept with a geometric area as referent. (Presenting seatwork first will allow the faster workers to be finishing the station activities before the slower students begin them.)

[Continued on page 188]

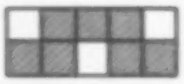
PROBLEM SOLVING **UNIT 6, EXPERIENCE 2**

Activity 1 Name ______________________

Answer the questions in the spaces provided.

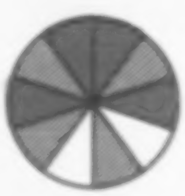


1. Six-tenths (.6) of the first rectangle is shaded. What fractional part of the rectangle is unshaded? .4



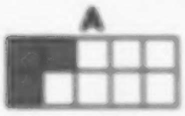
2. What fractional part of the second rectangle is unshaded? .7

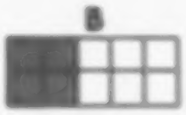
Shade 4 more parts of this circle green.

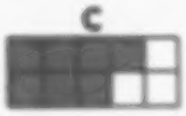


3. What fractional part of the circle is shaded either black or green? .8
4. What fractional part of the circle is not shaded? .2

Color 3 small squares of A, 4 small squares of B, and 7 small squares of C.

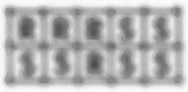






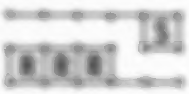
5. If A, B, and C are each considered to be 1.0, what fraction describes the 14 colored parts? 1.4
6. What fraction describes the uncolored parts? 1.6

Rita and Sally played a game of Complete the Square with the result shown here.



7. What fractional part of the squares did Rita complete? .4
8. What fractional part of the squares did Sally complete? .6

In another game, the results looked like this after each one had played 9 times:



9. What fractional part of the squares has been completed? .4

PROBLEM SOLVING

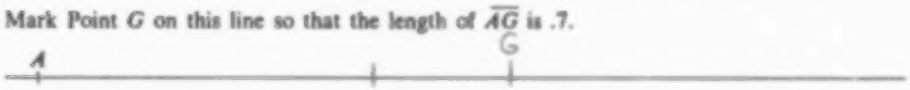
UNIT 6, EXPERIENCE 2

Name ______________________________

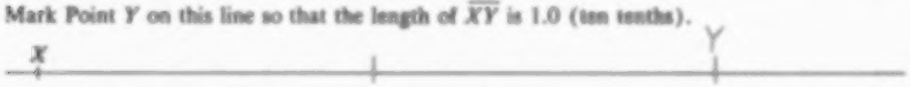
Activity 2, Station A

Use the sticks inside Envelope A for the following activities:

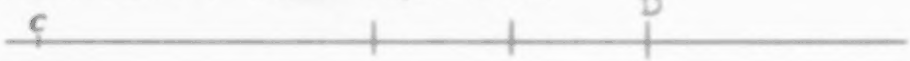
Mark Point *G* on this line so that the length of $\overline{AG}$ is .7.



Mark Point *Y* on this line so that the length of $\overline{XY}$ is 1.0 (ten tenths).



Mark Point *D* on this line so that the length of $\overline{CD}$ is .9.

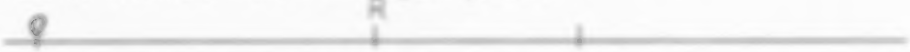


Replace the sticks in Envelope A.

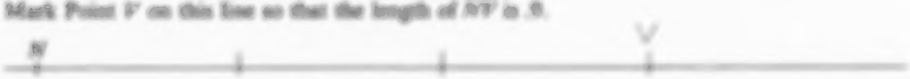
Activity 3, Station B

Use the sticks inside Envelope B for the following activities:

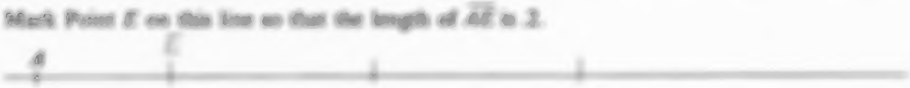
Mark Point *R* on this line so that the length of $\overline{QR}$ is .5.



Mark Point *V* on this line so that the length of $\overline{NV}$ is .9.



Mark Point *E* on this line so that the length of $\overline{AE}$ is .2.



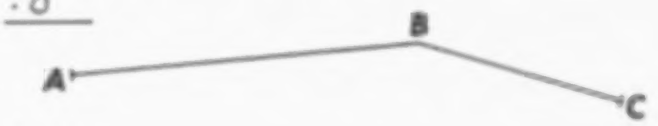
Replace the sticks in Envelope B.

PROBLEM SOLVING UNIT 6, EXPERIENCE 2

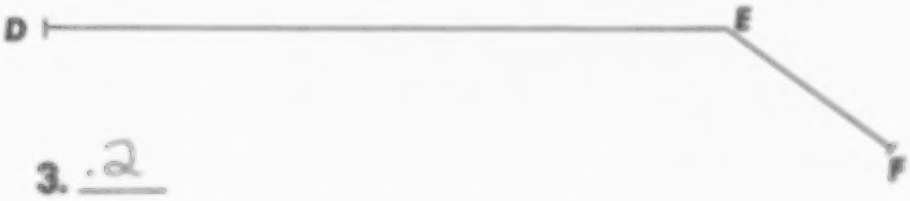
Activity 4, Station C Name ______________________________

Use the sticks inside Envelope C to measure each of these "paths." Record your measure for each one in the space provided.

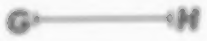
1. .8



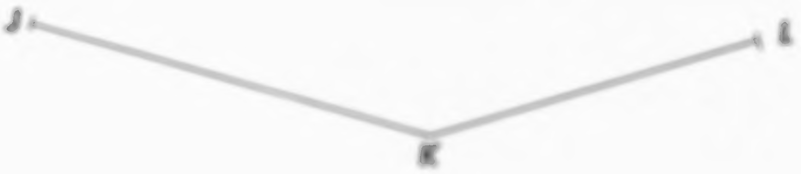
2. 1.3



3. .2



4. 1.1



Replace the sticks in Envelope C.

PROBLEM SOLVING

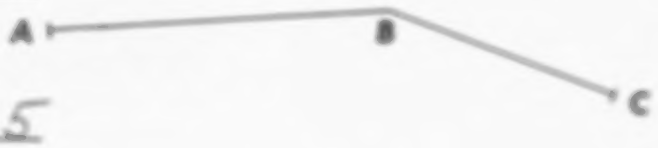
UNIT 4, EXPERIENCE 2

Activity 5, Station D

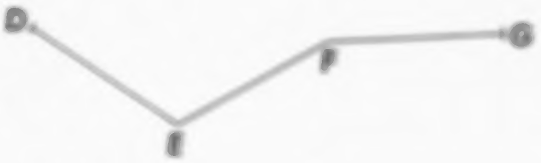
Name ______________________________

Use the sticks inside Envelope D to measure each of the "paths." Record your measure for each one in the space provided.

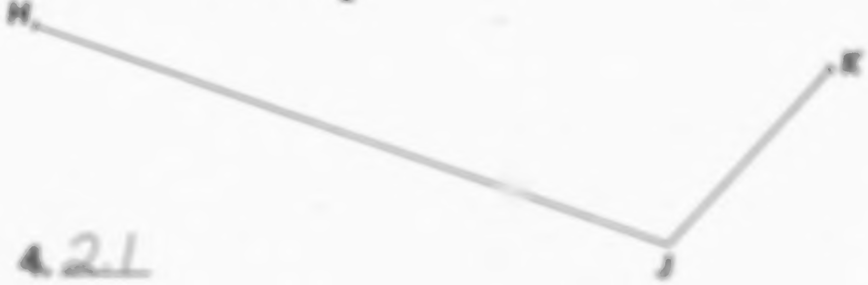
1. 1.7



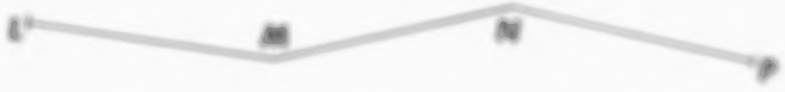
2. 1.5



3. 2.6



4. 2.1



Replace the sticks in Envelope D.

PROBLEM SOLVING **UNIT 6, EXPERIENCE 2**

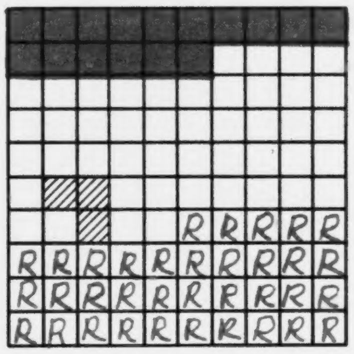
Activity 6 **Name** ____________________

Answer the questions for all three parts of this activity. Part 3 is on another sheet.

Part 1

Three-hundredths (.03) of the big square is colored black.

1. **Color 16 of the small squares green.**
2. **What fractional part of the big square is colored green?** .16
3. **What fractional part is colored either green or black?** .19
4. **Color 35 of the little squares red.**
5. **What fractional part of the big square is colored green, black, or red?** .54
6. **What fractional part of the big square is not colored?** .46



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■	■	■	■	■	■				
	▨	▨							
		▨			R	R	R	R	R
R	R	R	R	R	R	R	R	R	R
R	R	R	R	R	R	R	R	R	R
R	R	R	R	R	R	R	R	R	R

Part 2

1. **How many little squares have only 1 mark?** 23
2. **What fractional part of the big square is this?** .23
3. **How many little squares have 2 marks in them?** 12
4. **What fractional part of the big square is this?** .12
5. **How many little squares have either 1 or 2 marks in them?** 35
6. **What fractional part of the big square has no marks?** .65

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[Continued]

PROBLEM SOLVING **UNIT 6, EXPERIENCE 2**

Activity 6–*Continued* Name ______________________

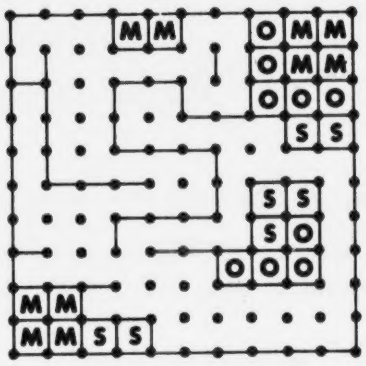
Part 3

Sam, Max, and Oscar played a game of Complete the Square, with the results shown here.

S	S	S	O	O	O	O	O	O	O
O	O	S	M	M	M	M	M	M	O
M	O	S	O	O	O	O	S	M	O
M	O	S	S	S	S	S	S	M	O
M	O	O	O	O	O	O	O	O	O
M	M	M	M	M	M	S	S	S	S
S	S	S	S	S	S	S	M	O	O
O	O	S	M	M	M	M	M	O	S
M	O	S	M	O	S	S	S	S	S
M	S	S	M	O	O	O	O	O	O

1. What fractional part of the squares did Sam complete? .33
2. What fractional part of the squares did Max complete? .27
3. Which one completed more squares? Sam

They began another game, and in a few minutes the results looked like this:



4. What fractional part of the squares has been completed? .26
5. What fractional part of the squares has not been completed? .74

The next activities are to be performed at four different stations in the room. They need not be worked on in any particular order. The stations should be separated as much as possible to avoid unnecessary confusion. If more than six students are working on any one activity, duplicate materials will be needed.

Activity 6 is designed to provide a geometric area as a referent for hundredths. It is an extension of the tenths concept developed in Activity 1. The slower students will probably not complete this activity (if they even get to it the first day). Whether or not all students have worked on it, this activity may be used as the basis for a class discussion before the class begins work on Experience 3.

EVALUATION

If the student is able to identify correctly the lengths of the paths in either Activity 4 or Activity 5, he will have achieved the minimum objectives of the experience. Hopefully he will also be successful with Activities 2 and 3.

EXPERIENCE 3

Area Model for Hundredths

OBJECTIVE

Given a rectangle ten units by ten units which is named 1.00, the student should be able to identify a five-by-seven-unit shaded portion as .35.

MATERIALS

3 envelopes for each 5 students
Materials for above (reproduced from Teaching Package)
4 worksheets for each student

Stations. At each station there is an envelope labeled with the station identification—one envelope for each group of five students or less who will be working at that station at one time. Envelope A contains a set of rectangles (fig. 8) that have been cut apart. Envelope

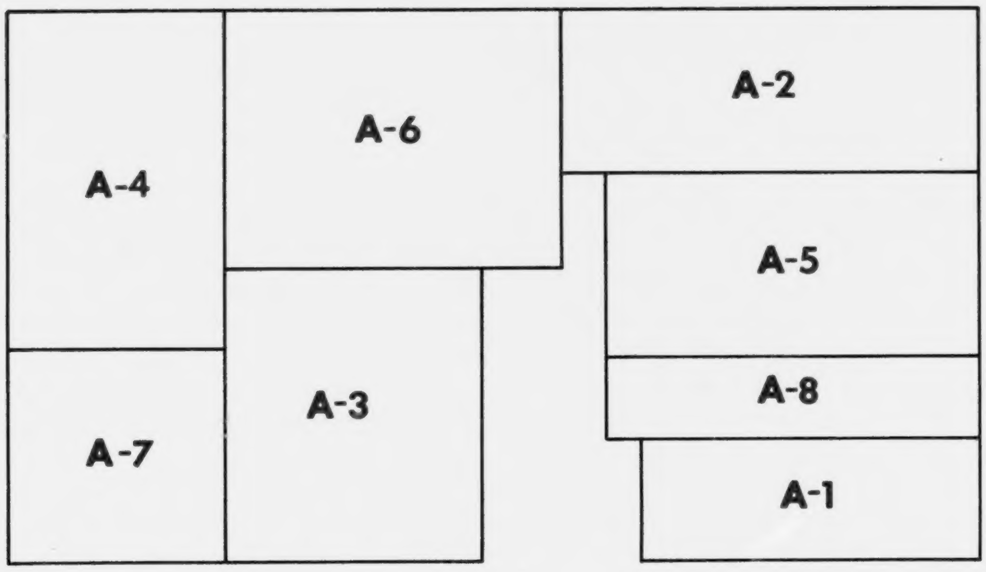


Fig. 8

B contains pieces of squared paper, and Envelope C contains a single piece of squared paper. These materials are all reproduced from the same sheet in the Teaching Package.

You might like to mount the copies on tagboard for greater durability.

TEACHER STRATEGY

These activities develop area as the referent for the hundredths concept. The focus is on a unit of area (1.00) which has been subdivided into 100 congruent parts. The physical activities involved force the student to make a visual as well as a numerical comparison between the unit and each of the various regions that are to be assigned decimal names.

Activity 1 is seatwork. You should circulate among the students, encouraging the use of the decimal numerals when necessary. Do not be concerned if some students count to find their answers. Hopefully, by the end of Experience 3 most will recognize that multiplication is a definite aid in these activities.

Activities 2, 3, and 4 are done at the three stations in the room. They need not be done in order. Multiplication is subtly pushed in each activity.

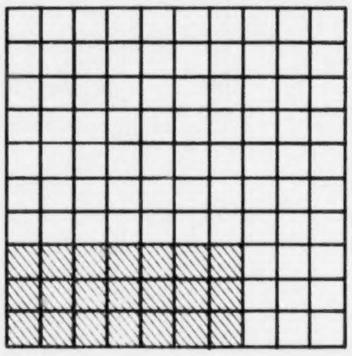
Activity 5 is seatwork. The student is given a brief encounter with a referent for hundredths. Both involve decimals greater than one and both use the area model.

[*Continued on page 194*]

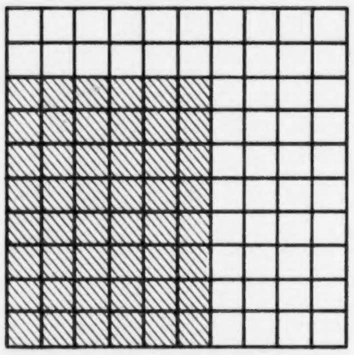
AREA MODEL FOR HUNDREDTHS UNIT 6, EXPERIENCE 3

Activity 1 Name ______________________

For each grid, tell what fractional part is shaded.

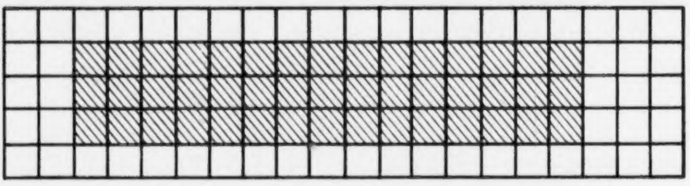


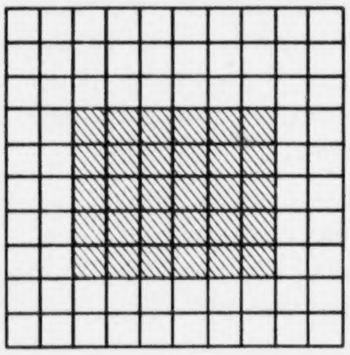
1. .21



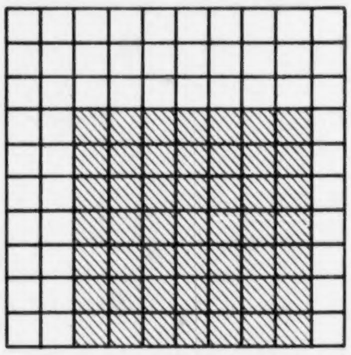
2. .48

3. .45





4. .30



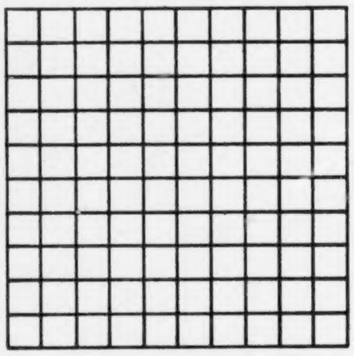
5. .49

AREA MODEL FOR HUNDREDTHS **UNIT 6, EXPERIENCE 3**

Name ______________________

Activity 2, Station A

If we assign the number 1.00 to this rectangle, we can assign a number to each piece of paper in the envelope at this station. Write your answers in the spaces below.

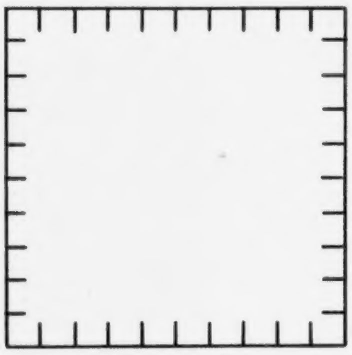


A-1 .24
A-2 .40
A-3 .42
A-4 .40
A-5 .36
A-6 .48
A-7 .25
A-8 .18

Return the pieces of paper to Envelope A.

Activity 3, Station B

If we assign the number 1.00 to this rectangle, we can assign a number to each piece of paper in the envelope at this station. Write your answers in the spaces below.



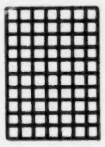
B-1 .24
B-2 .35
B-3 .36
B-4 .12
B-5 .56
B-6 .21
B-7 .36
B-8 .40

Return the pieces of paper to Envelope B.

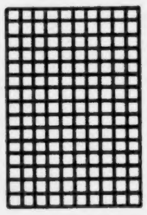
AREA MODEL FOR HUNDREDTHS UNIT 6, EXPERIENCE 3

Activity 4, Station C Name ______________________

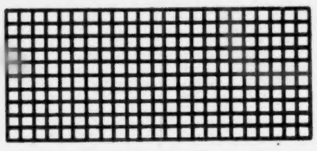
If we assign the number 1.00 to the square in Envelope C, we can use it to assign a number to each of the rectangles below. Write your answers in the spaces.



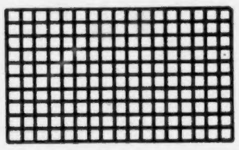
C-1 .70



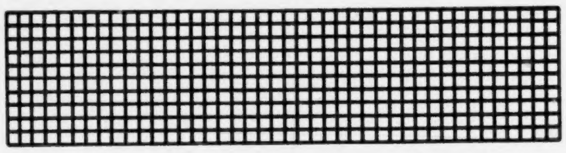
C-2 1.50



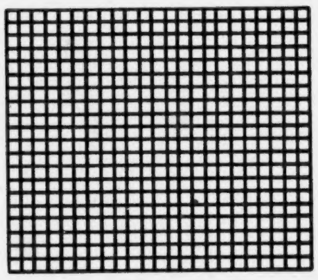
C-3 2.30



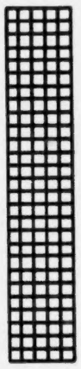
C-4 1.70



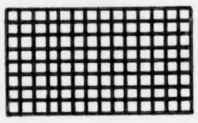
C-5 4.20



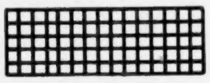
C-6 4.60



C-7 1.35



C-8 1.12

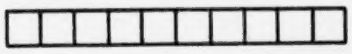


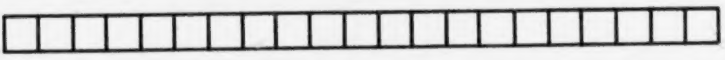
C-9 .75

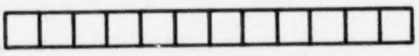
AREA MODEL FOR HUNDREDTHS UNIT 6, EXPERIENCE 3

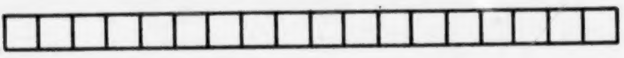
Activity 5 Name ______________________

Part 1

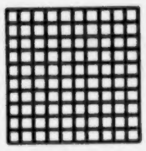
If  represents 1.0, assign a number to each of the lengths shown below.

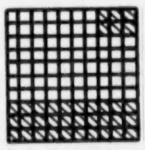
1. 2.1 

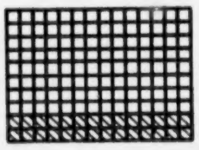
2. 1.2 

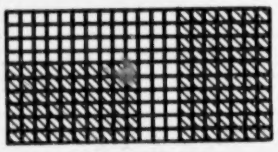
3. 1.8 

Part 2

If  represents 1.00, assign a number to the total of the shaded areas in each of the figures shown below.

1. .36 

2. .28 

3. 1.30 

EVALUATION

Students should be able to get five out of six answers correct in Activity 5. Errors arising from incorrect counting of individual squares are easily noted in Part 2 of this worksheet.

EXPERIENCE 4

Problems from the Physical World

OBJECTIVE

Given a set of 100 four-digit numerals, the student should be able to name the fractional part that contains the numeral 8 and to use the correct decimal name for the fraction.

MATERIALS

8 books
1 newspaper for each student
5 telephone books
105 index cards (some used; includes 5 blanks for Station C)
100 metal washers (some bent)
100 paper clips (some large size)
3 boxes
2 worksheets for each student

Stations. The books for Station A should be labeled (with tape) R, S, T, U, V, W, X, and Y. Each newspaper at Station B is to be identified by the name of the student using it. The telephone books and blank index cards are to be placed at Station C. The boxes should be labeled D-1, D-2, and D-3 and placed at Station D with the remaining index cards and the washers and paper clips.

Each student is given two worksheets, which cover the four activities. These are to be duplicated from the Teaching Package. (Since answers will vary according to the materials used, no answers have been added to the reproductions of the worksheets.)

TEACHER STRATEGY

Each of the four parts of Experience 4 engages the student in a physical situation that is to be described by a decimal fraction. The major focus is on hundredths. Since all activities relate the fraction concept to a subset of a set with either ten or one hundred members, the decimal names are appropriate.

In introducing this experience you should go to each station and briefly describe the activities at each. For Station A an agreement must be made as to just what a picture is.

For Station B you should be certain that everyone knows what a column is. You should also explain that the "first ten words," in the directions, means words of text—that headlines should be disregarded and that some stories may be in lines covering two columns.

At Station C the students should be warned against making marks in the telephone books. The index cards may be used to indicate the last number in the set of one hundred.

The cooperation of the students is required at Station D. As each student finishes using the index cards, washers, or paper clips, he must return them to the appropriate box. If you feel that it is necessary, you could string the washers and paper clips on loops of wire with the ends securely twisted together to discourage removal of the objects and to establish a beginning point for counting the set. However, this precautionary measure should be avoided if possible, since it is desirable that the students be able to partition the set physically.

The stations may be visited in any order.

EVALUATION

If a majority of the students arrive at a realistic answer for each of the activities and use a decimal numeral to indicate their answer, the experience can be considered a success.

PROBLEMS FROM THE PHYSICAL WORLD **UNIT 6, EXPERIENCE 4**

Name ________________________________

Station A

1. Take the books marked R, S, T, and U and look at pages 21 through 30 in each of them. What decimal fraction of these 10 pages has pictures?

   ______ in R ______ in S ______ in T ______ in U

2. Take any *one* of the books marked V, W, X, and Y and look at pages 101 through 200.

   What decimal fraction of these 100 pages has pictures? ______

   There are pictures on ______ of these pages in book ______.

3. Take another book marked V, W, X, or Y and look at pages 101 through 200.

   What decimal fraction of these 100 pages has pictures? ______

   There are pictures on ______ of these pages in book ______.

Station B

Take one of the newspapers at this station and write your name on it.

1. Draw a ring around the first 10 words in the first column at the left on page 1. Underline every *the*, *a*, and *and* inside this ring. What decimal fraction of the 10 words is underlined?

   ______

2. Repeat for each of the other columns on that page.

   ______ Col. 2 ______ Col. 3 ______ Col. 4 ______ Col. 5

   ______ Col. 6 ______ Col. 7 ______ Col. 8

3. Draw a ring around the first 100 words in the first column on page 2. Underline all words that start with the letter *t*. What fraction of the 100 words is underlined? ______

PROBLEMS FROM THE PHYSICAL WORLD UNIT 6, EXPERIENCE 4

Name ______________________________

Station C

Take one of the telephone books at this station.
Turn to the first full page of telephone numbers.

1. What decimal fraction of the first 10 telephone numbers ends in 2, 4, 6, 8, or 0? _____
2. What fraction of the first 100 telephone numbers ends in 5? _____
3. What fraction of the first 100 telephone numbers has a 5 as 1 of the last 4 digits? _____

Station D

1. What fraction of the file cards in Box D-1 has been used? _____
2. What fraction of the file cards in Box D-1 has *not* been used? _____
3. What fraction of the washers in Box D-2 is bent? _____

4. What fraction of the washers in Box D-2 is *not* bent? _____
5. What fraction of the paper clips in Box D-3 is of the larger size? _____

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SE 010 190

Teacher Materials

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

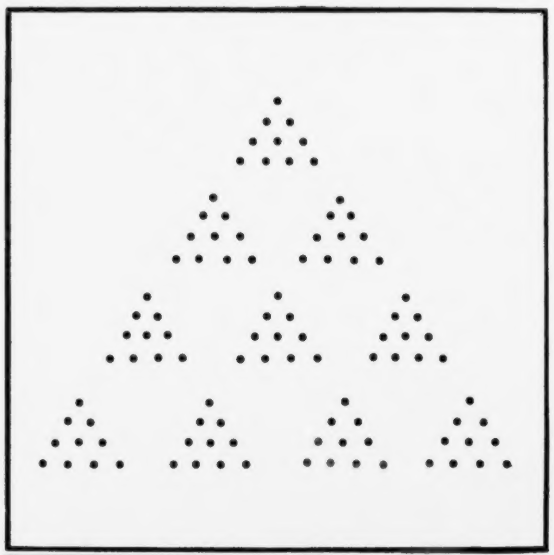
Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

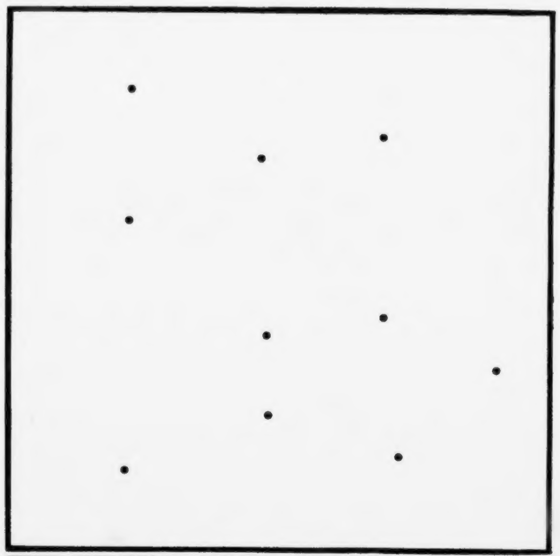
Krazy Kontest
My Guess ____________
Name ____________

Krazy Kontest
My Guess ____________
Name ____________

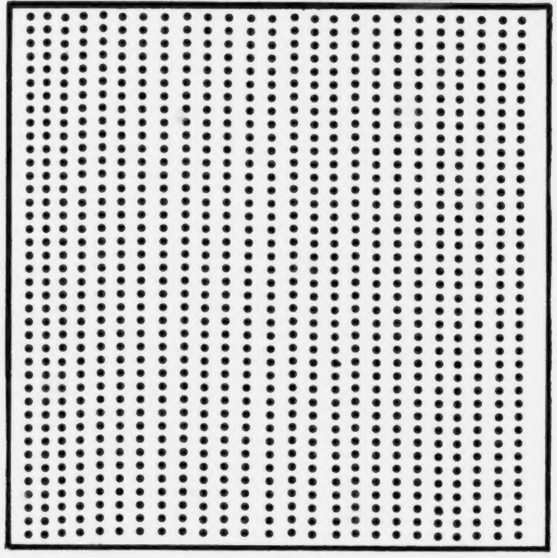
Teacher Materials



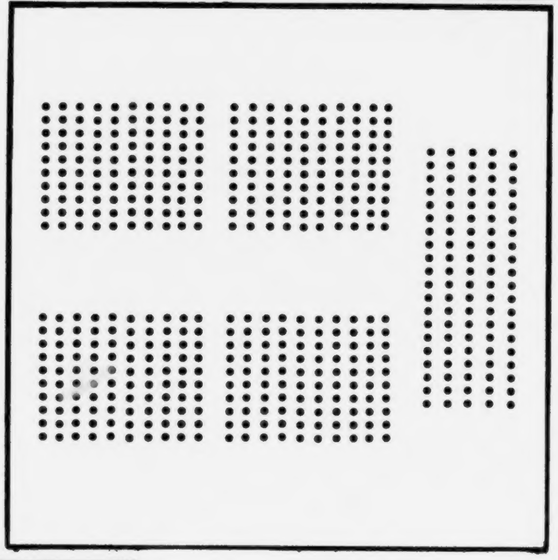
A



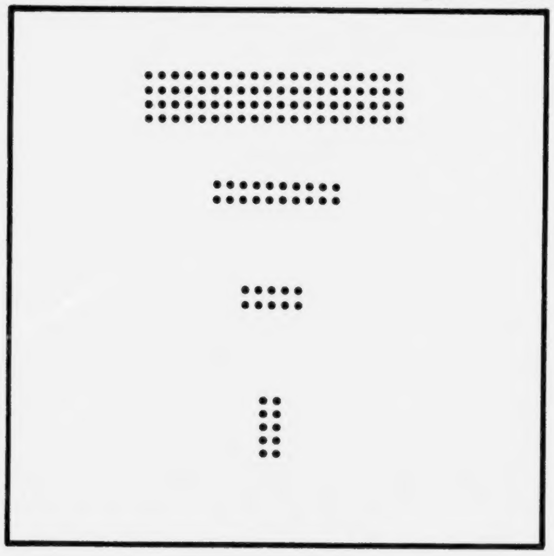
B



C



D



E

Name ____________________________________

Go to each station and find the sets equivalent to the numbers given, then write the letter for each of these sets in the proper space. As an example, the answer has already been given for 10 at Station A.

Visit the stations in the order indicated by your teacher.

Station A
10 .... Set E
100 .... Set ☐

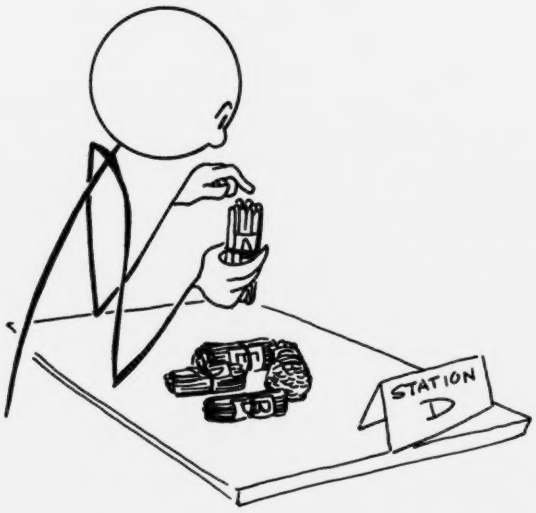
Station B
10 .... Set ☐
100 .... Set ☐
1,000 .... Set ☐

Station C
10 .... Set ☐
100 .... Set ☐

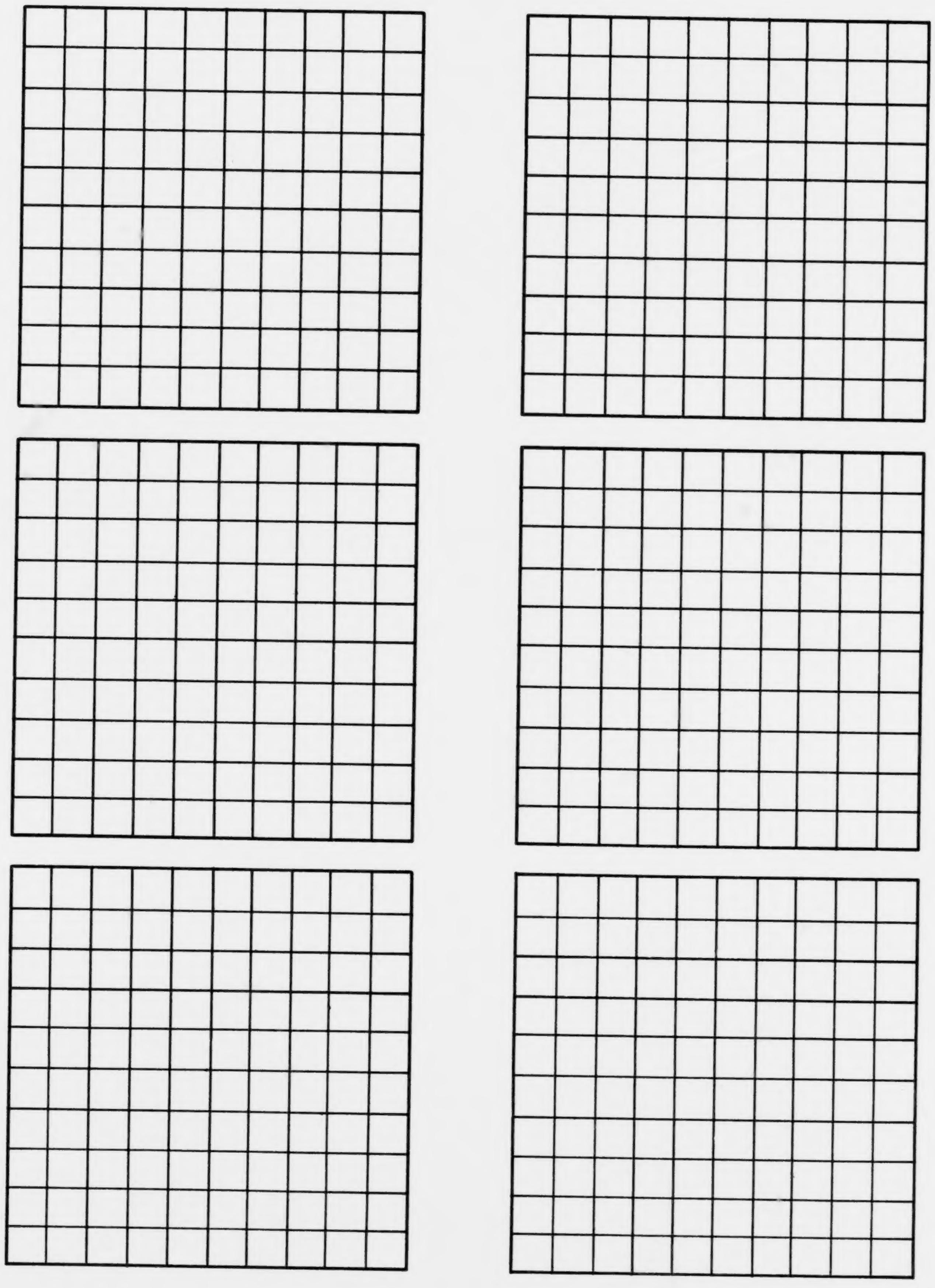
Station D
10 .... Set ☐
100 .... Set ☐

Station E
1 .... Set ☐
10 .... Set ☐
100 .... Set ☐

Station F
10 .... Set ☐
100 .... Set ☐
1,000 .... Set ☐



Teacher Materials

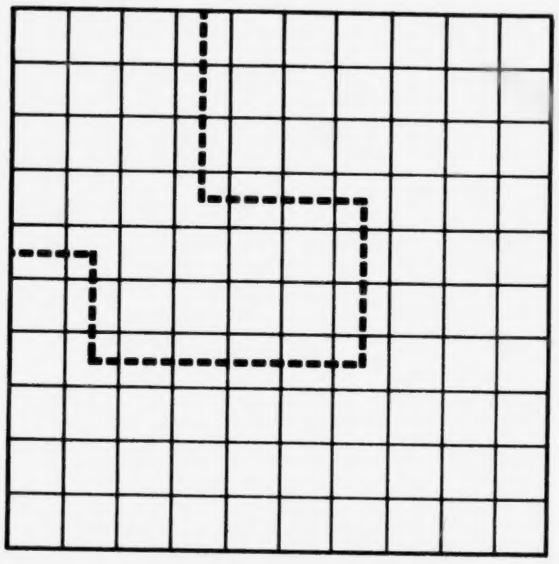


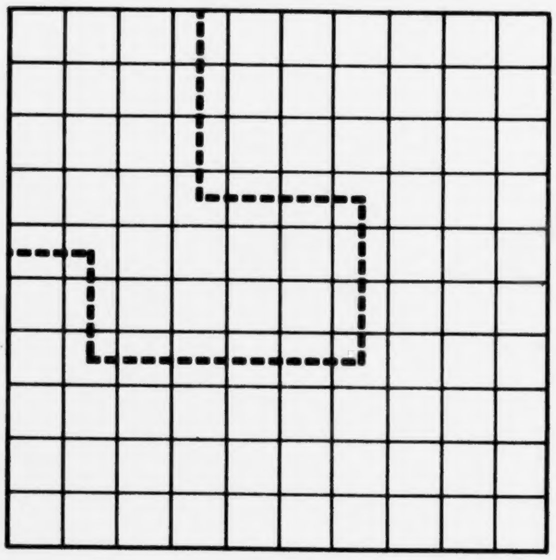
Teacher Materials

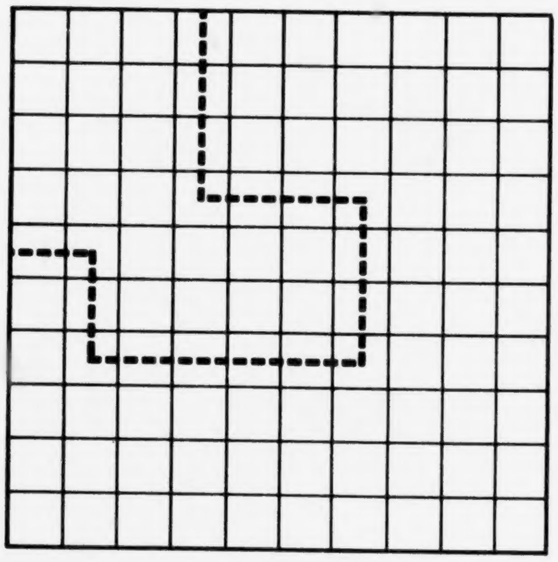
0									9
			13						
			23				27		
				54					
						86			
90									99

0									9
			13						
			23				27		
				54					
						86			
90									99

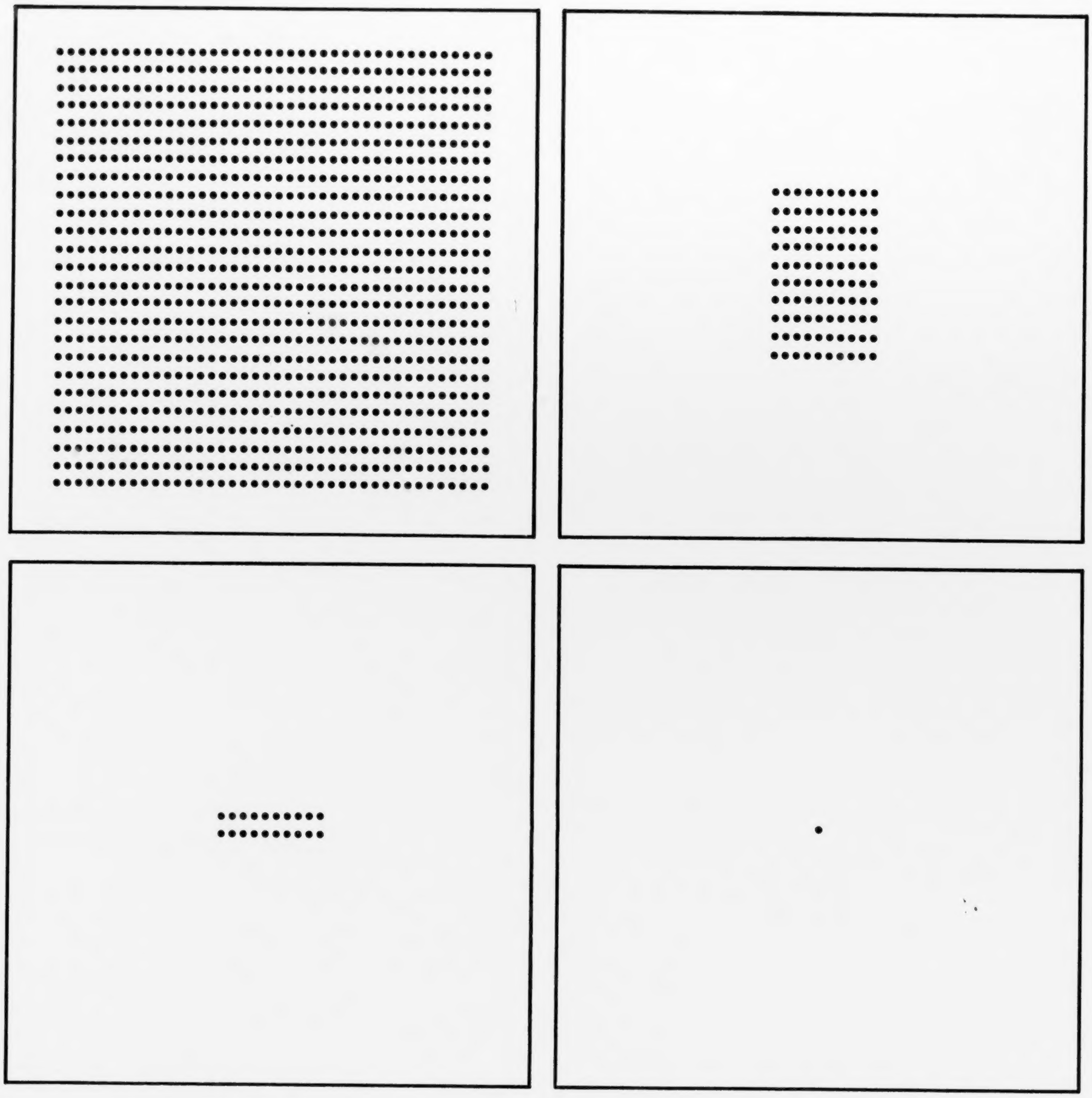
0									9
			13						
			23				27		
				54					
						86			
90									99







Teacher Materials



Name ______________________________

Game Station A—*SINK THE SUB*

Place an X in each of ten cells in your unmarked grid. Then turn over the grid that shows the path of a submarine and see if you made any hits. If you did, list the numbers that show the values of the cells where the hits were made.

__________ __________ __________

__________ __________ __________

__________ __________ __________

__________ __________ __________

Game Station B—*BREAK THE CODE*

Below each numeral write the letter that has that value in the grids at this station. When you are through, you will discover a secret message!

346	459	297

205	490	333

504	35	937	401	104

582	285	100	101

942	83	639	718

Name ______________________________

Game Station C—*HOW MANY DOTS?*

Study the four cards with dots. Color the squares below so that you will have the right number of dots, following the example below:

$$342 = \underbrace{\square\square\square}_{\text{Orange}}\underbrace{\square\square\square\square}_{\text{Yellow}}\underbrace{\square\square}_{\text{White}}$$

162 = ☐☐☐☐☐☐☐☐☐

359 = ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

902 = ☐☐☐☐☐☐☐☐☐☐☐

209 = ☐☐☐☐☐☐☐☐☐☐☐

1,234 = ☐☐☐☐☐☐☐☐☐☐

1,342 = ☐☐☐☐☐☐☐☐☐☐

Game Station D—*HOW MANY BEANS?*

Find the smallest number of bags of beans for each number below. Following the example given, show the number of beans in each bag and the number of bags.

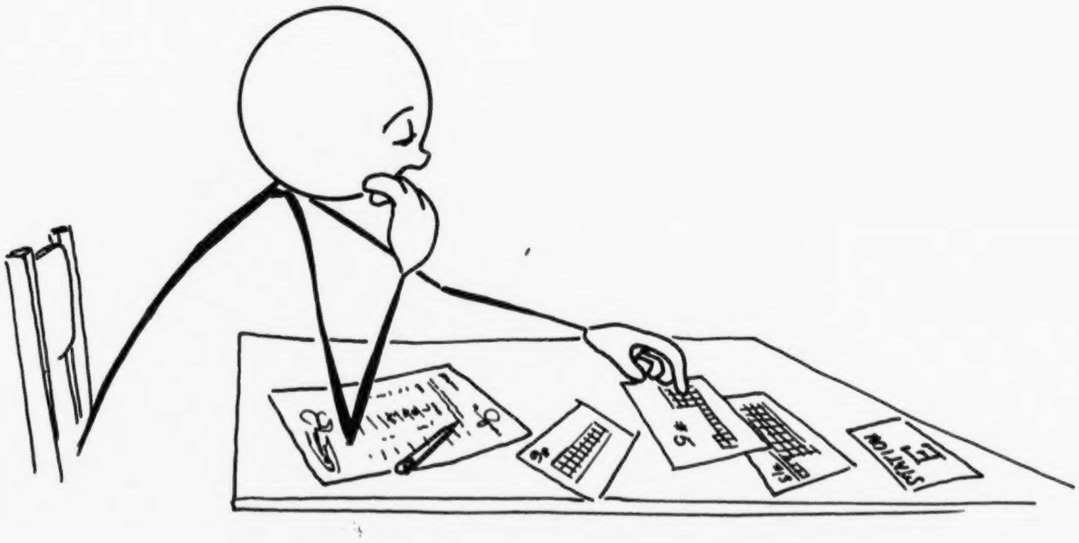
Number	Bags	Number of Bags
167	100, 40, 20, 4, 2, 1	6
132		
483		
461		
400		
306		
1,301		

Name ______________________________

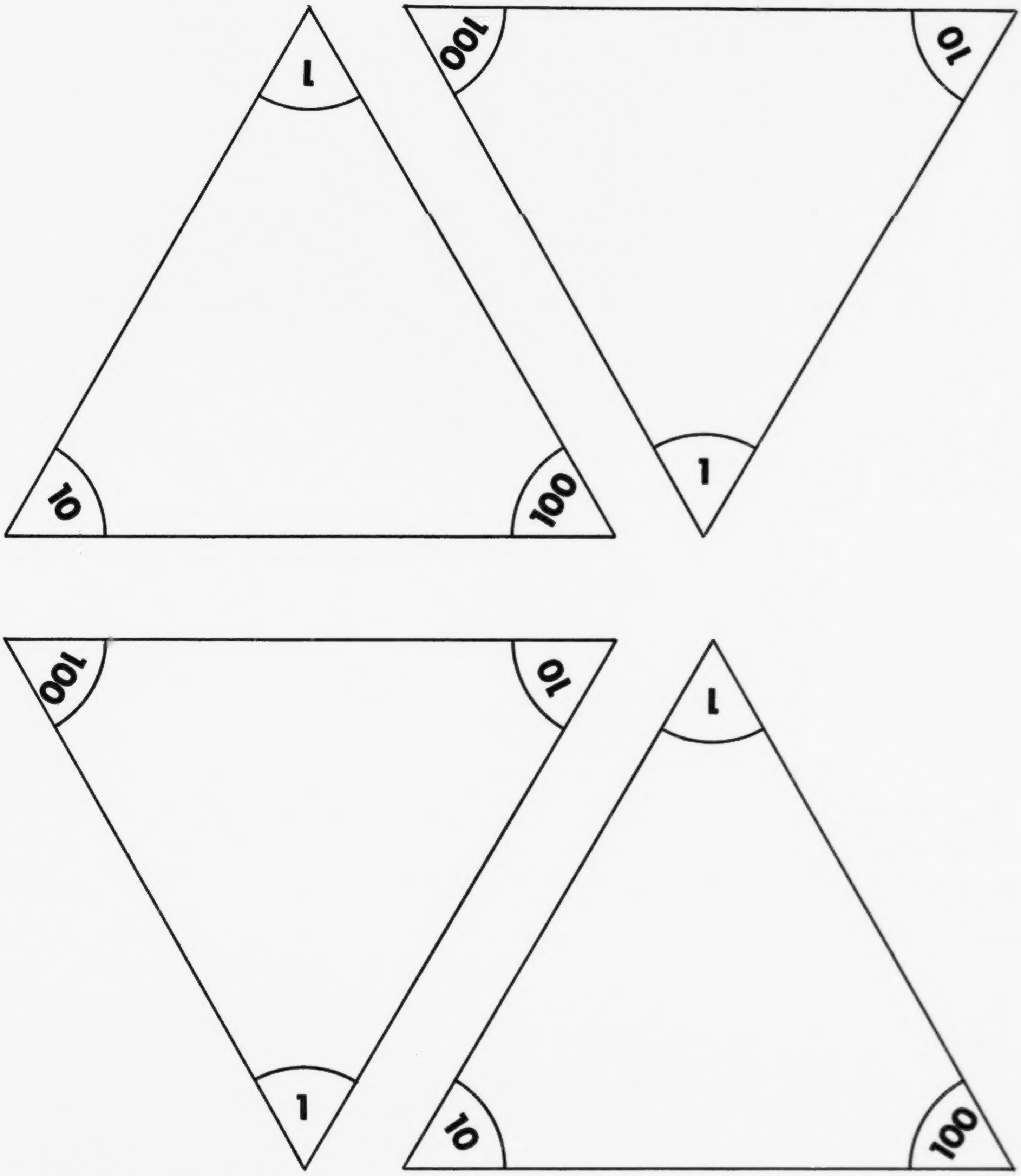
Game Station E–*HOW MANY SQUARES?*

Fill in after the number of each pattern the number of squares it contains.

Pattern	Number of Squares	Pattern	Number of Squares
1	________	11	________
2	________	12	________
3	________	13	________
4	________	14	________
5	________	15	________
6	________	16	________
7	________	17	________
8	________	18	________
9	________	19	________
10	________	20	________



Teacher Materials


1
10
100
100
10
1
100
10
1
1
10
100

EXPANDED NOTATION

UNIT 1, EXPERIENCE 4
Game Station D

Teacher Materials

LUCKY NUMBERS

5
13
24
33
42
50
65
72
104
111
115
123
132
141
145
150
160
165
202
211
226
230
231
240
241
301
305
314
320
321
330
331
334
415
422
423
441
442
453
500
510
512
513
524
525
535
545
552
563
571
572
573
574
580
581
583
585
600
601
621
624
625
630
642
645
652
700
701
702
724
842
851
921

Recorder's Name ______________________

GAME RULES AND REPORT OF WINNERS

Game A

Mix the cards and lay them face down on the table in neat rows. The first person turns over any two cards. If the cards match in value, the player keeps them. If the cards do not match, he turns them face down again. Then the player to the left turns up two cards. Play continues until there are no more unclaimed cards. The player with the most cards at the end wins.

WINNERS

Round 1 ______________________

Round 2 ______________________

Round 3 ______________________

Round 4 ______________________

Game B

Mix the deck of cards and spread them out face down on the table in neat rows. One player turns a card face up. Then the next person on the left does the same thing. If this makes a pair of cards that match in value he can claim both cards; if it does not, both cards are kept face up and again play passes to the left. The winner is the person who has the most cards when the cards have all been claimed.

WINNERS

Round 1 ______________________

Round 2 ______________________

Round 3 ______________________

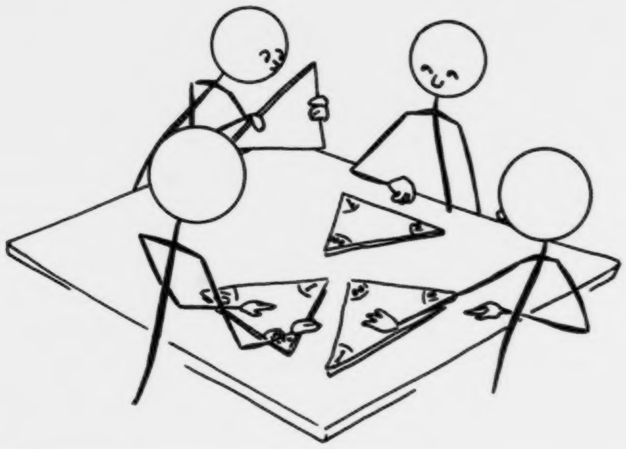
Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

Game C

Each person takes one of the triangles and holds it in front of him with the numbers hidden from the other players. He turns it around several times so no one knows which number is at the top.

Each player then guesses the sum of the numbers at the top and writes the guess on a piece of paper. After everyone has recorded his guess, players hold their triangles at the lower corners and place them face up in the middle of the table. The person who has the best guess about the sums of the corners in the middle is the winner of that round. If there is a tie, both students are winners.



WINNERS

Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Round 6			
Round 7			
Round 8			

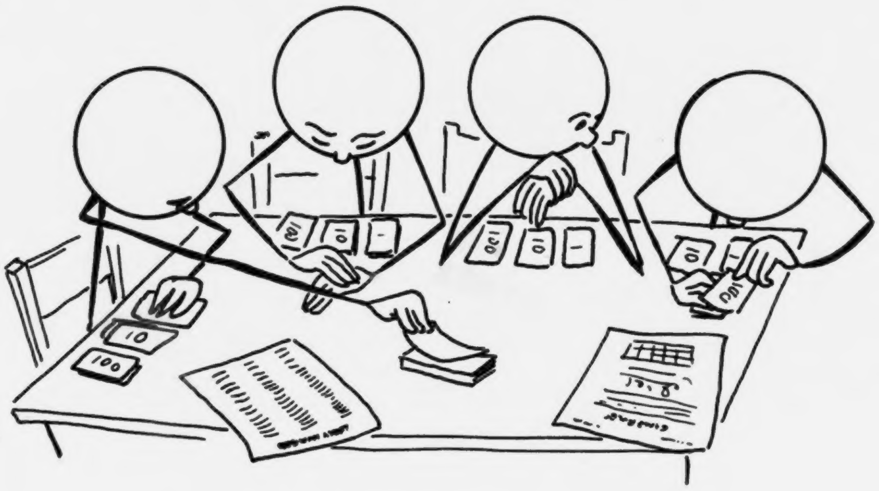
Recorder's Name ____________________

GAME RULES AND REPORT OF WINNERS

Game D

Mix the deck of cards and place the cards face down in a pile in the center of the table. Each player draws a card in turn. The drawing of cards continues until someone can match the values of the cards he has drawn to one of the lucky numbers.

When drawing cards you should make piles of ones, tens, and hundreds.



WINNERS

Round 1 ____________________

Round 2 ____________________

Round 3 ____________________

Round 4 ____________________

Round 5 ____________________

Round 6 ____________________

COMPARING NUMBERS

UNIT 1, EXPERIENCE 5

Name ______________________________

Station A

The deck of cards you will use at this station has numerals on one side. Arrange these in the order of their value from least to greatest, then turn them over and write down in the spaces below whatever you find on the other side.

Then mix the cards up for the next person who will use them.

Station B

Mix the deck of cards you find at this station. You will see that each card has two numbers written on it. If the number on the right-hand side is greater than the one of the left, put it in a pile on your right. If the number on the left-hand side is greater, put it in a pile on your left. When you have finished, count the number of cards in each pile and put the answers in the squares below:

Left-Hand Number Greater	Right-Hand Number Greater
No. of Cards __________	No. of Cards __________

Name ________________________________

Station C

You will find a set of cards spread out on the table. Make sure the numerals are facing up. On the reverse side of each card is written one of the following:

L (for "less than")
G (for "greater than")
E (for "equal to")

The object of this experience is to turn over as few cards as possible before you come to the card with E on the reverse side. Turn over as many cards as you need to, one at a time, making a tally mark in the space below each time you turn over a card. After you have found the E card, add up the tally marks and give the total.

Number of Marks __________

COMPARING NUMBERS UNIT 1, EXPERIENCE 5

Name ______________________________

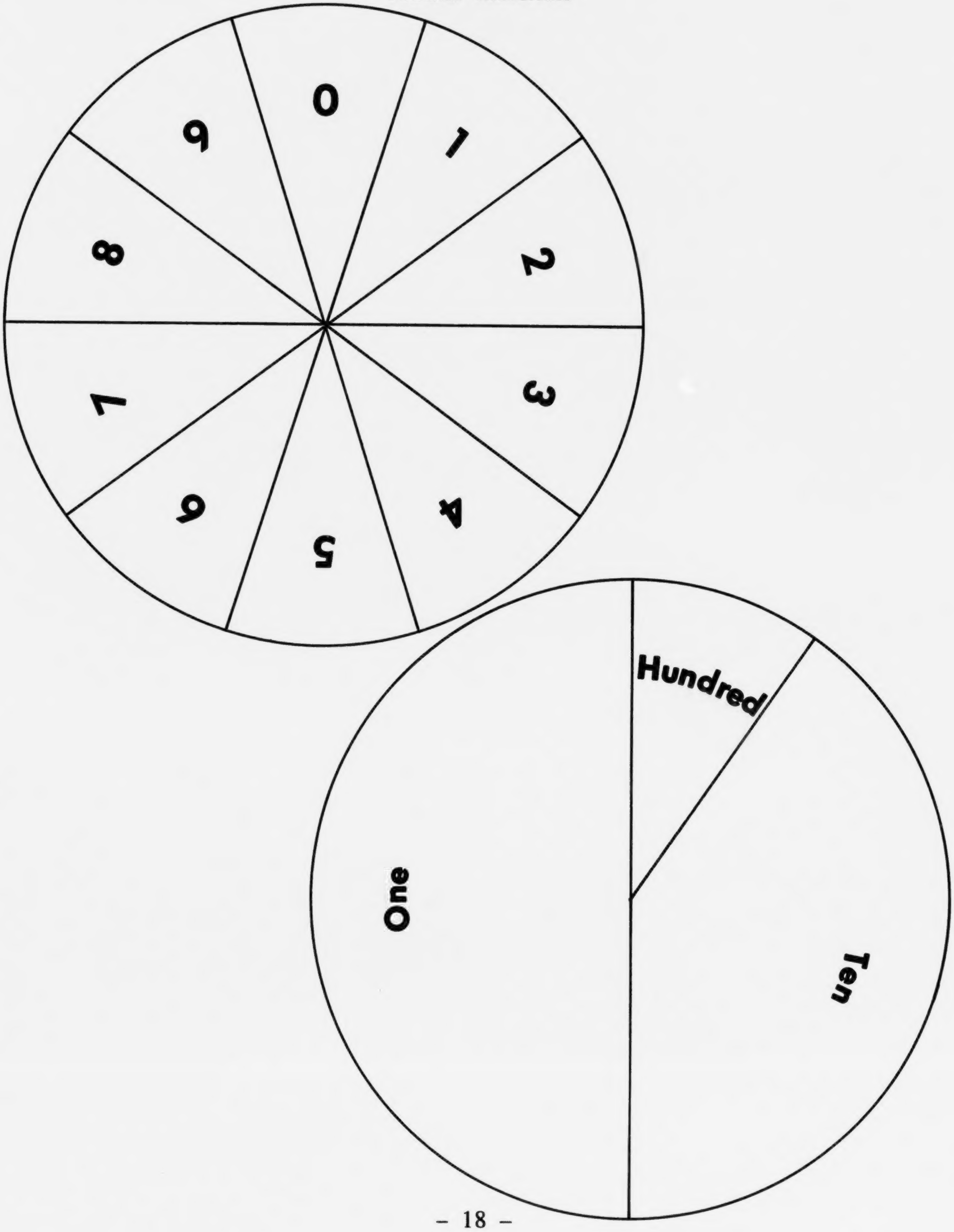
Station D

You will find three cubes with numerals on them at this station. Each cube is different. The numerals on them are shown below:

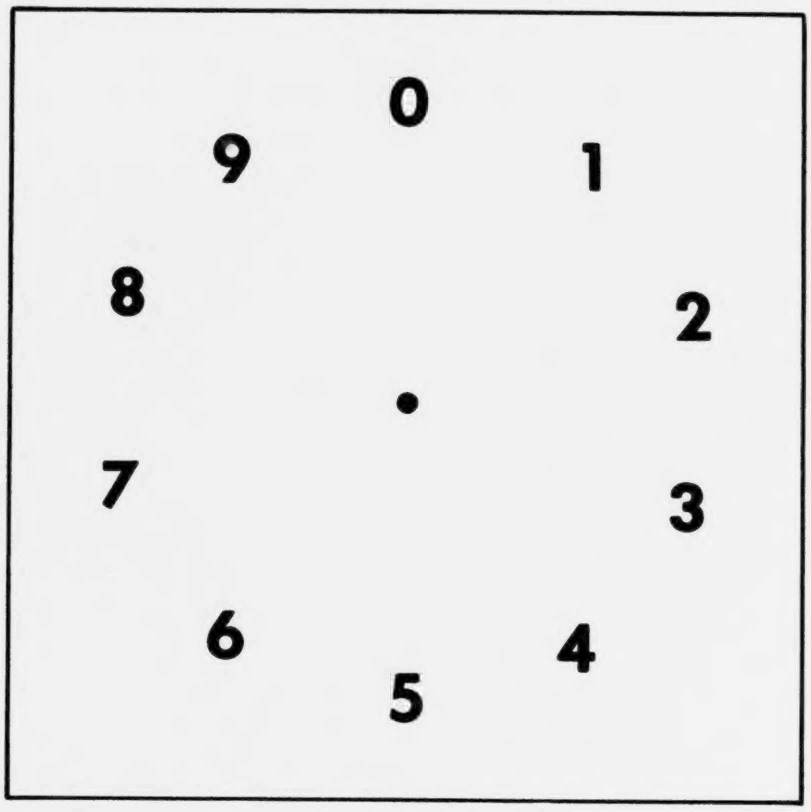
1, 2, 3, 4, 5, 6
3, 5, 6, 7, 8, 9
1, 2, 3, 7, 8, 9

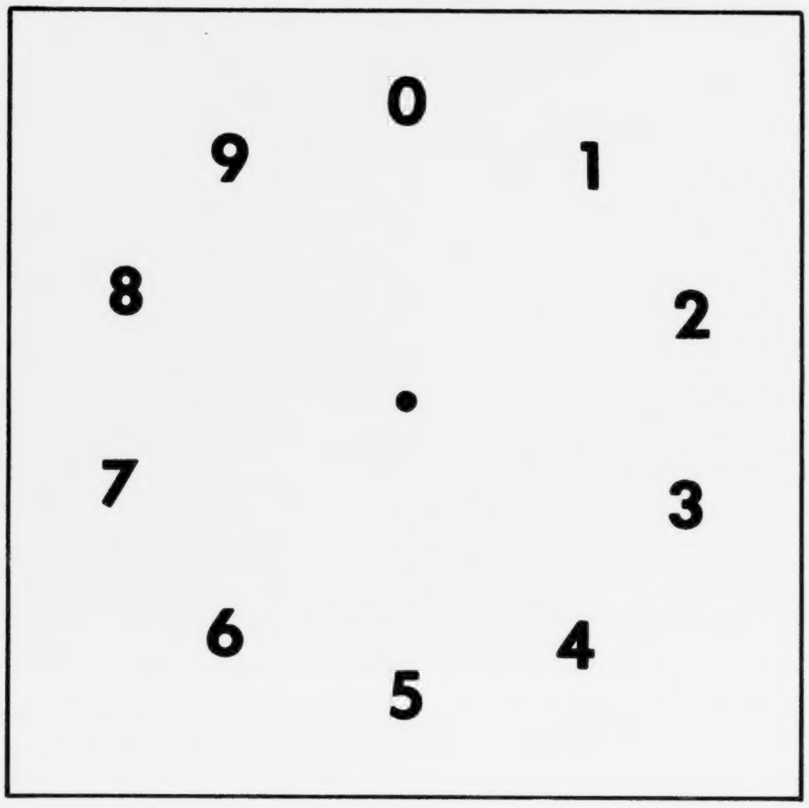
1. What is the greatest number you can make by lining up the cubes next to each other?		9	
2. What is the greatest number with 7 in the tens place?		7	
3. What is the greatest number with 1 in the tens place?		1	
4. What is the smallest number with 9 in the tens place?		9	
5. What is the smallest number with 7 in the tens place?		7	
6. What is the smallest number with 1 in the tens place?		1	

Teacher Materials


0
1
2
3
4
5
6
7
8
9
Hundred
Ten
One

Teacher Materials


0
1
2
3
4
5
6
7
8
9


0
1
2
3
4
5
6
7
8
9

EXCHANGING GROUPS OF TEN UNIT 2, EXPERIENCE 1

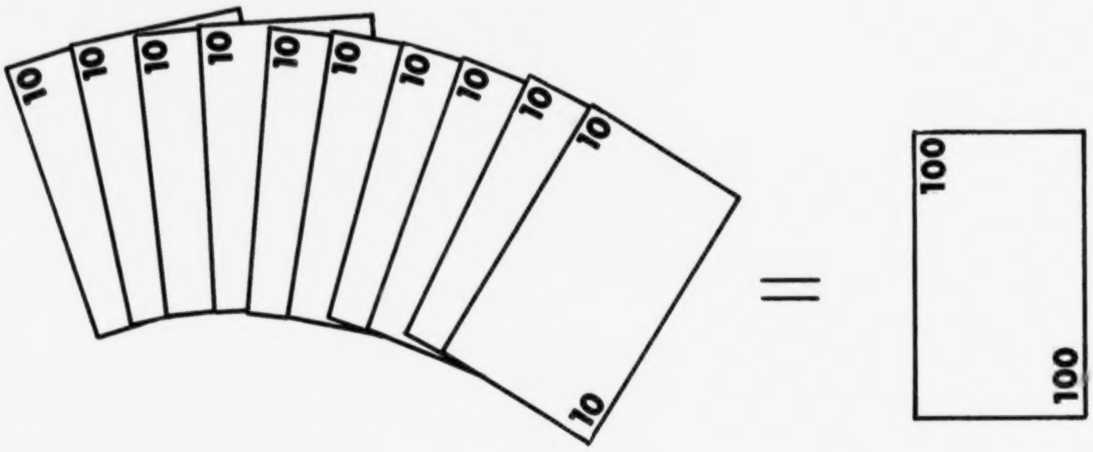
Game Station A Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

At this station there are two spinners and some cards labeled 1, 10, 100, and 1,000. The cards should be separated into four piles.

Take turns spinning the two spinners. The spinner on the left tells you how many cards to pick up, and the one on the right tells you which kind of card to pick up. If there are not enough cards left in the bank when you spin, then take another turn. When you get ten cards of either 1, 10, or 100 you must make an exchange.

The first person to reach 1,000 points wins.



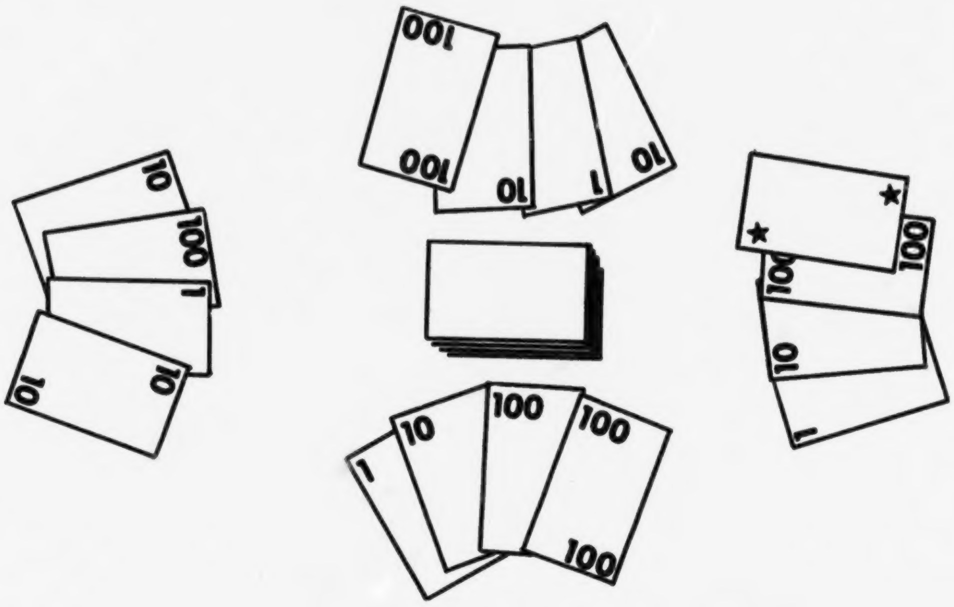
WINNERS	
Round 1	______________________________
Round 2	______________________________
Round 3	______________________________
Round 4	______________________________
Round 5	______________________________

Game Station B Recorder's Name ______________________

GAME RULES AND REPORT OF WINNERS

At this station there is a deck of cards labeled with 1s, 10s, 100s, and stars. Shuffle the deck and put it in the center of the table. Take turns drawing one card at a time. When a card with a star is drawn, each student adds the numbers on his cards to find his score.

The student with the highest score wins.


100
100
10
1
10
10
100
1
10
10
★
★
100
100
10
1
1
10
100
100
100

WINNERS

Round 1 ______________________

Round 2 ______________________

Round 3 ______________________

Round 4 ______________________

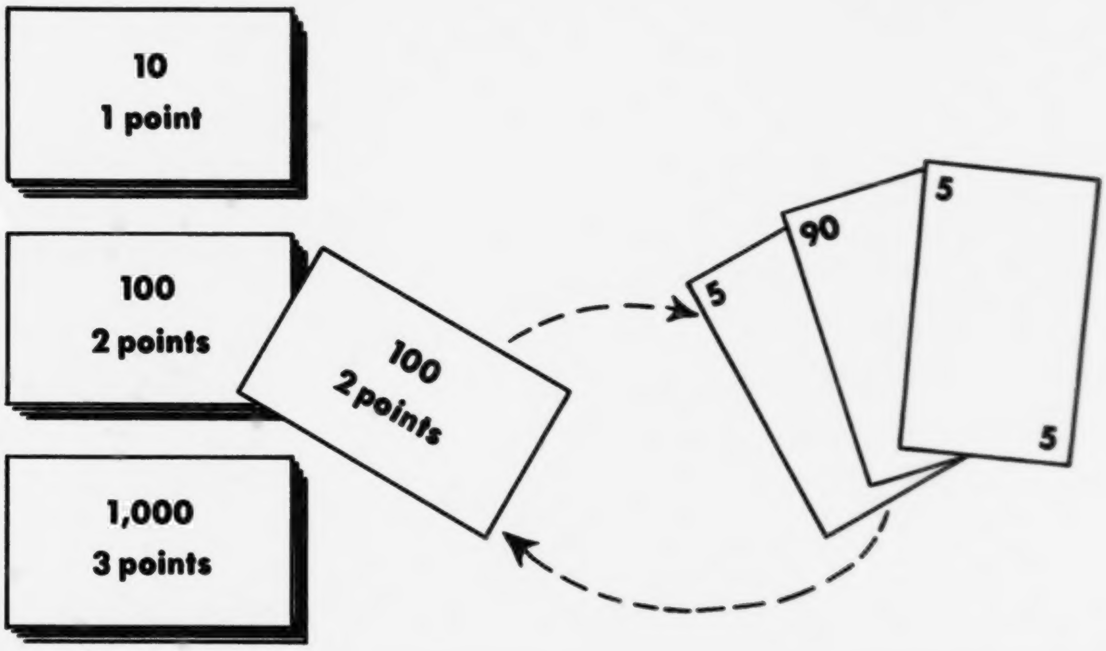
Round 5 ______________________

Game Station C Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

At this station there is a deck of white cards and a set of colored score cards. The score cards should be divided into three piles according to color. The white cards are mixed up and dealt to the players until the deck is gone. Hold your cards so other players cannot see them.

If you can form a sum of 10, 100, or 1,000 by using two or more cards, you should exchange them for the score card with the correct value. Always exchange a group of white cards for one colored score card. After each player has made all the exchanges he can, everyone must pass one card to the person on his left. Again make any possible exchanges, then everyone pass one card to the person on his left. Again make exchanges. Continue the game until no more exchanges can be made. The player with the greatest number of points wins.



WINNERS

Round 1 ______________________________

Round 2 ______________________________

Round 3 ______________________________

Round 4 ______________________________

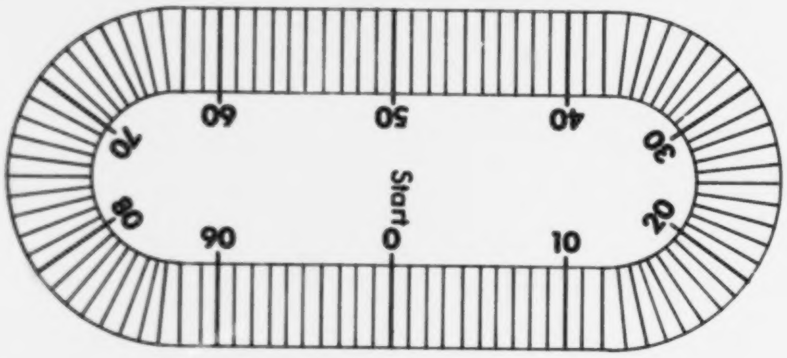
EXCHANGING GROUPS OF TEN UNIT 2, EXPERIENCE 1

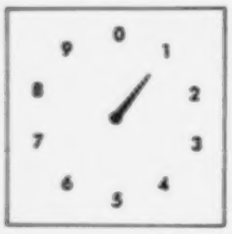
Game Station D Recorder's Name ______________________________

GAME RULES AND REPORT OF WINNERS

At Station D you will find the following materials:

- a racetrack
- markers
- a deck of cards
- lap counters





Each player takes a marker and a counter. Put the markers on Start. Shuffle the deck and put it in the middle of the table.

Take turns drawing cards. Move your marker the number of spaces indicated on the card you draw. Every time you pass Start, move the pointer on your counter to show one more lap.

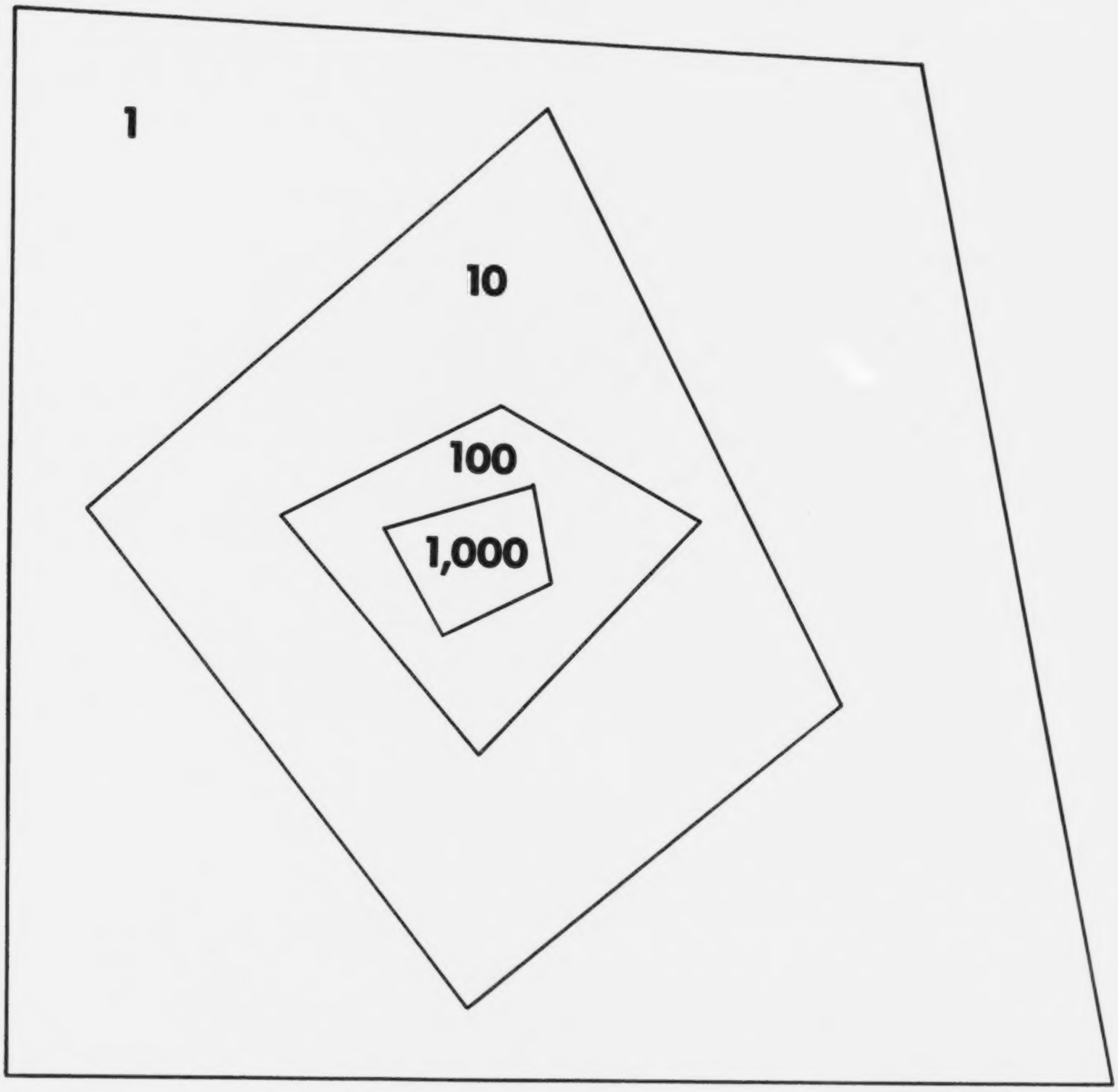
The first player to complete ten laps wins.

WINNERS

Round 1 ______________________________

Round 2 ______________________________

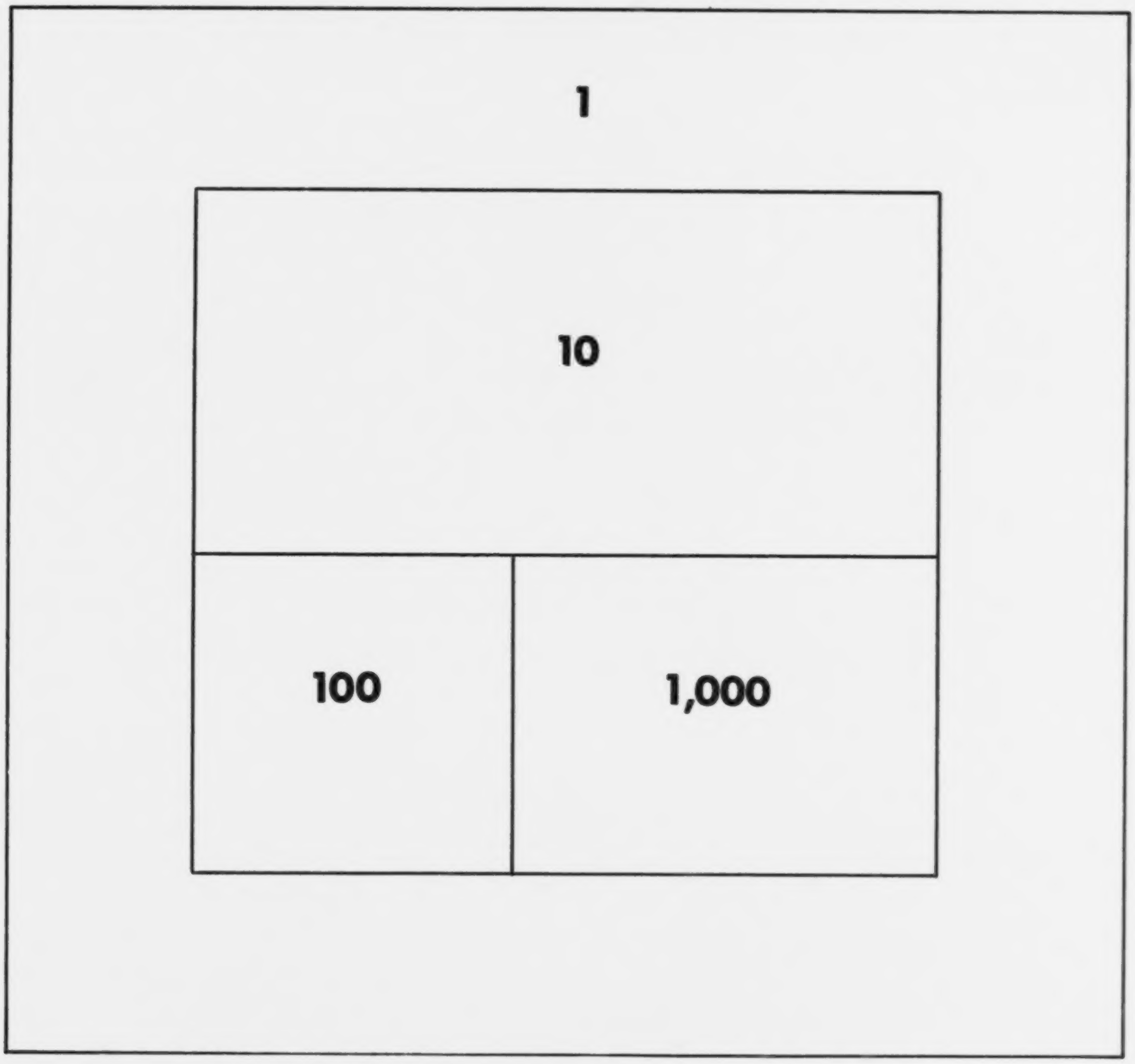
Teacher Materials


1
10
100
1,000

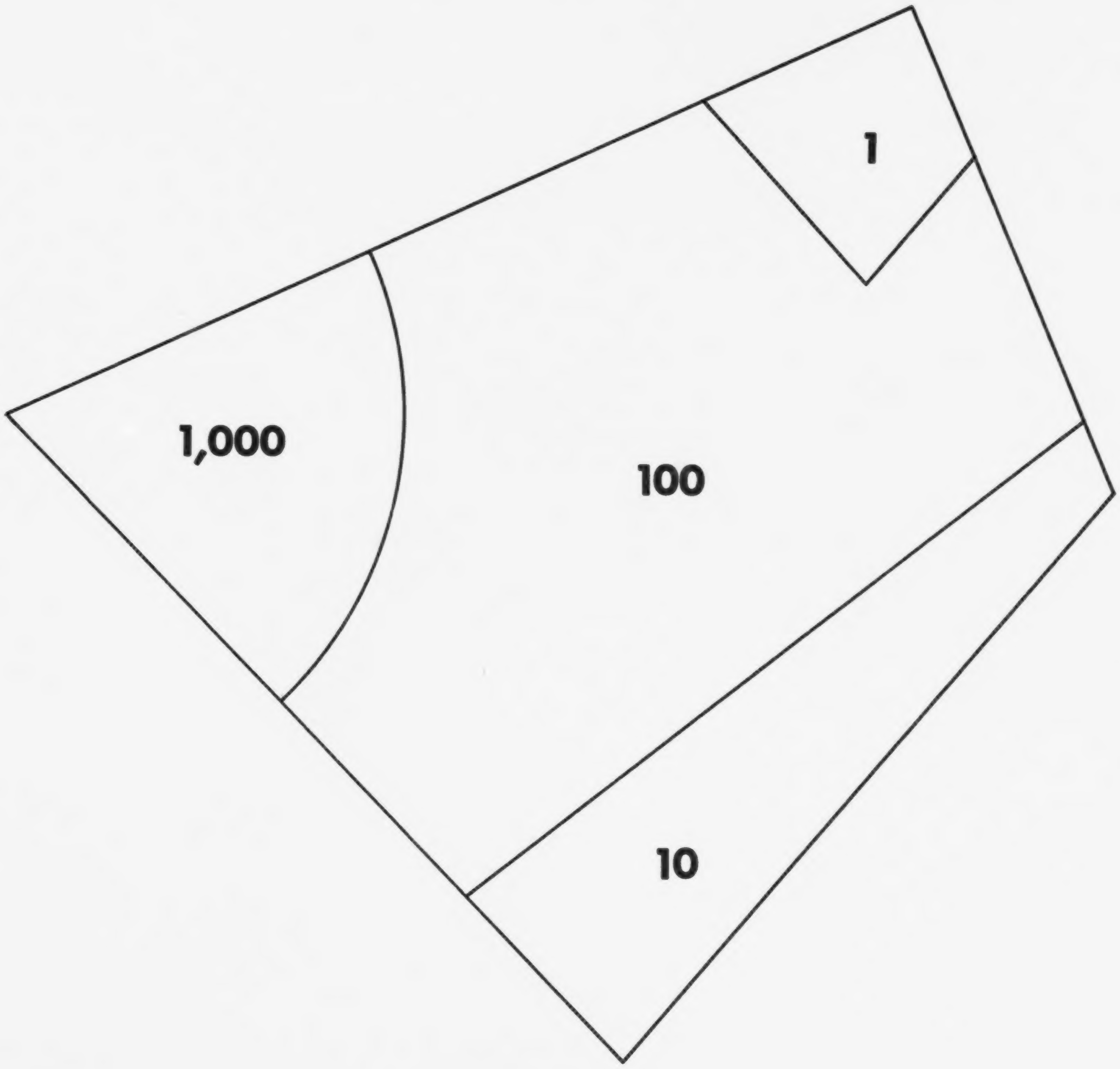
RECORDING NAMES FOR GROUPS OF TEN

UNIT 2, EXPERIENCE 2
Station B

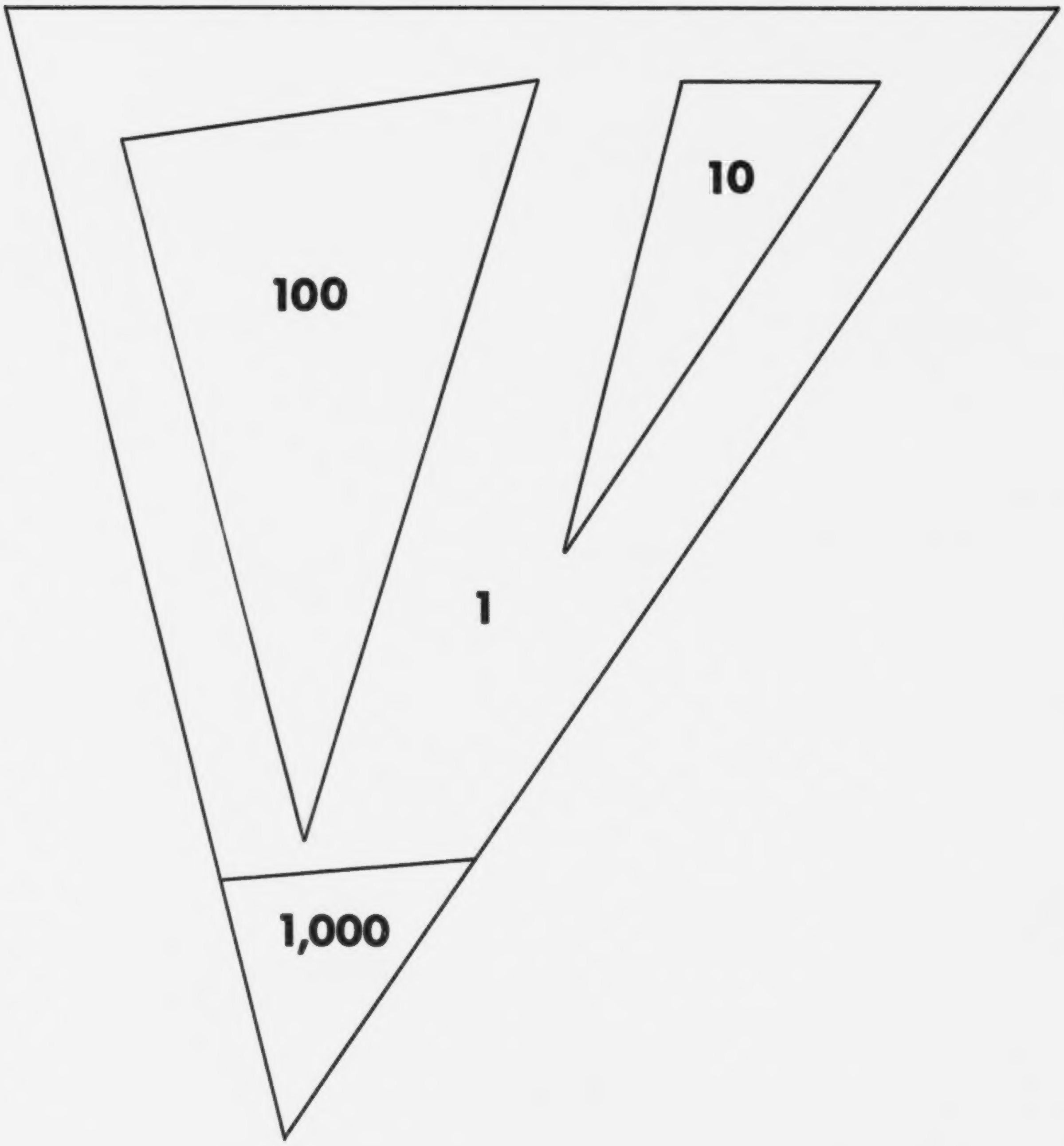
Teacher Materials


1
10
100
1,000

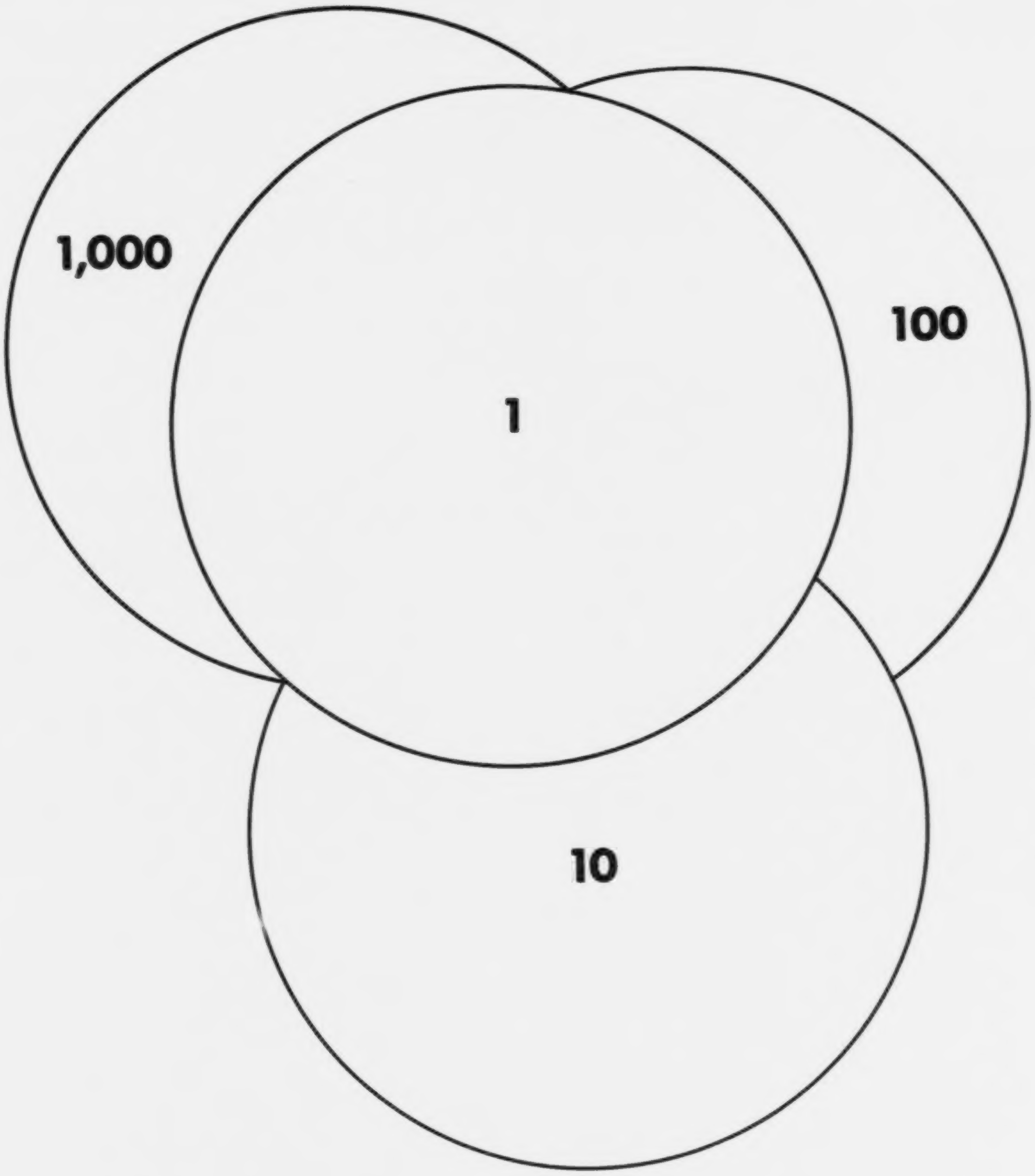
Teacher Materials


1
1,000
100
10

Teacher Materials


10
100
1
1,000

Teacher Materials


1,000
100
1
10

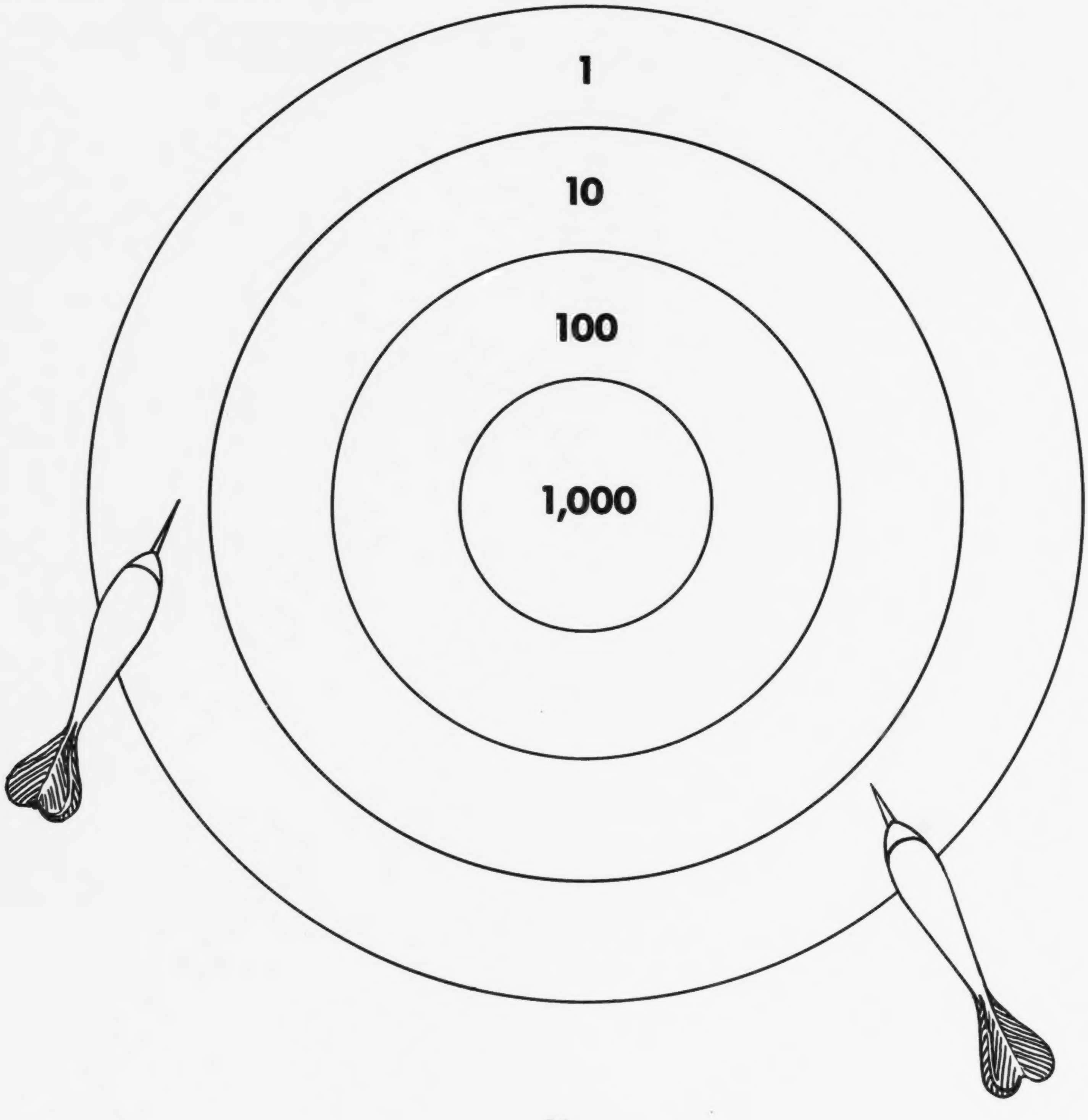
RECORDING NAMES FOR GROUPS OF TEN — UNIT 2, EXPERIENCE 2

Activity 1 — Name ____________________

Form the following scores by placing the letters on the target:

342 with *a*'s
621 with *b*'s
1,832 with *c*'s
1,002 with *d*'s
3,013 with *e*'s
204 with *f*'s
402 with *g*'s

Use as few letters as possible.



RECORDING NAMES FOR GROUPS OF TEN UNIT 2, EXPERIENCE 2

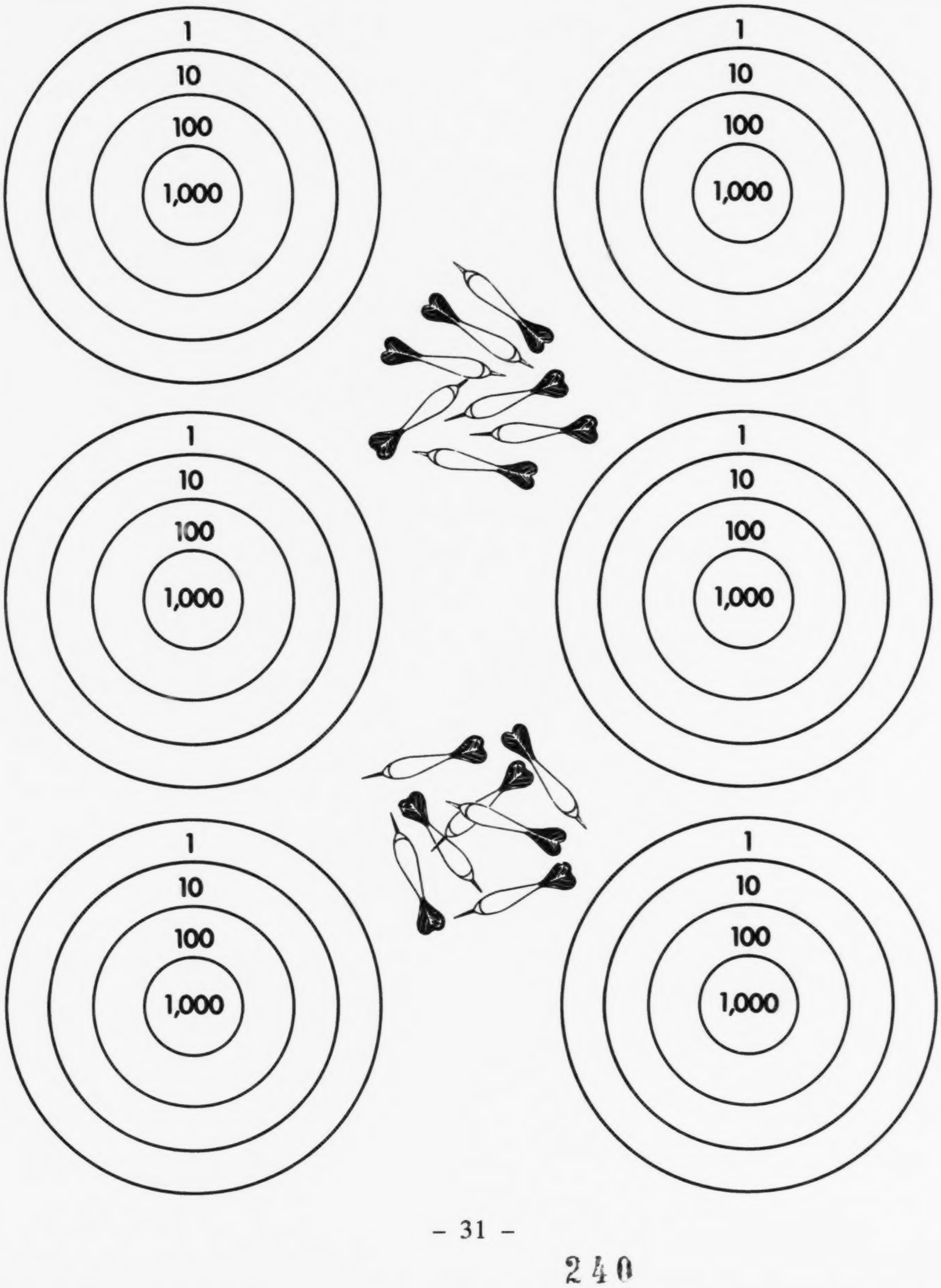
Activity 2 Name ____________________

Draw twenty Xs in the box. At each station place the transparency over this sheet and determine your score. Record your scores below and then find the total score.

	Thousand	Hundred	Ten	One	Score
Station A					
Station B					
Station C					
Station D					
Station E					
				TOTAL SCORE	

Activity 1 Name ______________________________

Show how a score of 142 can be formed in different ways. Use small *x*'s to show where the darts land.


1
10
100
1,000
1
10
100
1,000
1
10
100
1,000
1
10
100
1,000
1
10
100
1,000
1
10
100
1,000

Activity 2 Name ________________

Fill in the scores.

	Thousand	Hundred	Ten	One	Score
1.	2	3	1	4	2,314
2.			2	0	
3.		2	0	1	
4.		1	10	0	
5.			1	12	
6.		2	10	3	
7.		10	2	3	
8.			23	1	
9.		1	25	2	
10.		10	10	1	
11.		10	20	30	
12.		0	20	40	
13.		1	26	42	
14.		1	73	17	

RENAMING NUMBERS

UNIT 2, EXPERIENCE 3

Activity 3

Name ______________________

Fill in the blanks.

	Thousand	Hundred	Ten	One	Score
1.				4	24
2.		6		13	683
3.		1		14	164
4.		1		8	248
5.		6		1	781
6.		1		43	163
7.			22		222
8.			67		683
9.		17	2		1,834
10.		14		8	1,688

Activity 4 Name ______________________

Fill in the blanks to show how the score can be achieved in different ways.

	Thousand	Hundred	Ten	One	Score
1.			2	4	24
2.					24
3.					24
4.			5	10	60
5.					60
6.					60
7.			17	3	173
8.					173
9.					173
10.					173
11.					173
12.	3	3	0	42	3,342
13.					3,342
14.					3,342
15.					3,342
16.					3,342

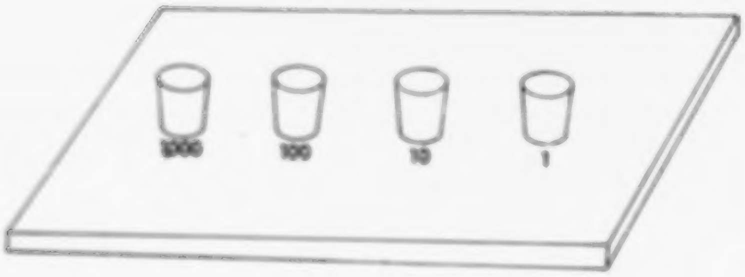
Activity 1

Name ______________________

Use your lima-bean calculator to find the answers to the problems below.

1. 163
142
+ 178

2. 113
142
+ 199



3. 113
162
193
184
+ 188

4. 1,162
1,381
1,294
1,163
+ 2,194

5. 6,831
1,294
1,193
1,239
4,128
+ 1,188

6. 1,138
1,924
1,238
1,949
1,199
+ 1,488

Activity 2 Name ______________________

Draw a line through pairs of numbers that add up to ten.

1

2 1

3 ~~4~~ 5

~~6~~ 2 6 8

4 8 7

6

9 5 4 3

7 7

3 9

4

8 7 6

3 2

Draw a line through triples of numbers that add up to ten.

1 1 2 3 2

3 5

4 7

1

4 3 4 ~~6~~

3 ~~1~~ ~~3~~

3

5 1

4 8 6

1 6

2 1

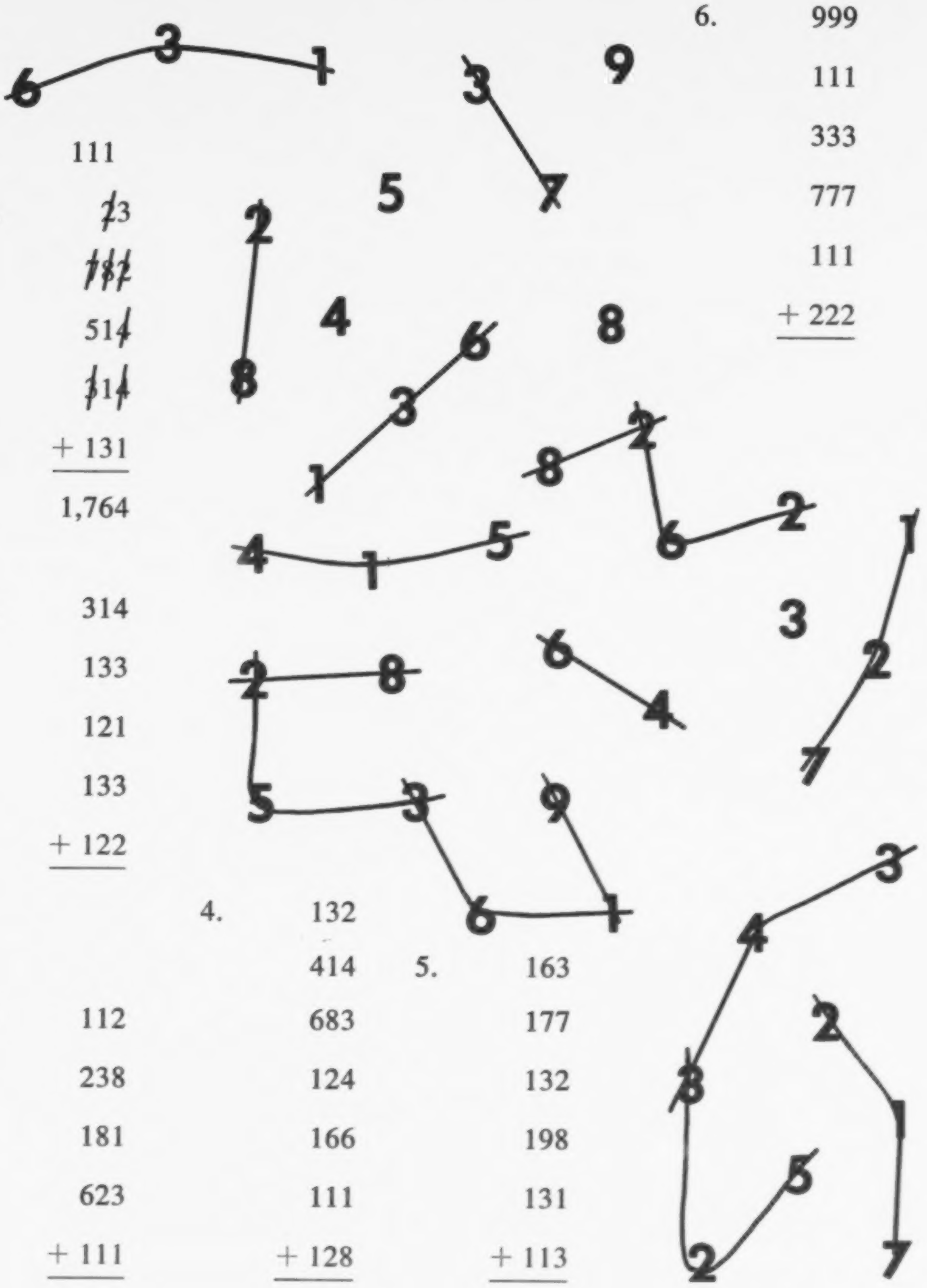
1 3

2

4

Activity 3 Name ____________________

Find the answers to the problems below by exchanging ten in one column for one in the next. Check your answers with the lima-bean calculator.


6
3
1
3
9
7
5
2
4
6
8
8
3
1
2
8
4
1
5
6
2
1
3
2
8
6
4
2
7
5
3
9
6
1
3
4
2
3
1
5
2
7

1.
111
23
782
514
314
+ 131
1,764

2.
314
133
121
133
+ 122

3.
112
238
181
623
+ 111

4.
132
414
683
124
166
111
+ 128

5.
163
177
132
198
131
+ 113

6.
999
111
333
777
111
+ 222

7.
1,342
1,738
1,438
1,122
+ 1,111

8.
1,138
9,999
1,112
1,181
+ 1,143

9.
1,234
4,321
1,111
2,222
+ 8,888

EXCHANGES IN SUBTRACTION

UNIT 2, EXPERIENCE 5

Activity 1

Name ____________________

Carry out these subtractions with the aid of your lima-bean calculator.

1. 1,274
   – 131

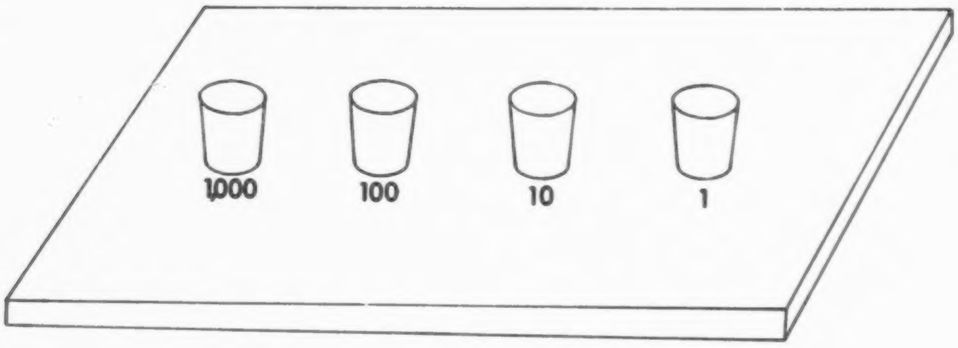
2. 1,263
   – 172

3. 1,621
   – 118

4. 1,631
   – 480

5. 1,631
   1,298

6. 7,823
   – 938



EXCHANGES IN SUBTRACTION

UNIT 2, EXPERIENCE 5

Activity 2

Name ____________________

Carry out these subtractions *without* the aid of your lima-bean computer. Renaming has already been shown, with slashes, in Problems 3, 4, and 5. Show with slashes any other renaming that you use in solving the problems.

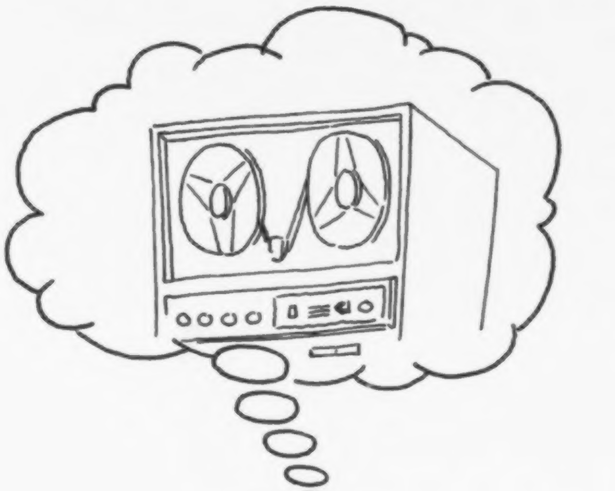
1. 372
 – 161

2. 6,931
 – 811

3. 1/11/2
 – 8 1

4. 2/13/4
 – 1 8 1

5. 1/12/13
 – 8 5



6. 223
 – 86

7. 1,631
 – 290

8. 1,821
 – 489

9. 1,611
 – 188

10. 1,231
 – 998

ARRAYS

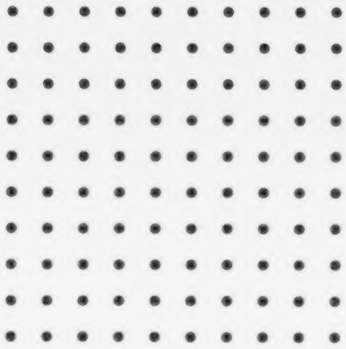
UNIT 3, EXPERIENCE 1

Activity 1

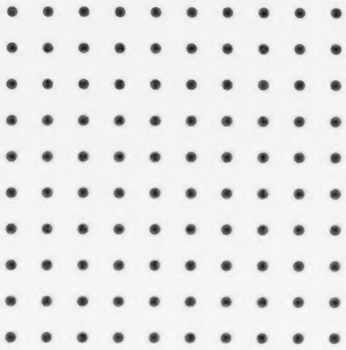
Name ______________________________

Outline rectangles to show the arrays and answer the questions that follow.

1. Outline a 4 × 5 array.



Outline a 6 × 4 array.

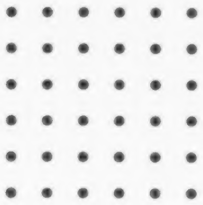


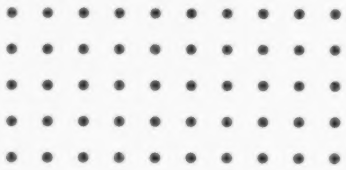
2. How many dots are in each array outlined above?

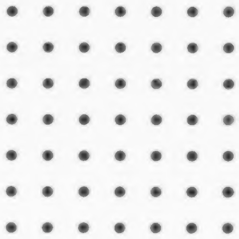
__________ __________

3. How many different arrays can you make with 12 objects? __________

4. How many dots are shown in each of these arrays?







__________ __________ __________

5. Two other arrays with 36 dots are 12 × 3 and 3 × 12. Name two more arrays with 36 dots.

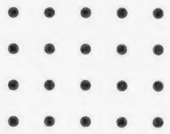
____

ARRAYS

UNIT 3, EXPERIENCE 1

Activity 2

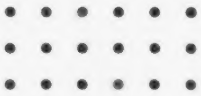
Name ____________________________



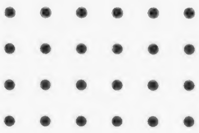
4 × 5 = 20.

The number of dots in this array can be described with the number sentence given underneath it. Write a sentence for each array given below.

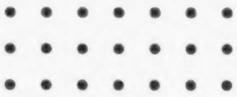
1.



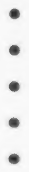
2.



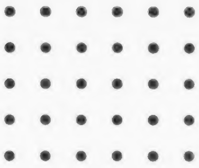
3.



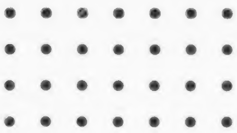
4.



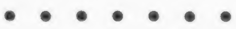
5.



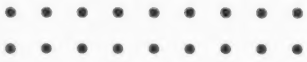
6.



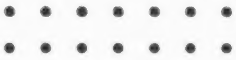
7.



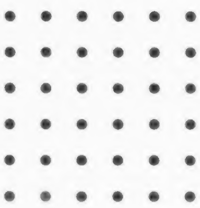
8.



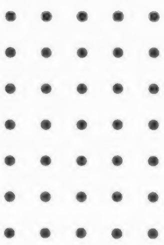
9.



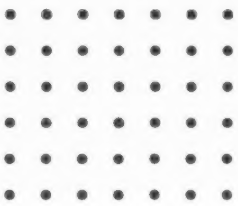
10.



11.



12.

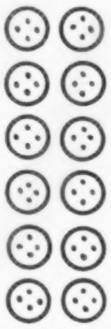
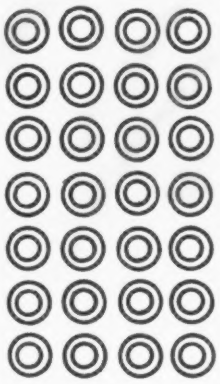


ARRAYS UNIT 3, EXPERIENCE 1

Activity 3 Name ______________________

Complete the tables.

Number of Rows	4	5		8	7		4
Number of Columns	3	4	6		6	8	
Number of Objects	12		24	24		56	28

Picture of the Array					
Name of the Array	6×2		3×4	4 × 8	6×__
Number of Objects in the Array	12				42

SOLVING EQUATIONS USING ARRAYS

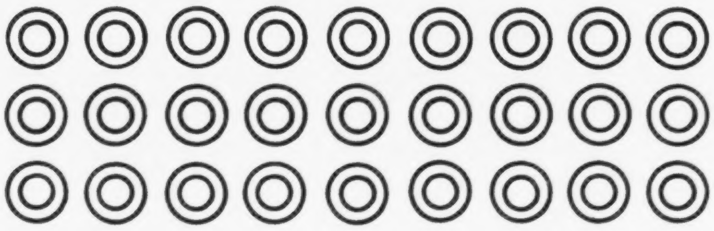
UNIT 3, EXPERIENCE 2

Activity 1

Name ______________________________

Write the name of the correct number in each box in the number sentences. If you are not sure of any result, use your set of objects to form the appropriate array.

$3 \times \square = 27.$



$3 \times \square = 21.$

$3 \times \square = 15.$

$4 \times \square = 8.$

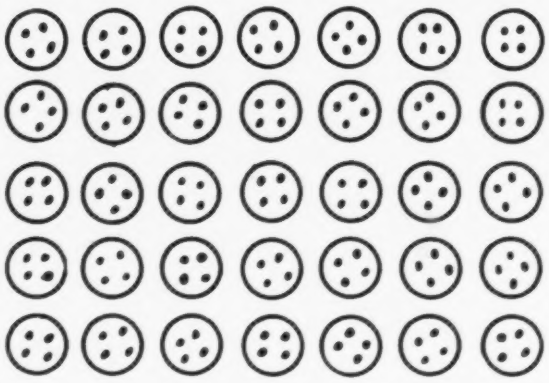
$4 \times \square = 16.$

$4 \times \square = 24.$

$4 \times \square = 28.$

$5 \times \square = 40.$

$5 \times \square = 35.$



$5 \times \square = 30.$

$\square \times 7 = 35.$

$\square \times 6 = 36.$

$\square \times 6 = 18.$

$\square \times 6 = 30.$

$\square \times 6 = 6.$

$\square \times 7 = 21.$

SOLVING EQUATIONS USING ARRAYS

UNIT 3, EXPERIENCE 2

Activity 2

Name ______________________________

Write the name of the correct number in the box in each number sentence. Use an array only when you are not sure of your answer.

$\square \times 8 = 24.$

$2 \times \square = 14.$

$\square \times 4 = 24.$

$3 \times \square = 18.$

$\square \times 7 = 21.$

$2 \times \square = 18.$

$5 \times \square = 35.$

$\square \times 9 = 18.$

$\square \times 9 = 27.$

$8 \times \square = 8.$

$8 \times \square = 16.$

$6 \times \square = 48.$

$6 \times \square = 54.$

$\square \times 9 = 36.$

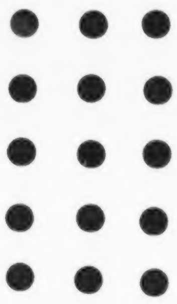
$\square \times 5 = 45.$

$9 \times \square = 81.$

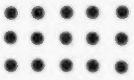
Teacher Materials

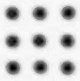
Demonstration Card

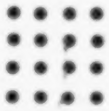
Multiplication-Bingo Cards

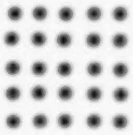
9	**5×4**	**35**
7×8		**5×9**
24	**7×6**	**18**

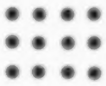
The following four sheets contain bingo cards to be reproduced for student use.

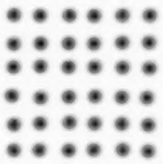
42	7×6	20
5×8		9×7
27	7×5	18

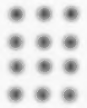
32	6×7	72
3×5		3×4
24	9×7	24

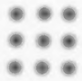
48	4×8	15
7×6		6×6
24	8×3	81

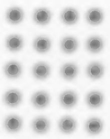
24	3×7	72
9×5		8×8
12	8×6	36

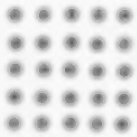
36	3×3	18
6×7		9×9
32	5×4	32

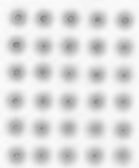
54	5×5	48
9×6		5×4
49	5×9	27

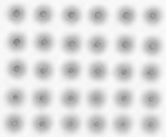
32	3×8	30
4×5		7×3
28	7×9	81

42	9×4	24
6×8		3×6
63	7×8	15

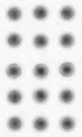
12	9×8	32
35		5×6
54	4×6	18

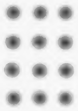
32	4×7	54
4×9		7×4
20	9×6	48

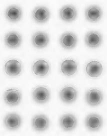
24	8×3	27
5×6		9×5
16	7×7	48

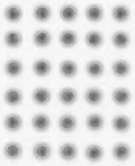
36	8×7	30
7×7		7×3
54	7×5	32

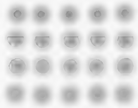
MULTIPLICATION BINGO

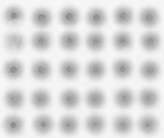
35	8×7	42
7×5		5×6
27	4×8	16

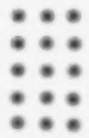
36	7×9	48
6×8		4×7
21	8×3	20

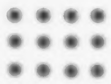
24	5×5	28
7×6		8×9
12	6×5	21

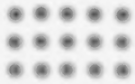
42	5×6	12
32		9×9
36	6×9	40

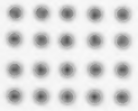
63	7×9	30
6×8		4×8
36	8×5	36

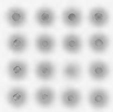
49	6×9	48
24		9×4
32	6×8	28

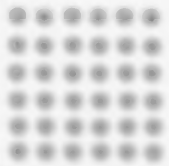
9	5×4	35
7×8		5×9
24	7×6	18

28	3×6	20
5×8		9×7
30	7×9	18

30	4×8	6×3
6×4		4×8
20	4×4	24

56	9×8	32
5×8		4×8
64	8×6	36

48	4×3	54
9×8		8×3
63	6×3	32

45	9×3	20
32		8×6
24	9×6	25

Teacher Materials

Multiplication-Bingo Announcements

The following products or pairs have been determined from random tables.

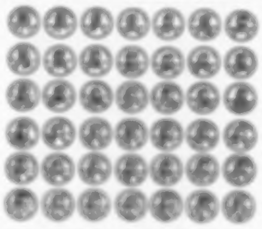
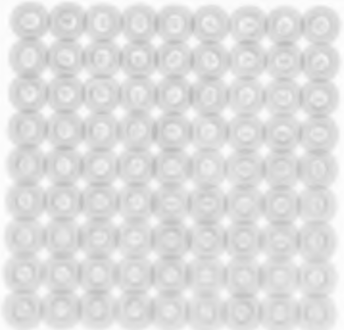
3 × 5 (Read as "three *times* five")

9	28	18	32	35	72	24
5 × 7	3 × 6	5 × 3	4 × 7	9 × 6	6 × 7	8 × 3
45	20	56	54	48	32	81
6 × 3	4 × 3	5 × 5	7 × 4	6 × 6	6 × 8	6 × 6
35	63	63	48	40	21	15
3 × 8	3 × 6	6 × 8	9 × 6	4 × 9	8 × 3	4 × 8
56	54	12	20	32	36	48
6 × 5	7 × 9	9 × 8	4 × 9	7 × 9	5 × 4	5 × 8
16	28	32	24	63	48	27
4 × 8	5 × 6	5 × 6	5 × 6	7 × 6	4 × 7	7 × 5
18	40	18	24	12	48	18
3 × 8	8 × 4	4 × 6	3 × 9	6 × 5	7 × 9	9 × 7
20	24	54	45	21	36	20
6 × 4	6 × 6	7 × 5	8 × 6	8 × 9	6 × 7	7 × 6
48	30	56	49	28	32	35
4 × 3	7 × 3	9 × 8	4 × 4	5 × 5	3 × 6	3 × 9
54	81	32	24	24	20	42
8 × 3	4 × 7	4 × 8	9 × 4	3 × 5	8 × 4	4 × 8
32	20	36	56	24	63	16
6 × 3	6 × 7	8 × 6	6 × 5	9 × 7	3 × 3	5 × 6
63	36	64	21	28	36	42
9 × 8	8 × 8	6 × 6	8 × 4	5 × 8	7 × 6	8 × 7

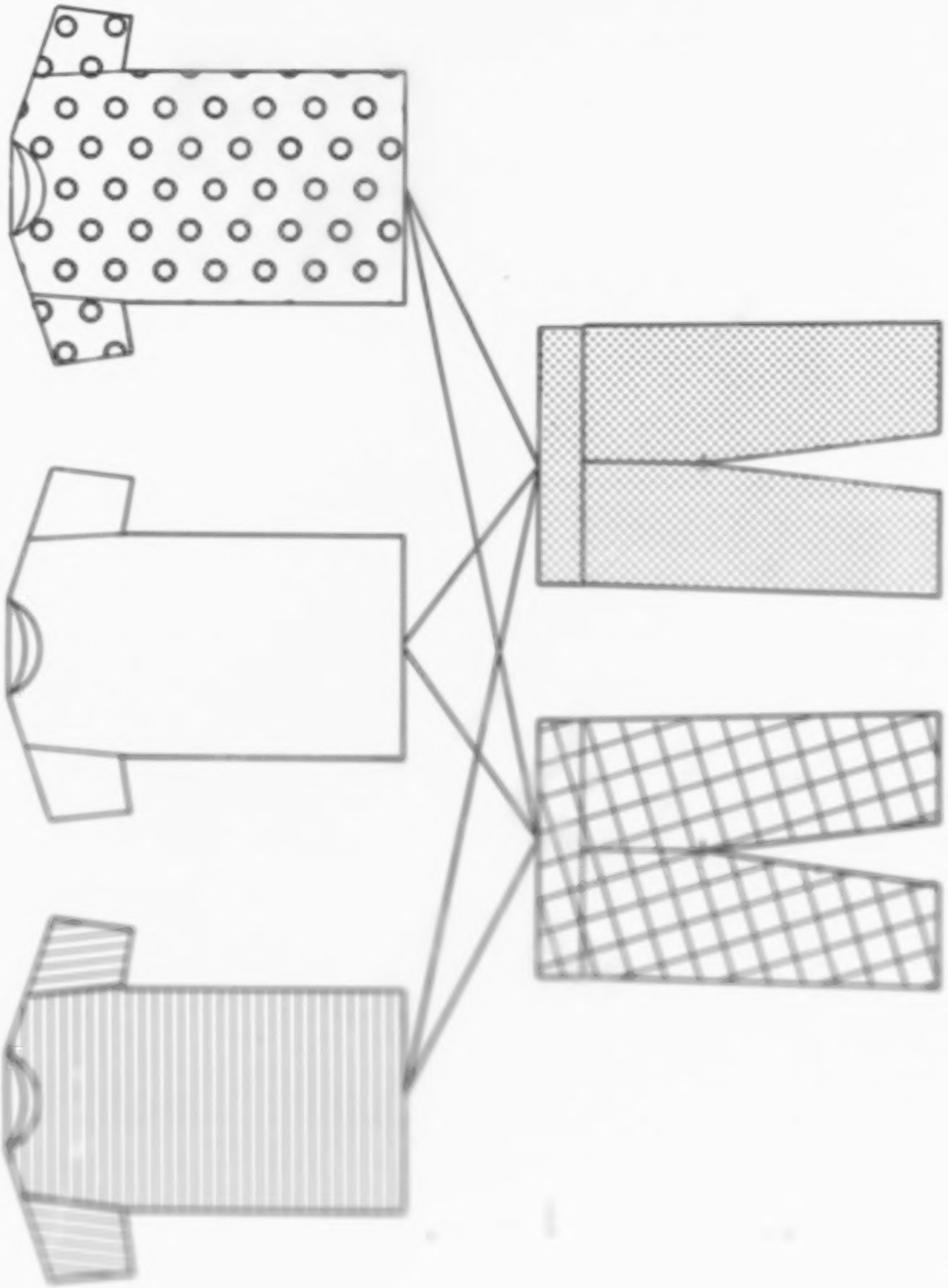
MULTIPLICATION BINGO UNIT 3, EXPERIENCE 3

Activity 2 Name ______________________________

Fill in each space with an array, a product, or missing factor(s).

Array	Factors	Product
	___ × ___	___
	3 × 8	___
	6 × ___	48
	___ × 4	36
	___ × ___	___

Teacher Materials



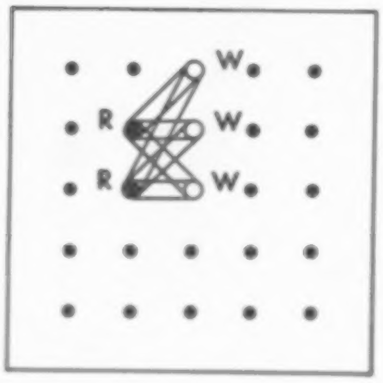
CROSS PRODUCTS

UNIT 3, EXPERIENCE 4

Activity 1

Name ______________________

Connect each red post with each white post by means of rubber bands. Record your answers, remove the bands, and repeat for the next geoboard.



Number of red posts 2
Number of white posts 3
Number of rubber bands 6
2 × 3 = 6.

A

Number of red posts ____
Number of white posts ____
Number of rubber bands ____
1 × 4 = ____

B

Number of red posts ____
Number of white posts ____
Number of rubber bands ____
2 × ____ = ____

C

Number of red posts ____
Number of white posts ____
Number of rubber bands ____
____ × 3 = ____

D

Number of red posts ____
Number of white posts ____
Number of rubber bands ____
____ × ____ = ____

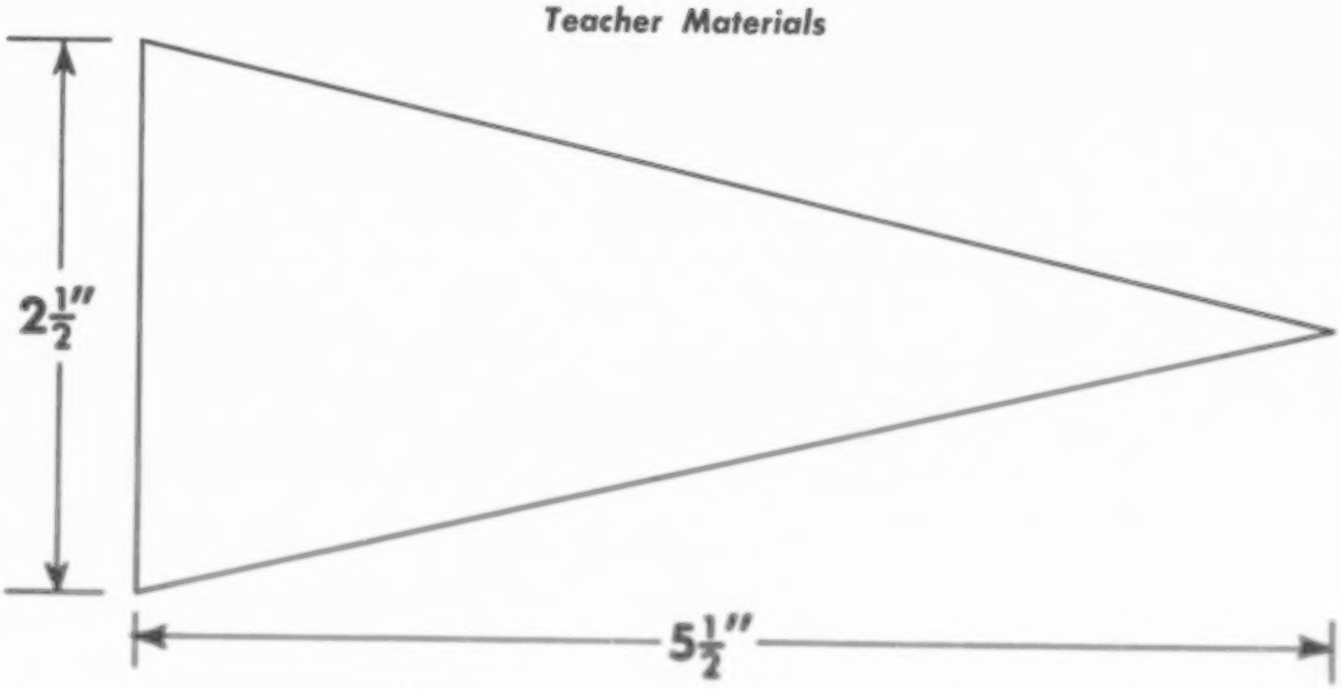
E

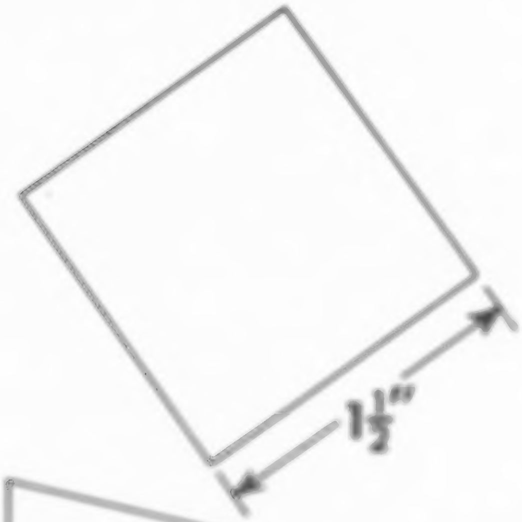
Number of red posts ____
Number of white posts ____
Number of rubber bands ____
____ × ____ = ____

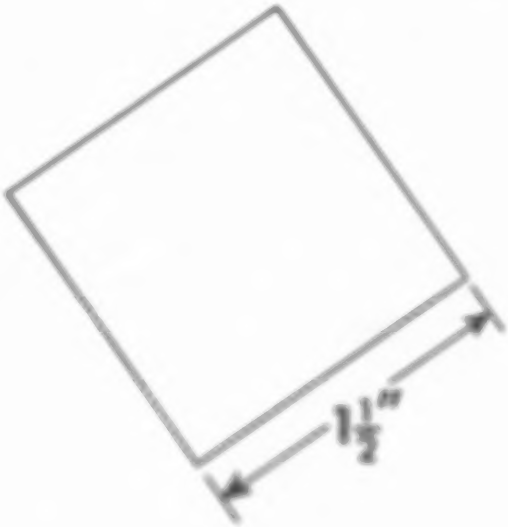
F

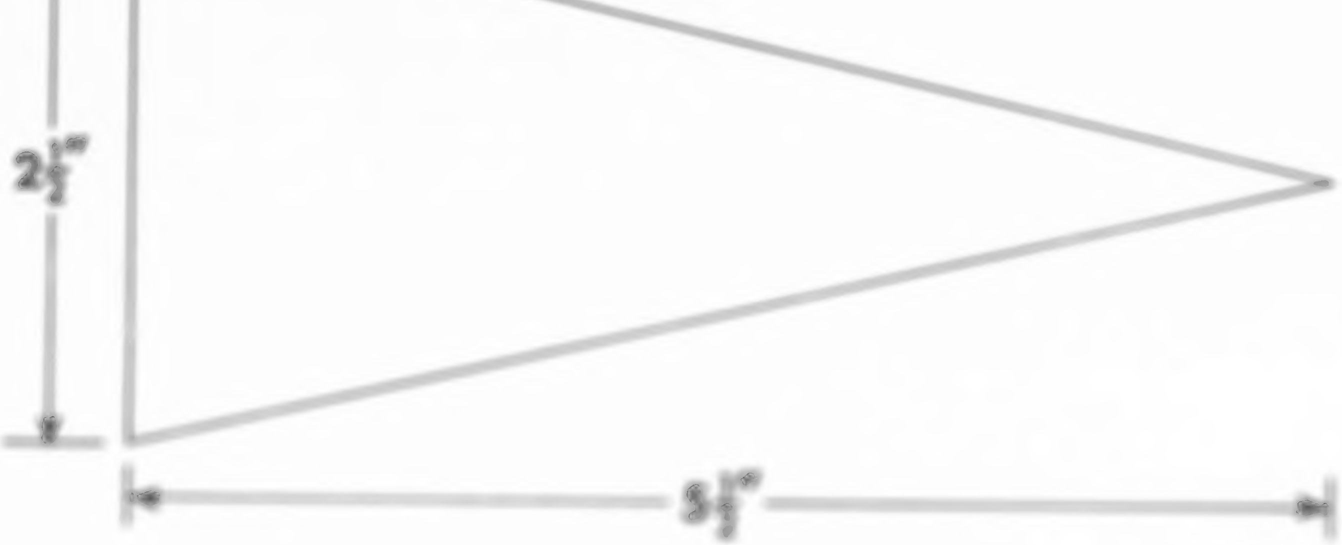
Number of red posts ____
Number of white posts ____
Number of rubber bands ____
____ × ____ = ____

Teacher Materials


$2\frac{1}{2}''$
$5\frac{1}{2}''$

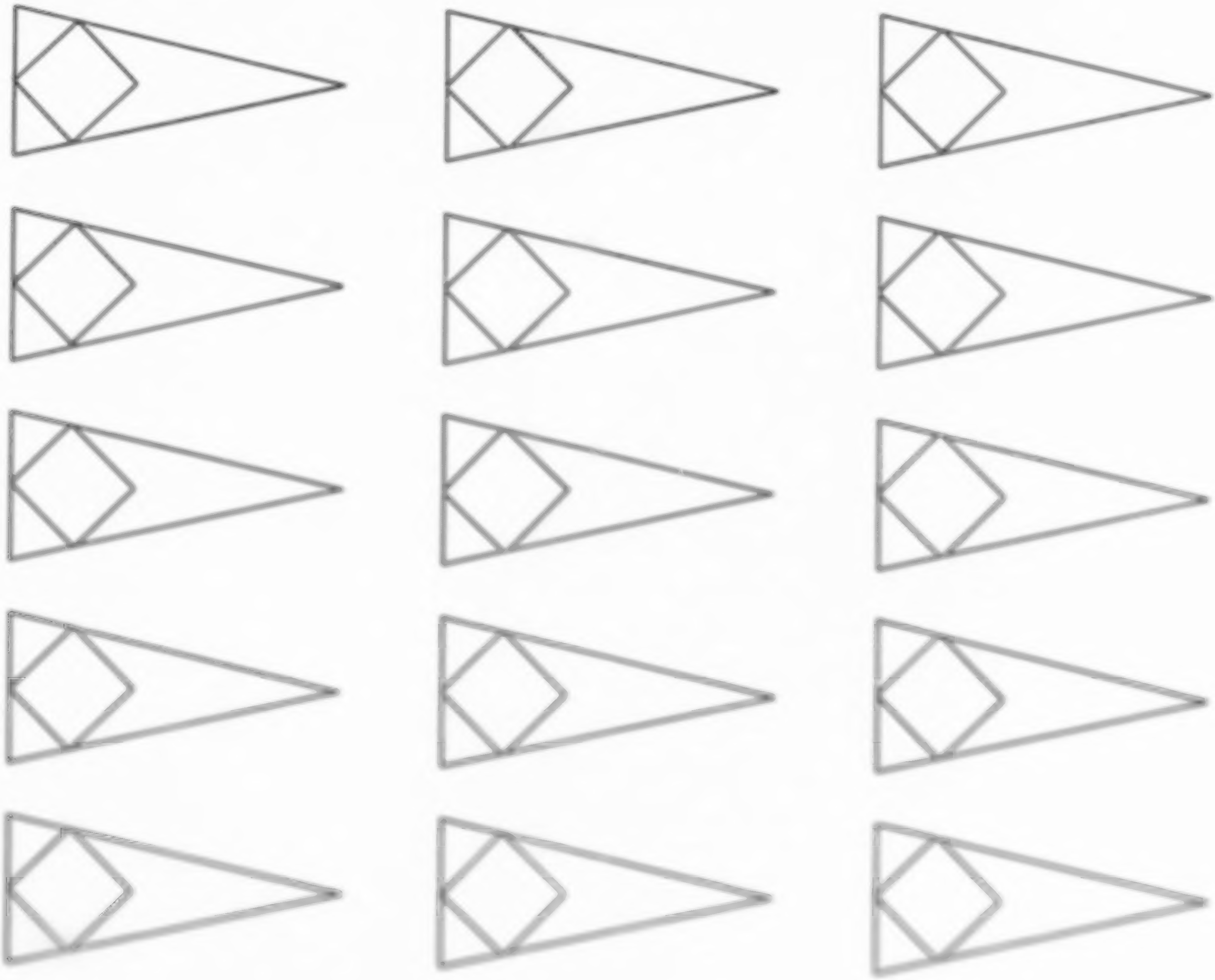





$2\frac{1}{2}''$
$5\frac{1}{2}''$

Activity 2 Name ______________________________

Make as many different pennants as you can, using the colored triangles and squares in the envelope, and color the pennants below to match. Color only as many pennants as you can really make.



How many different shapes like this, ▷, do you have? __________

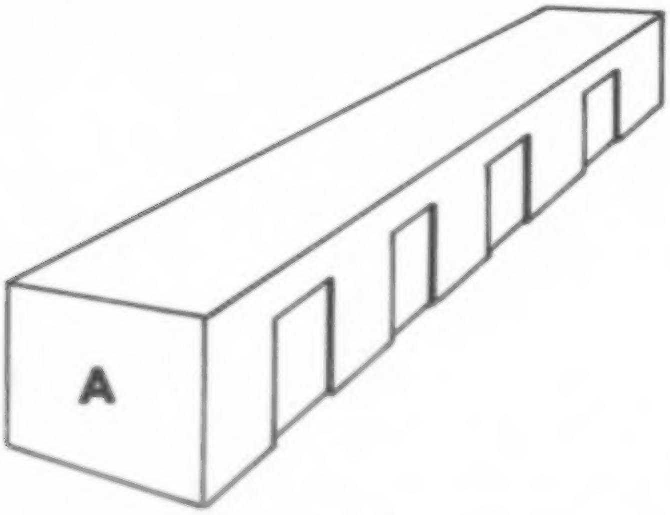
How many different shapes like this, ◇, do you have? __________

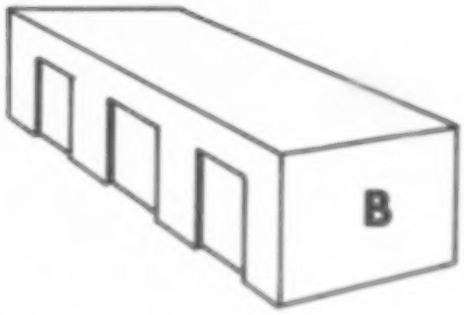
How many different pennants did you make? __________

__________ × __________ = __________

Activity 3 Name ______________________________

1. Draw a sidewalk from each door in Building A to each door in Building B, then answer the questions and write a number sentence.

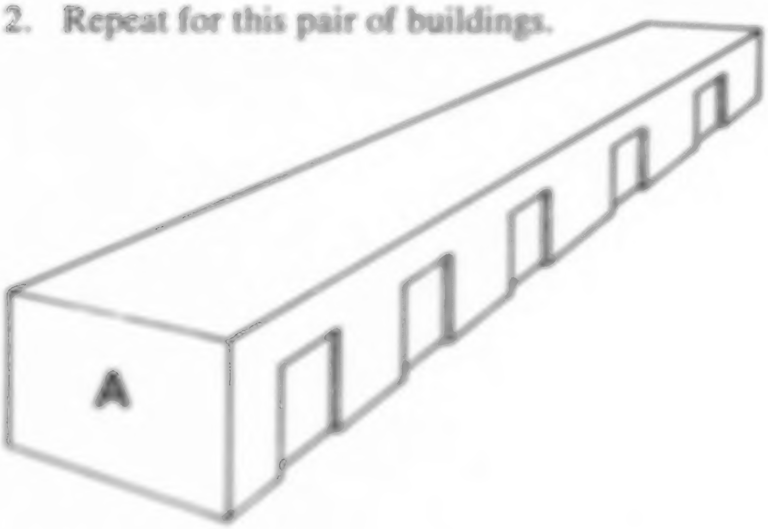


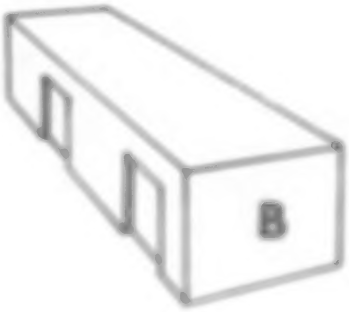


How many doors in Building A? ________ How many sidewalks? ________

How many doors in Building B? ________ ________ × ________ = ________.

2. Repeat for this pair of buildings.





How many doors in Building A? ________ How many sidewalks? ________

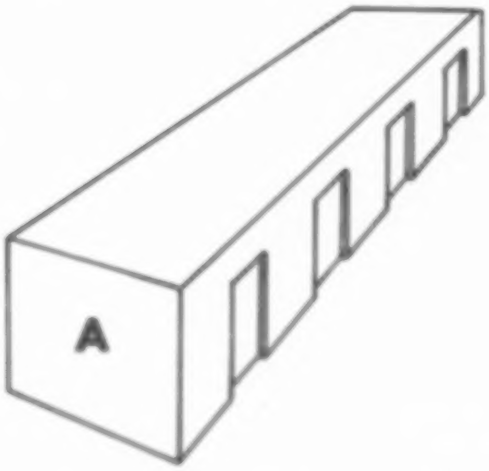
How many doors in Building B? ________ ________ × ________ = ________

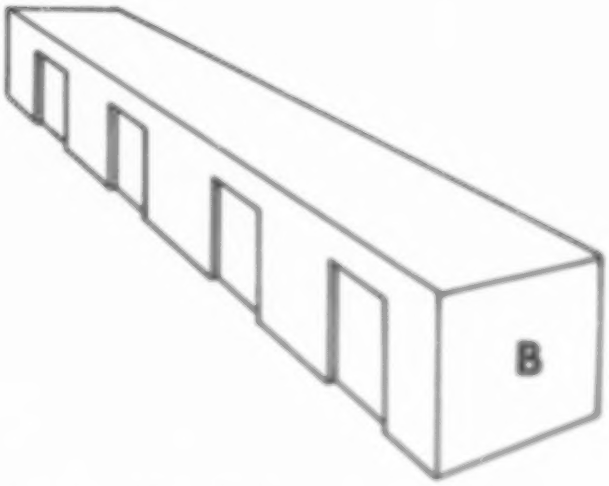
[Continued]

Activity 3–*Continued*

Name ______________________________

3. Repeat for this pair of buildings.





How many doors in Building A? ________

How many doors in Building B? ________

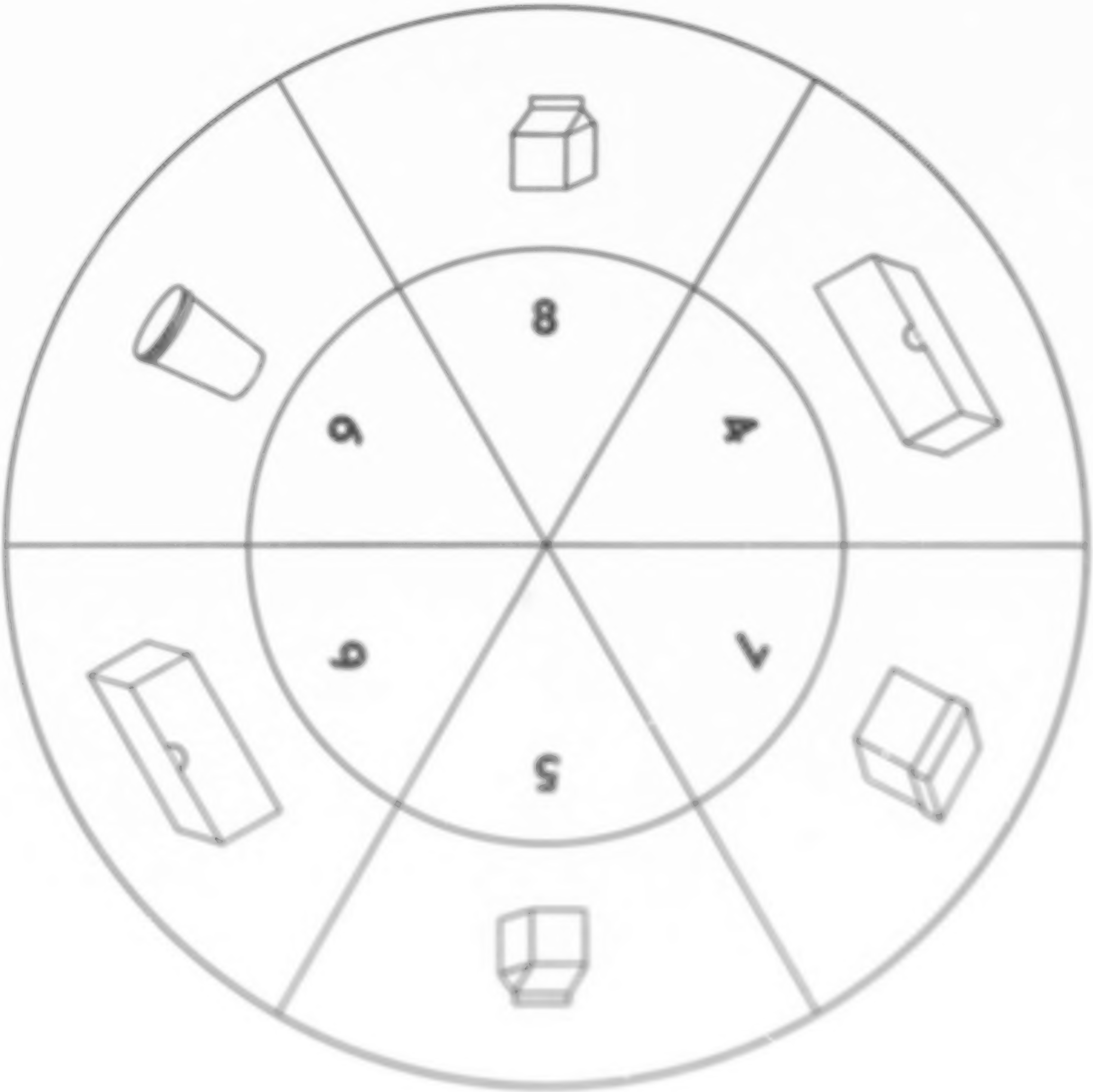
How many sidewalks? ________

________ × ________ = ________.

4. In the chart below different buildings labeled A and B have sidewalks connecting each door in one to each door in the other. Fill in all the blanks.

Doors in Building A	Doors in Building B	Total Sidewalks	Number Sentences
1	4		1 × 4 =
5		20	× =
	5	15	× =
6	3		× =
	5	25	× =
7		21	× =
4	6		× =

Teacher Materials

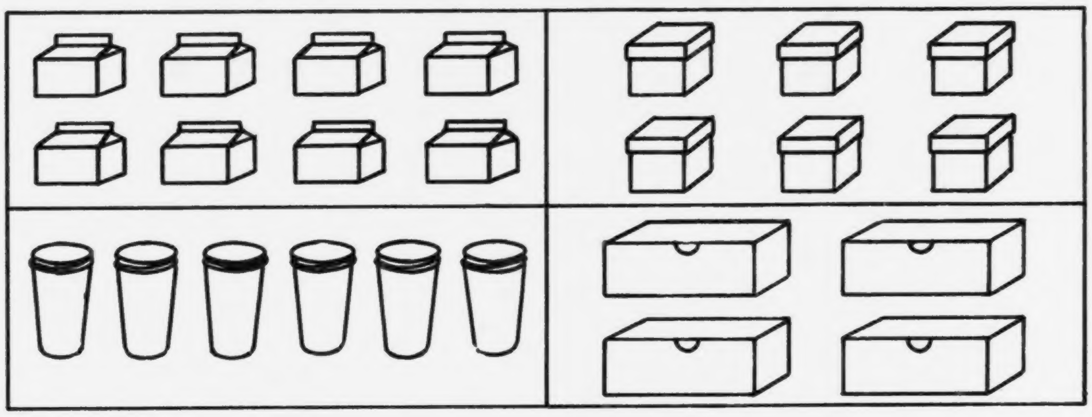


Activity 1 Name ______________________________

BOX THE CIRCLE

All players are to keep score. The game is completed when each player has had one turn; the one with the highest score wins.

1. Spin the spinner once to select a box from the outer ring. Match this with the number of boxes, shown below, and enter that number on the scoresheet.
2. Spin the spinner a second time to select, from the inner ring, the number of objects in each box.
3. Enter the total number of objects in the last column.
4. At the end of the game, mark the winner's name with a star.



Game 1

Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

[*Continued*]

Name ____________________________

BOX THE CIRCLE—Continued

Game 2

Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

Game 3

Player	Number of Boxes	Number of Objects in Each Box	Total
1.			
2.			
3.			
4.			
5.			
6.			
7.			

REPEATED ADDITION AND MULTIPLES

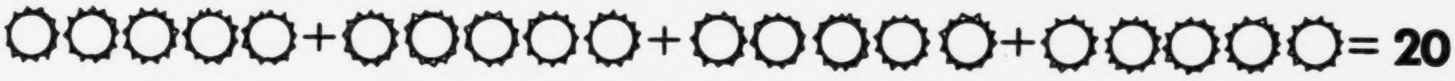
UNIT 3, EXPERIENCE 5

Activity 2

Name ____________________

Complete the number sentences below by writing the standard name for the answer.

Example: 5 + 5 + 5 + 5 = 20.



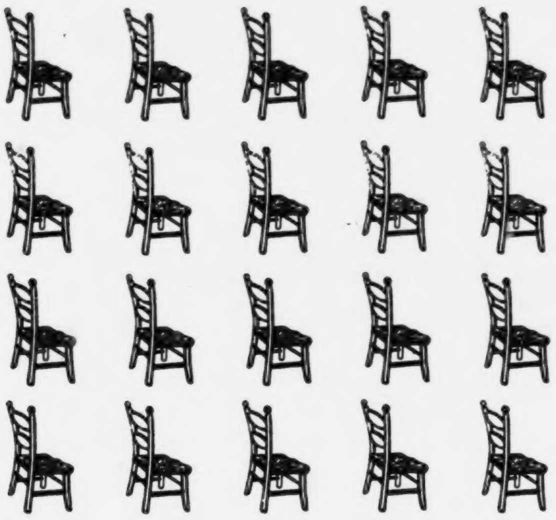
1. 6 + 6 + 6 + 6 = _____.
2. 8 + 8 + 8 + 8 = _____.
3. 7 + 7 + 7 + 7 = _____.
4. 9 + 9 + 9 + 9 = _____.

5. 3 + 3 + 3 + 3 + 3 + 3 = _____.
6. 5 + 5 + 5 + 5 + 5 + 5 = _____.
7. 4 + 4 + 4 + 4 + 4 + 4 = _____.
8. 7 + 7 + 7 + 7 + 7 + 7 = _____.

9. 6 + 6 + 6 = _____.
10. 8 + 8 + 8 = _____.
11. 7 + 7 + 7 = _____.
12. 9 + 9 + 9 = _____.

13. 2 + 2 + 2 + 2 + 2 + 2 + 2 = _____.
14. 9 + 9 + 9 + 9 + 9 = _____.
15. 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = _____.
16. 9 + 9 = _____.

Activity 1 Name ________________________________

1.  This classroom has 4 rows of chairs. There are 5 chairs in each row. How many chairs in all? ____ Write a sentence for this problem.

____

2. A classroom has 7 rows of desks. There are 4 desks in each row. How many desks in all? _________ Use the space at the right to draw a picture for this problem. Write a sentence for it.

____

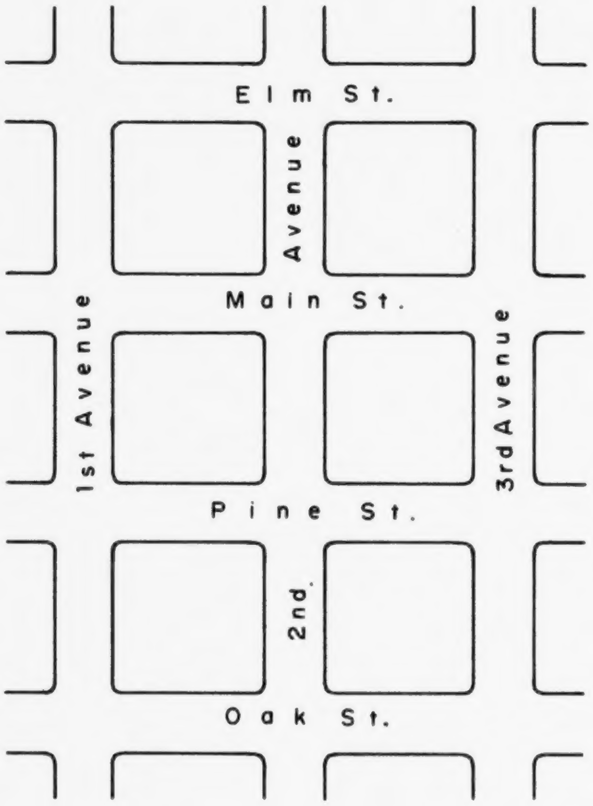
3. An orchard has 42 trees. There are 6 rows of trees. How many trees in each row? _________ Use the space at the right to draw a picture for this problem. Write a sentence for it.

____

Activity 2

Name ______________________________

1.



This is a map of Tiny Town. Each place a street and avenue cross is called an intersection. Place an X on each intersection.

How many avenues? ______ How many streets? ______ How many intersections? _____

Write a sentence for this problem.

____

2. Big Town has 7 streets and 5 avenues. Use the space at the right to draw a map of Big Town. How many intersections in Big Town? ______ Write a sentence for this problem.

____

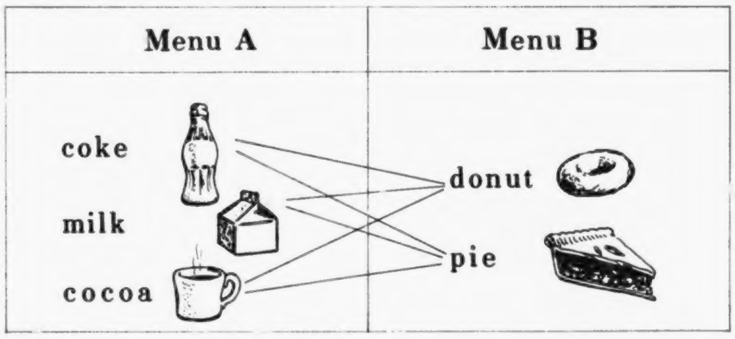
3. Little City has 49 intersections. It has 7 streets. Use the space at the right to draw a map of Little City. How many avenues in Little City? ______ Write a sentence for this problem.

____

PROBLEM SOLVING UNIT 3, EXPERIENCE 6

Activity 3 Name ______________________________

1.



In the space at the right, list all the possible pairs from Menu A and Menu B. You must choose one item from each menu for each pair you list. Write a sentence for this problem.

____

2.

Boys	Girls
John	Mary
Jim	Betty
David	Susan
Bill	
Don	

In the space at the left, list all the possible pairs from the boys and girls. You must use one boy's name and one girl's name for each pair you list. Write a sentence for this problem.

____

Activity 4 Name ______________________________

1. Try tossing a penny with a friend.

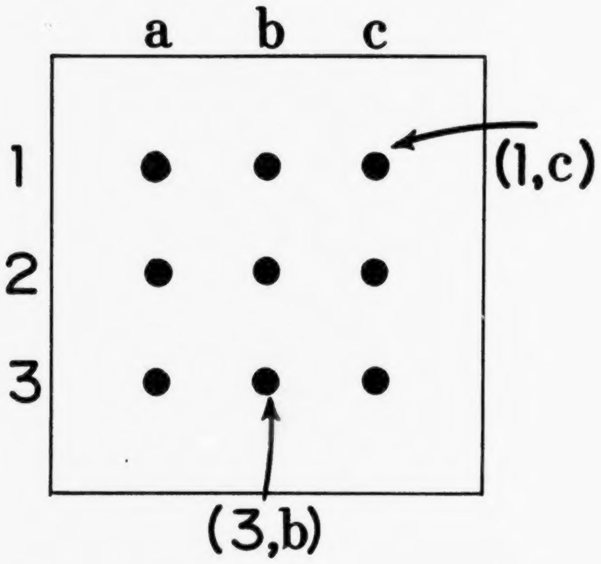
   List all the ways the penny could land in two throws.

Penny 1	Penny 2
heads 1	heads 2
tails 1	tails 2

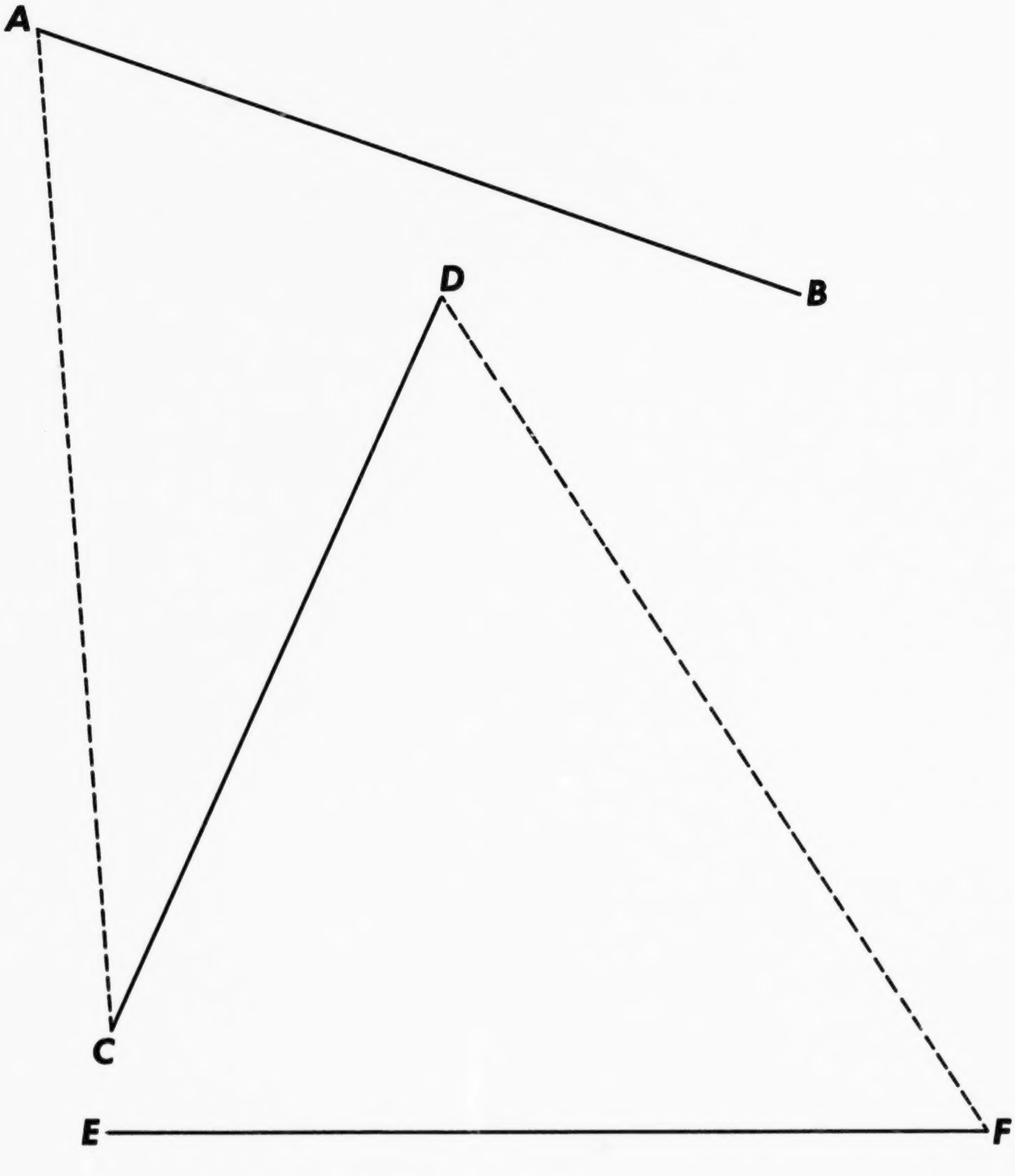
   Write a sentence for this problem. ________________

2. Each dot in the array is named with a number and a letter. List the names of all the dots in the array. Write a sentence for this problem.

   ____



Drawing 1



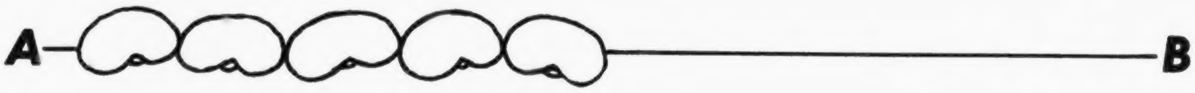
MEASURING LENGTH UNIT 4, EXPERIENCE 1

Name __________________________

Activity 1–Using Lima Beans to Measure Length

Use your lima beans to measure the line segments in Drawing 1 to get the answers to the following questions.

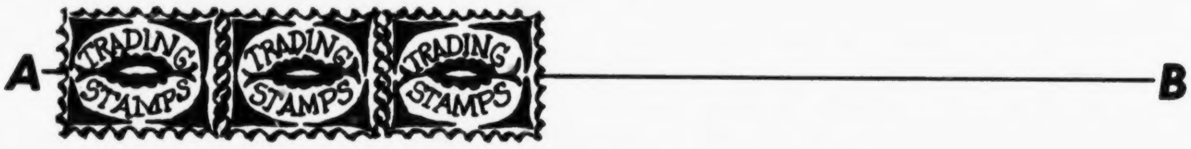
1. How many lima beans long is line segment $\overline{AB}$? __________ Segment $\overline{CD}$? __________ Segment $\overline{EF}$? __________
2. Which line segments are longer than line segment $\overline{AB}$? __________
3. Which line segments are shorter than segment $\overline{AB}$? __________
4. In Drawing 1, how far is it from point *A* to point *C* (in lima beans)? __________
5. Will all the other students get the same answers to Questions 1–4? __________



Activity 2–Using Trading Stamps to Measure Length

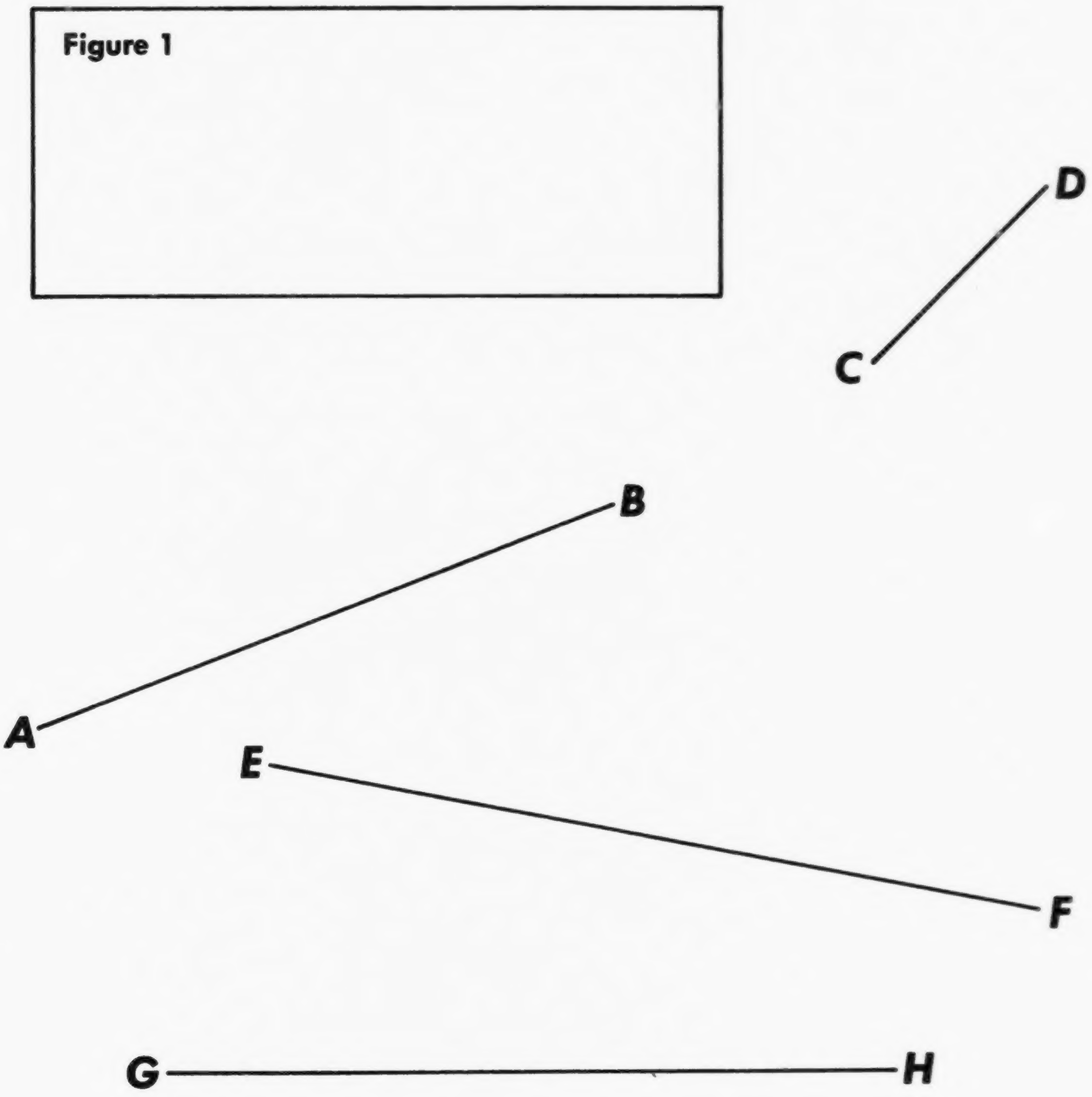
Use your trading stamps and Drawing 1 to answer the following questions.

1. How many trading stamps long is line segment $\overline{AB}$? ________ Segment $\overline{CD}$? ________ Segment $\overline{EF}$? __________
2. Which line segments are longer than segment $\overline{AB}$? __________
3. In Drawing 1, how far is it from point *D* to point *F* (in trading stamps)? __________
4. Will all the other students get the same answers to these questions? __________



Activity 3 Name ______________________________

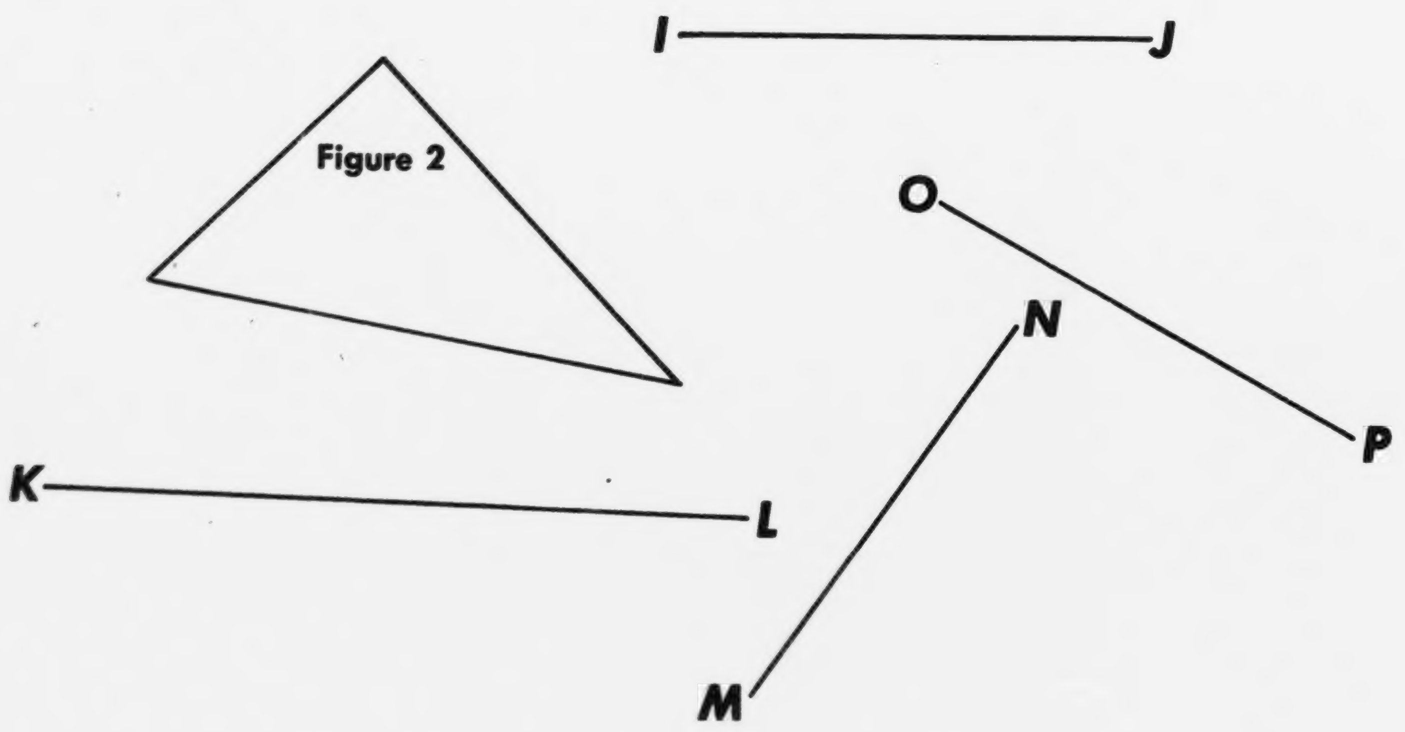
Each of the following three exercises shows one closed figure and several line segments. Draw a ring around those line segments that will fit inside the closed figure.

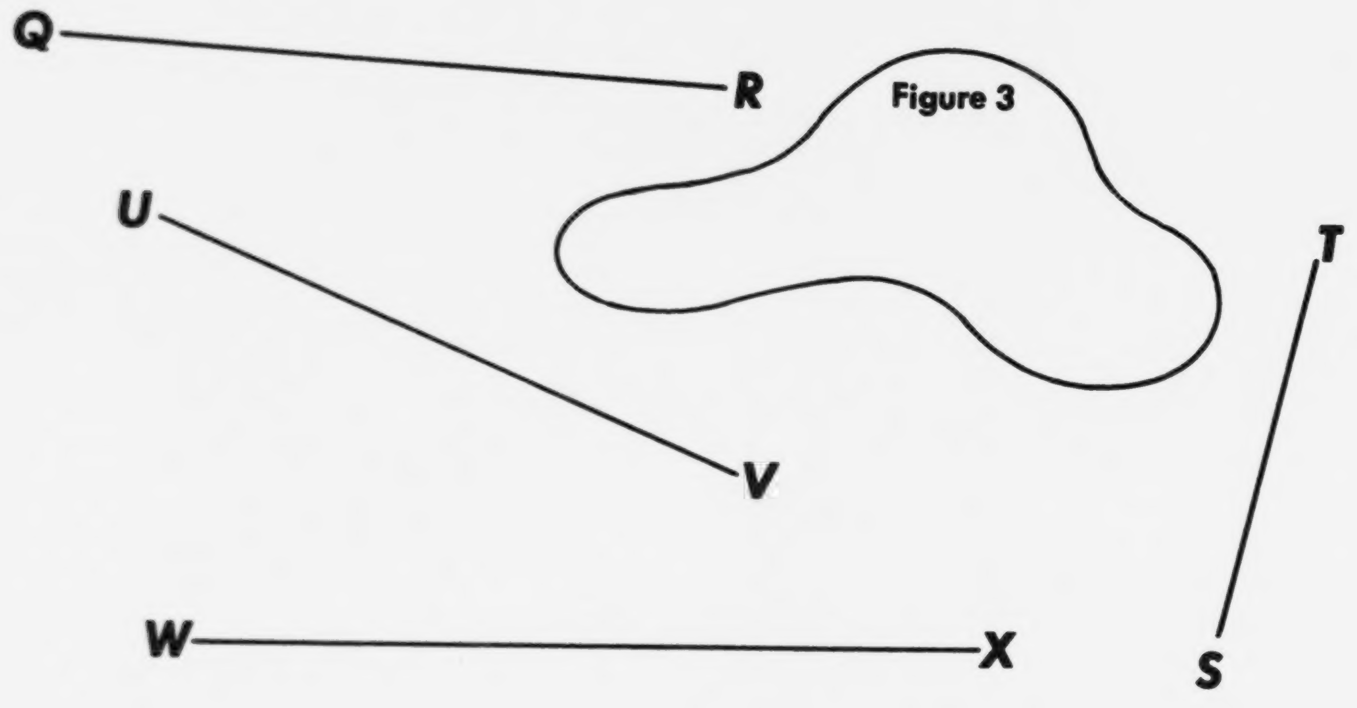


[*Continued*]

Activity 3–*Continued*

Name ______________________________


I
J
Figure 2
O
N
P
K
L
M


Q
R
Figure 3
U
T
V
W
X
S

Drawing 2

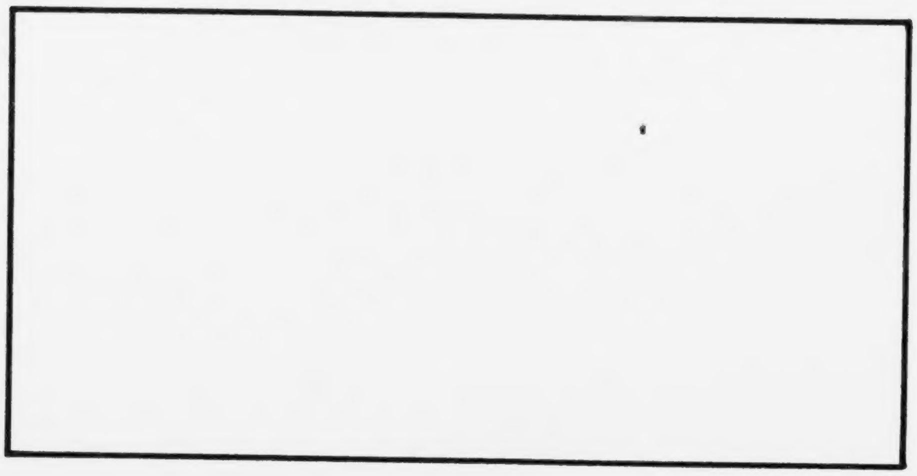


Figure 1

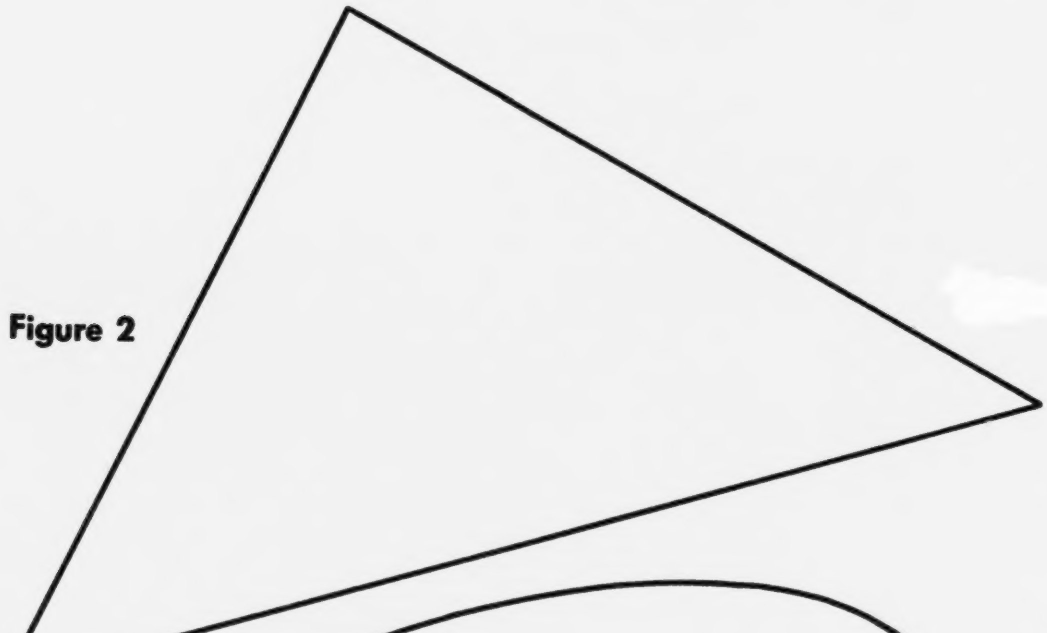


Figure 2

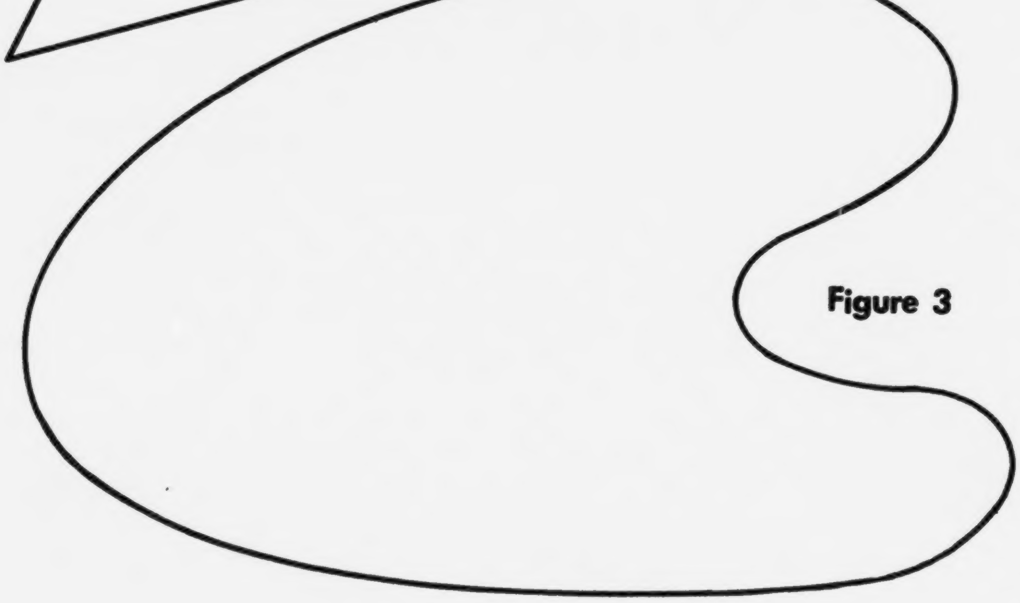


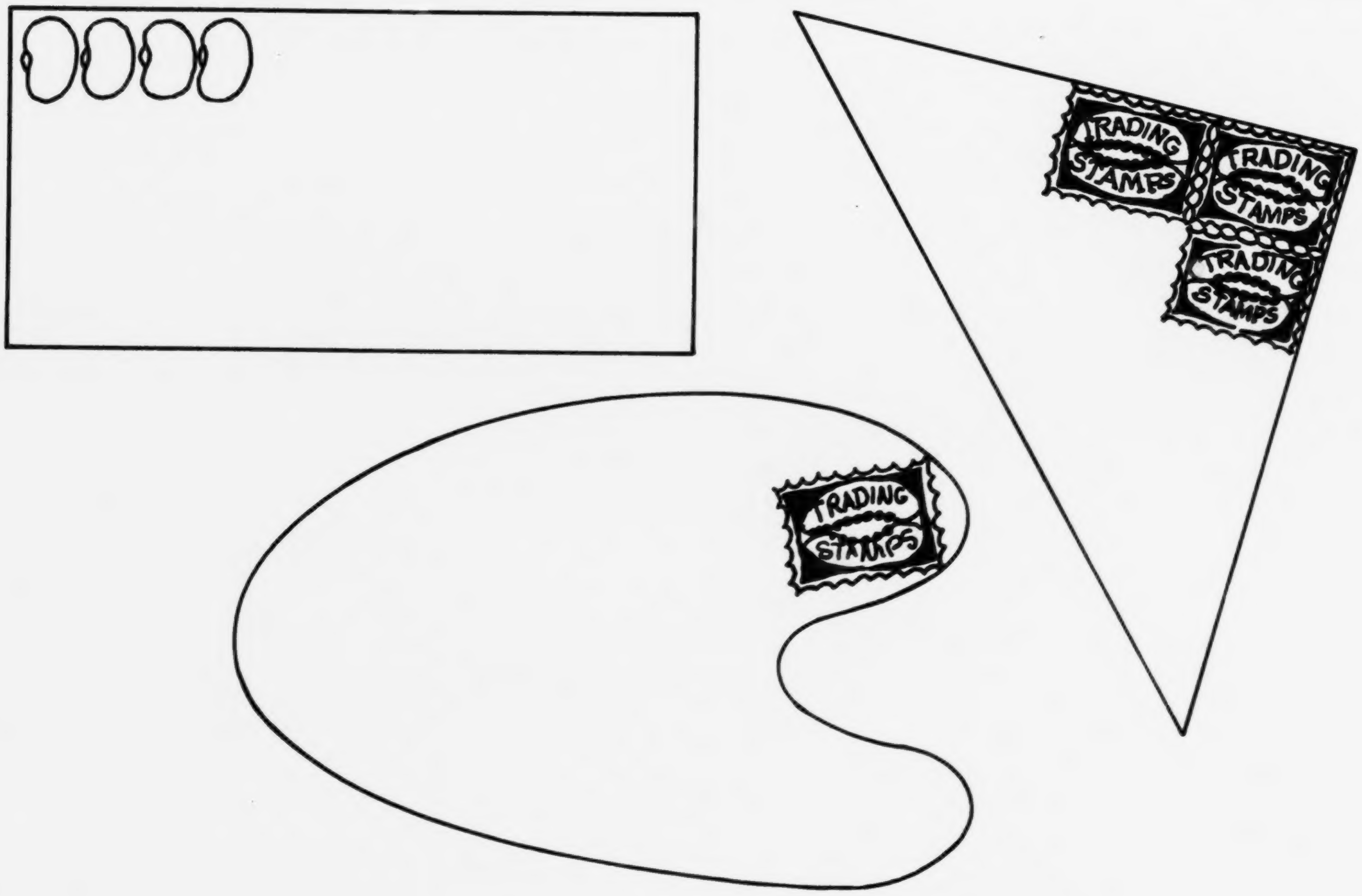
Figure 3

MEASURING AREA UNIT 4, EXPERIENCE 2

Name ______________________________

Activity 1: Use Drawing 2 and your lima beans to answer these questions.

1. How many lima beans can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________
2. Which figure contains the most lima beans? ________


TRADING STAMPS
TRADING STAMPS
TRADING STAMPS
TRADING STAMPS

Activity 2: Use Drawing 2 and your trading stamps to answer these questions.

1. How many trading stamps can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________

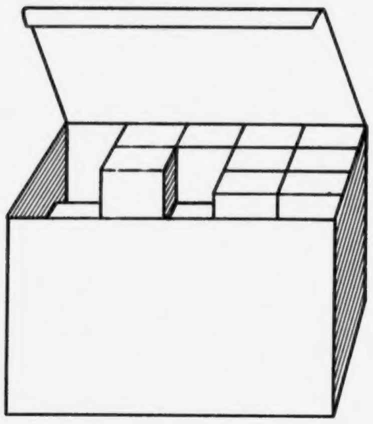
Activity 3: Use only one stamp to answer these questions. You may draw on Drawing 2 if you wish.

1. How many trading stamps like this one can you place within Figure 1? ________ Figure 2? ________ Figure 3? ________

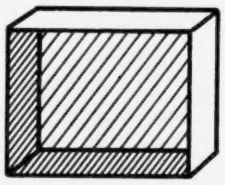
Station A

Name ____________________

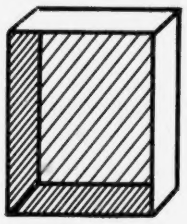
At this station are some blocks and Box A. Use them to answer these questions.



1. What is the largest number of blocks like these that you can fit into the box? __________
2. How many blocks can you fit into one layer? __________
3. How many layers can you get into the box? __________
4. If you put the box on its long side, how many blocks can you put in one layer? __________



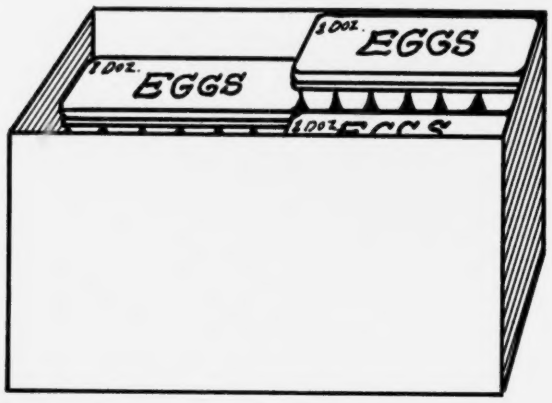
5. If you stand the box on end, how many blocks can you put in one layer? __________



MEASURING VOLUME UNIT 4, EXPERIENCE 3

Station B Name ______________________________

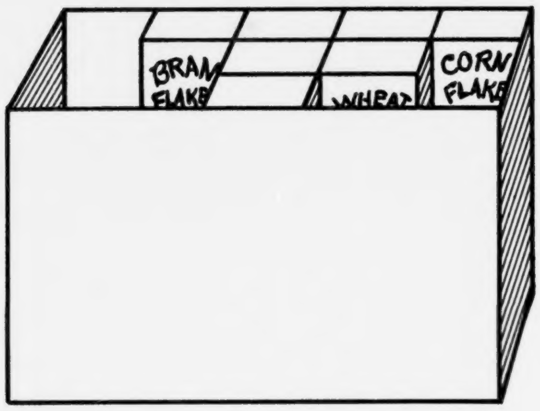
At this station you find a large cardboard box and some egg cartons. Use them to answer these questions.


1 DOZ.
EGGS
1 DOZ.
EGGS
1 DOZ.

1. How many egg cartons like this would it take to fill the box? __________
2. How many eggs does each carton hold? __________
3. If the box were filled with egg cartons and the egg cartons were filled with eggs, how many eggs would the box hold? __________
4. How many dozens of eggs are in the box when it is filled with egg cartons? __________

Station C Name ______________________________

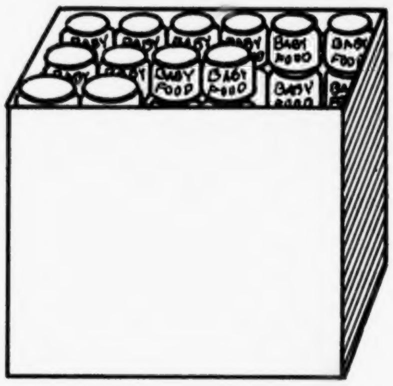
At this station there are small cereal boxes and a large cardboard box. Use these materials to answer the following questions.



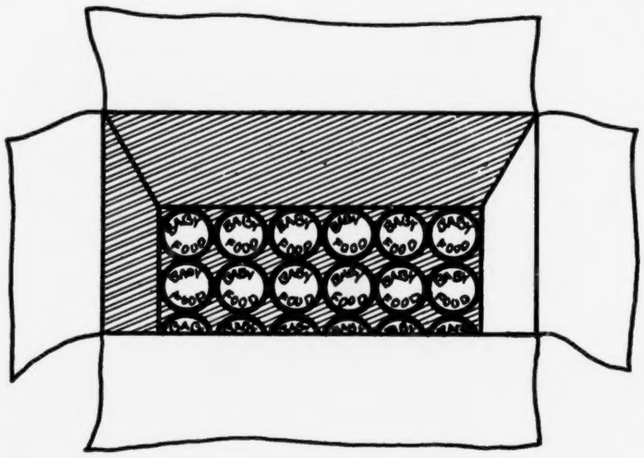
1. What is the *largest* number of cereal boxes that you can place in one layer on the bottom of the box? __________
2. What is the *smallest* number of cereal boxes that you can place in one layer and still cover the bottom of the box? __________
3. How many cereal boxes would it take to fill the large box? __________

Station D Name ______________________________

Baby-food jars and a cardboard box are at this station. Use them to answer the following questions.



1. How many baby-food jars high is the box? __________
2. How many of the jars can you put in one layer? __________

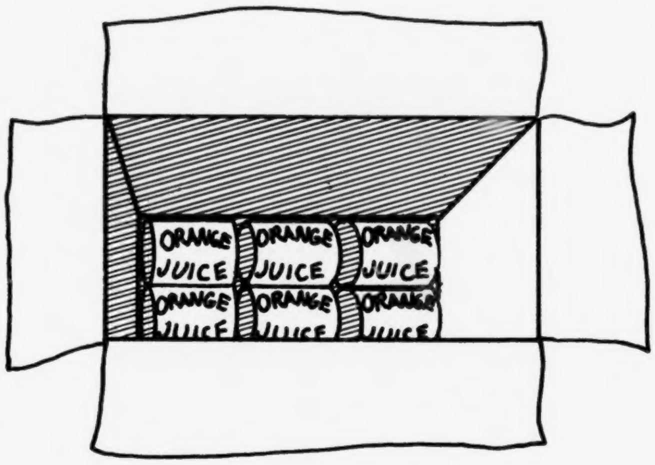


3. How many baby-food jars will the box hold? __________
4. If a jar of baby food weighs 4 ounces, how many pounds of baby food in jars will the box hold? __________

Station E Name ______________________________

At this station there are some frozen-juice cans and a box. Use them to answer the questions.

1. How many cans does it take to cover the bottom of the box? __________


ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE
ORANGE JUICE

2. How many cans will cover the bottom of the box if you lay the cans on their sides?

____

3. How many frozen-juice cans can you pack in the box? __________

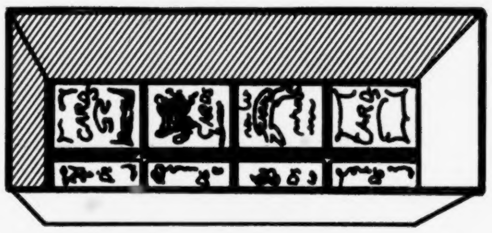
4. Can you pack in more cans by laying them down or by standing them up?

______________________________ How many more? __________

MORE ABOUT MEASURING VOLUME UNIT 4, EXPERIENCE 4

Station A Name ______________________________

You will find decks of cards and a shoe box at this station. Use them to answer the first question, then go on to the second question.



1. What is the largest number of decks of cards that you could put in the box? __________

   How many would you put in each layer? ____________

   How many layers would you have to make? ____________

2. A packing case has 4 layers and 18 boxes in each layer. How many boxes are there in the packing case? ____________

MORE ABOUT MEASURING VOLUME

UNIT 4, EXPERIENCE 4

Station B

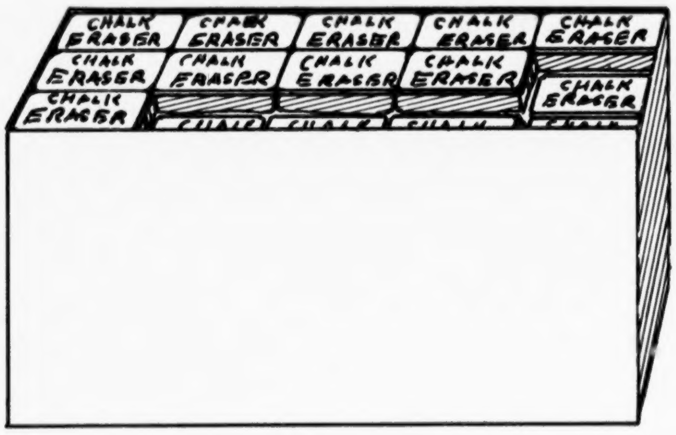
Name ____________________________

There are baby-food jars and a box at this station. Use them to help you answer the first question, then answer the second.

1. What is the largest number of jars you could put in the box? __________

   How many would you put in one layer? __________

   How many layers would you make? __________


CHALK ERASER
CHALK ERASER
CHALK ERASER
CHALK ERASER
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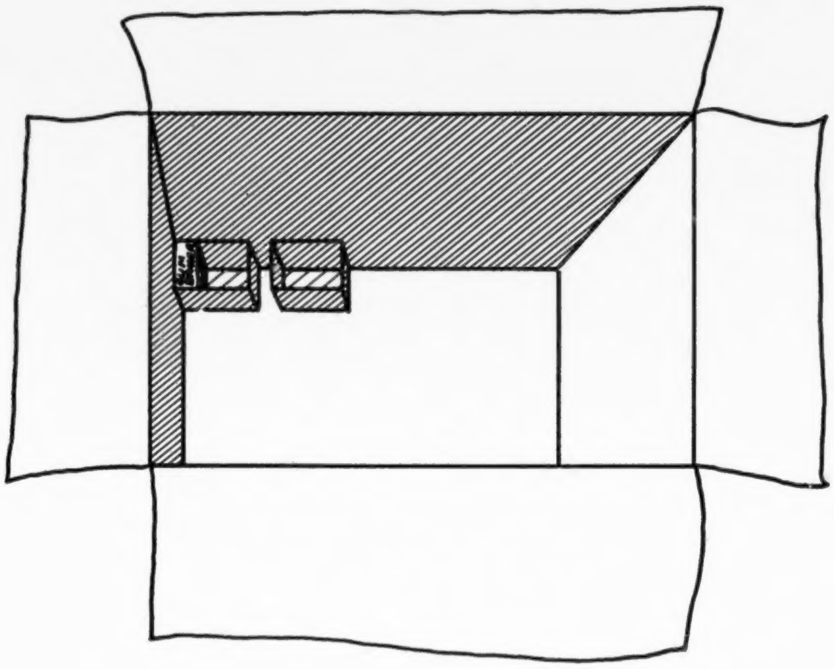
CHALK ERASER
CHALK ERASER
CHALK ERASER

2. Jim packed 100 erasers in a box. He started by putting 15 in each layer. How many layers did he make? __________

Station C Name ______________________

There are erasers and two boxes at this station. Use them to help you answer the first question, but do not stack the small boxes inside each other! Then go on to the second question.



1. What is the largest number of erasers that you could put in the small box? __________

   What is the largest number of small boxes that you could put in the large box? ________

   What is the largest number of erasers that you could put in the large box? __________

2. A carton of playing cards holds 10 decks with 52 cards in each deck. How many cards are there in the carton? __________

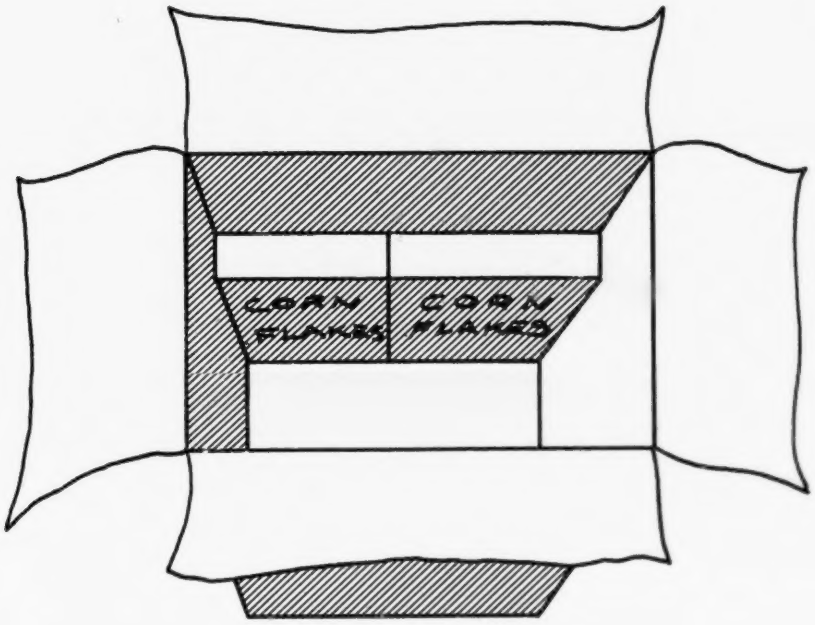
MORE ABOUT MEASURING VOLUME

UNIT 4, EXPERIENCE 4

Station D

Name ______________________________

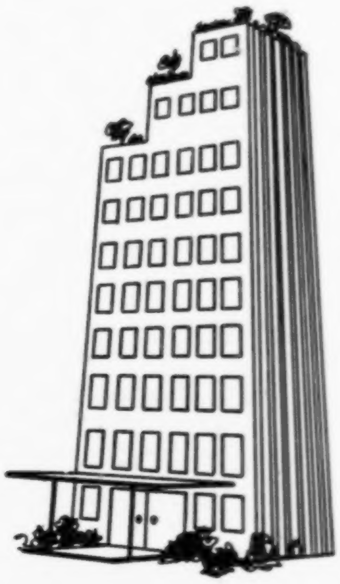
At this station there are two cereal boxes and one large box. Use them to answer the first question, then go on to the next one.



1. What is the largest number of cereal boxes that you could put in the large box? ________

   How many boxes would you put in each layer? __________

   How many layers would you have? __________



2. This apartment building has 25 apartments on each floor.

   If it is an eight-story building, how many apartments

   does the building have? __________

   If all of them are three-room apartments, how many rooms

   does the building have? __________

Station E Name ______________________________

At this station there are cubes and a box. Use them to help you answer the first question, then answer the second.

1. What is the largest number of cubes you could put in the box? __________

   How many cubes long is the box? __________

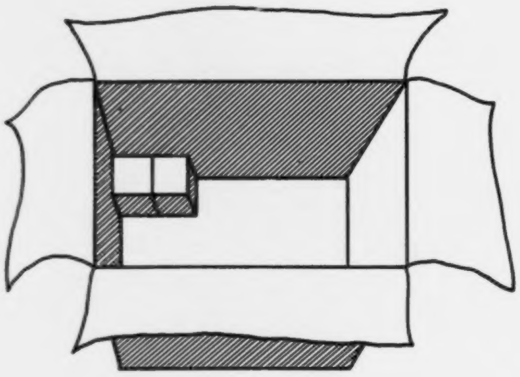
   How many cubes wide is the box? __________

   How many cubes high is the box? __________

   How many cubes would you put in each layer?

   ____

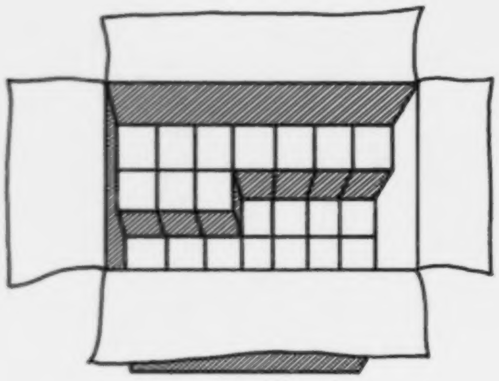
   How many layers would you have? __________



2. Dick filled two boxes with blocks. Box A had 6 layers with 28 blocks in each layer.

   Box B had 8 layers with 21 blocks in each layer. Which box held more blocks?

   ____



MORE ABOUT MEASURING VOLUME UNIT 4, EXPERIENCE 4

Station F Name ______________________________

Special–for experts only!

Use the cubes and the sealed cereal box to help you answer the first question, then go on to the second.

1. What is the largest number of cubes that you could put in the cereal box? __________

2. Tom visited his dad's warehouse. One stack of cases was 10 cases long and 15 cases wide. Tom figured out that there were 1,800 cases in the stack. How many cases high was the stack? __________

MEASURING WEIGHT

UNIT 4, EXPERIENCE 5

Name ______________________________

Use your balance and paper clips to help you answer the questions for these two activities.

Activity 1

1. If you use the paper clip as your unit of weight, what is the weight of the wooden cube? _____________ The eraser? _____________ 5 sheets of typing paper? _____________ 50 index cards? _____________
2. How many paper clips would it take to balance 2 cubes and 3 erasers? _____________
3. What is the total weight, in paper clips, of a ream (500 sheets) of typing paper? _____________

Activity 2

1. Find out how many paper clips are in any 5 of the sealed envelopes. DO NOT OPEN THEM!

Envelope A ________
Envelope B ________
Envelope C ________
Envelope D ________
Envelope E ________
Envelope F ________
Envelope G ________
Envelope H ________
Envelope I ________
Envelope J ________

2. Suppose you had as many of each of these 5 envelopes as you needed. Describe the envelopes you would need to have exactly 100 paper clips. ____
3. Describe the envelopes you would need to have exactly 51 paper clips. ____

Activity 3 Name ______________________

Use your balance and paper clips to help you answer these questions.

1. Find out how many nails are in Box A and in Box B. Then find the number of nails in as many of the other boxes as you have time for. DO NOT OPEN THE BOXES!

Box A __________ Box D __________
Box B __________ Box E __________
Box C __________ Box F __________

2. Suppose you had as many boxes of each kind as you needed. Could you make a stack of Box A's and a stack of Box B's so that both stacks would have the same number of nails? __________ If so, how would you do it? ______________________
____

Teacher Materials

RECORD CARD

Experience 1 Name ____________________________

Station 1
Envelope A ______
Envelope B ______
Envelope C ______

Station 2
Envelope D ______
Envelope E ______
Envelope F ______

Station 3
Box G ______
Box H ______

Station 4
Envelope I ______
Envelope J ______

RECORD CARD

Experience 1 Name ____________________________

Station 1
Envelope A ______
Envelope B ______
Envelope C ______

Station 2
Envelope D ______
Envelope E ______
Envelope F ______

Station 3
Box G ______
Box H ______

Station 4
Envelope I ______
Envelope J ______

RECORD CARD

Experience 1 Name ____________________________

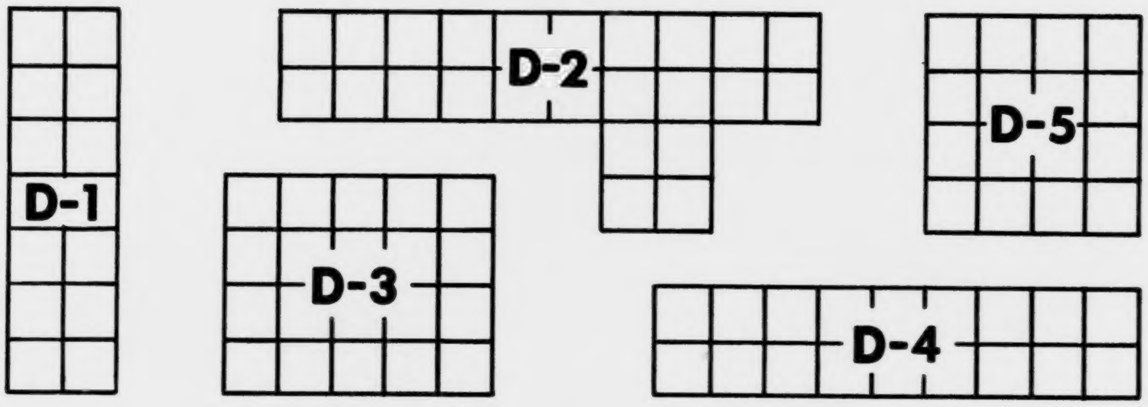
Station 1
Envelope A ______
Envelope B ______
Envelope C ______

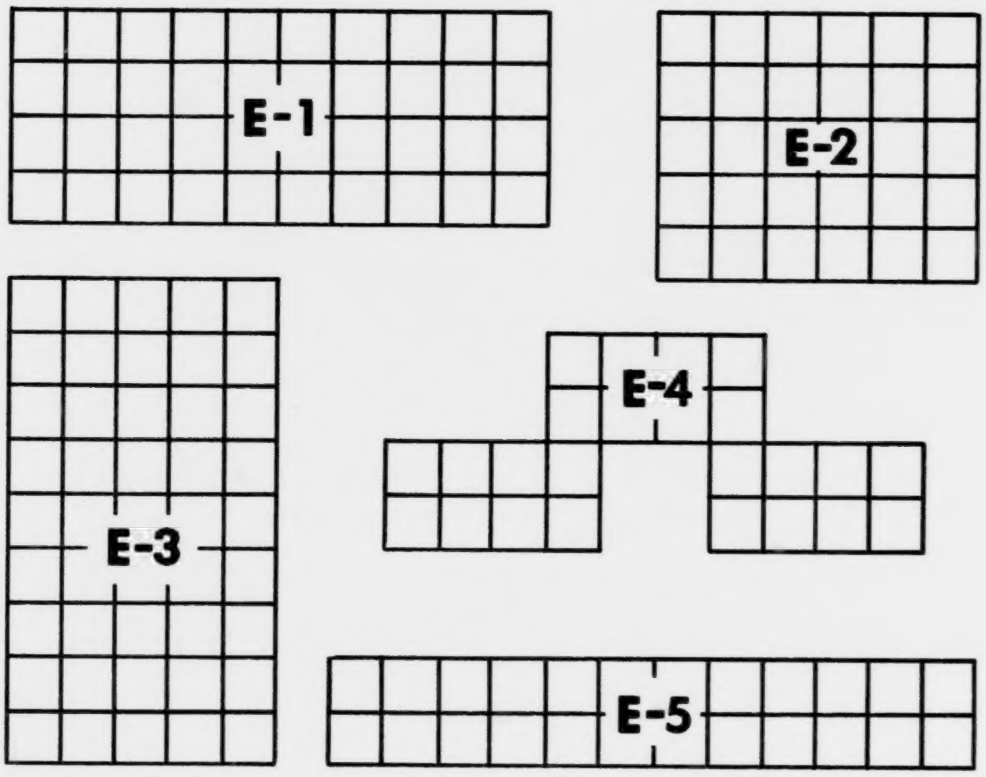
Station 2
Envelope D ______
Envelope E ______
Envelope F ______

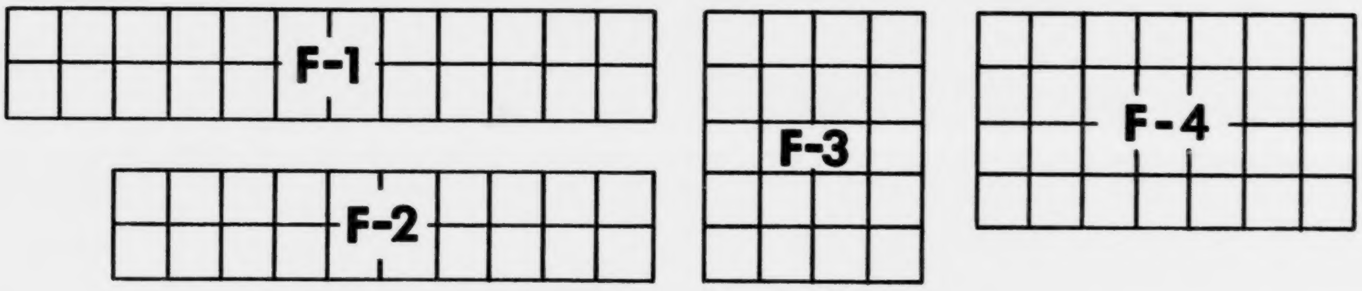
Station 3
Box G ______
Box H ______

Station 4
Envelope I ______
Envelope J ______

Teacher Materials


D-1
D-2
D-3
D-4
D-5


E-1
E-2
E-3
E-4
E-5



Teacher Materials

A

This wire is $\frac{1}{5}$ as long as one of the four pieces of wire in this envelope.

Find that piece of wire.

Write its name on your record card.

B

This toothpick is $\frac{1}{3}$ as long as one of the four pieces of ribbon in this envelope.

Find that piece of ribbon.

Write its name on your record card.

C

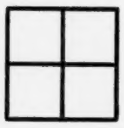
This piece of straw is $\frac{1}{4}$ as long as one of the four pieces of string in this envelope.

Find that piece of string.

Write its name on your record card.

D

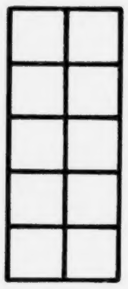
This piece of paper is $\frac{1}{4}$ as large as one of the five pieces of paper in this envelope.



Find that piece of paper.

Write its name on your record card.

E



This piece of paper is $\frac{1}{3}$ as large as one of the five pieces of paper in this envelope.

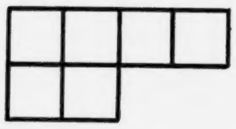
Find that piece of paper.

Write its name on your record card.

Teacher Materials

F

This piece of paper is $\frac{1}{4}$ as large as one of the four pieces of paper in this envelope.



Find that piece of paper.

Write its name on your record card.

G

The container labeled G–1 holds $\frac{1}{4}$ as much as one of the four containers in this box. You may use the supply of water to compare the capacities of the containers.

Find that container.

Write its name on your record card.

H

The container labeled H–1 holds $\frac{1}{4}$ as much as one of the four containers in this box. You may use the supply of water to compare the capacities of the containers.

Find that container.

Write its name on your record card.

I

This group has $\frac{1}{4}$ as many lima beans as one of the groups in this envelope.

Find that group.

Write its name on your record card.

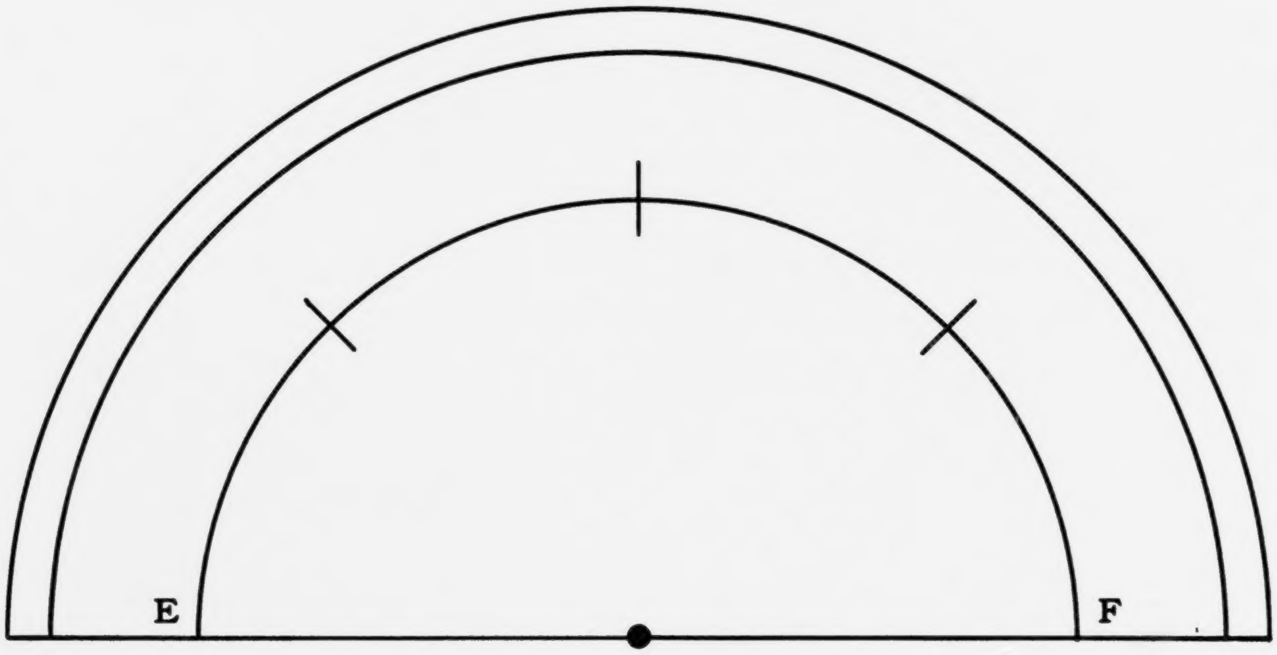
J

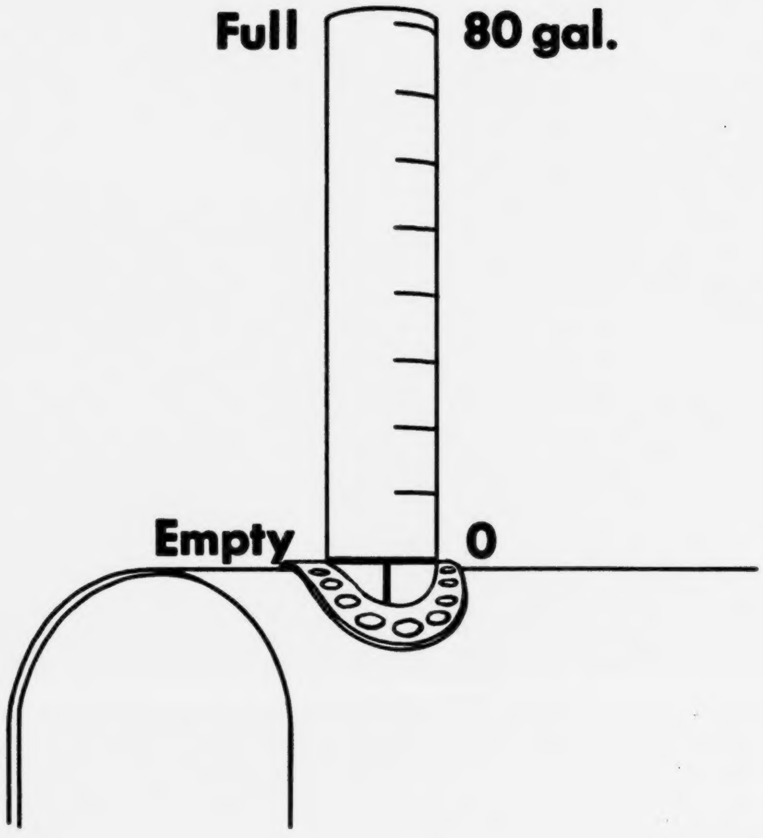
This group has $\frac{1}{3}$ as many lima beans as one of the groups in this envelope.

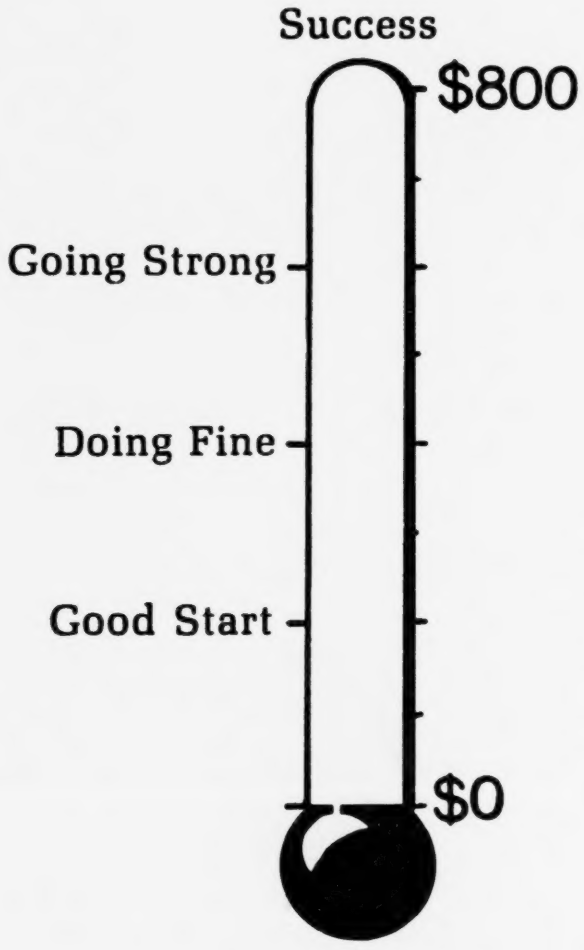
Find that group.

Write its name on your record card.

Teacher Materials



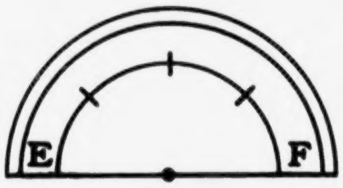

Full
80 gal.
Empty
0


Success
$800
Going Strong
Doing Fine
Good Start
$0

Name ________________________________

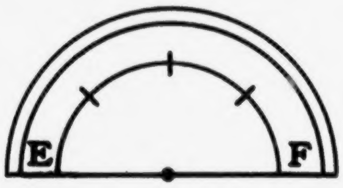
Draw an arrow on each gauge to show the situation described below the gauge.

1.



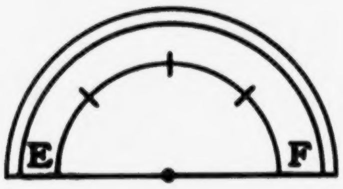
$\frac{3}{4}$ full

2.



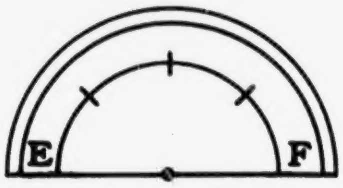
$\frac{1}{8}$ full

3.



$\frac{4}{8}$ full

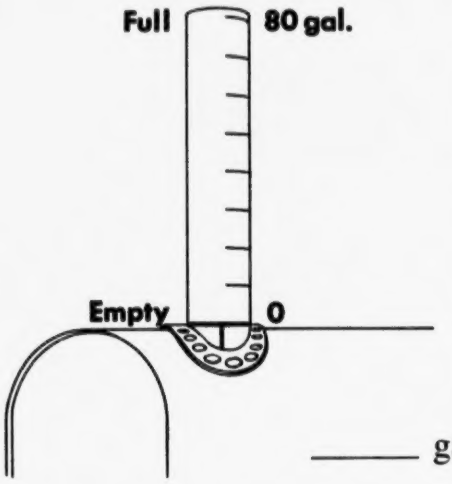
4.



$\frac{5}{8}$ full

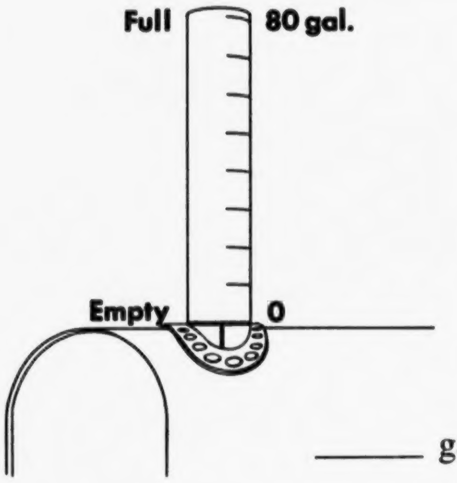
Shade each gauge to show the situation described at the left of it, then record the number of gallons in each case.

5. $\frac{1}{4}$ full


Full
80 gal.
Empty
0

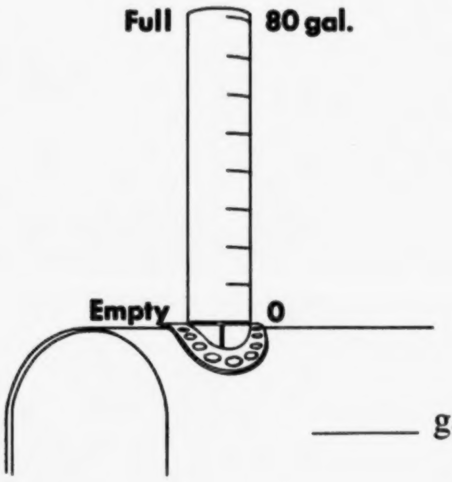
______ gal.

6. $\frac{1}{8}$ full


Full
80 gal.
Empty
0

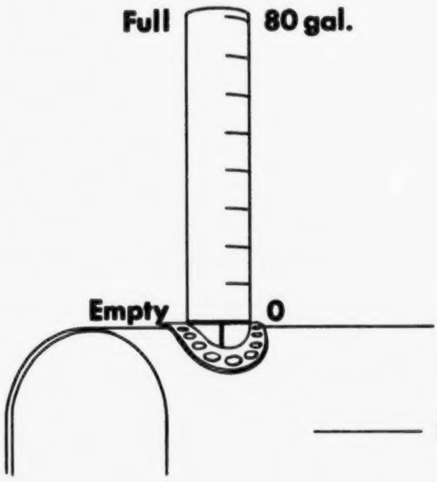
______ gal.

7. $\frac{3}{4}$ full


Full
80 gal.
Empty
0

______ gal.

8. $\frac{3}{6}$ full

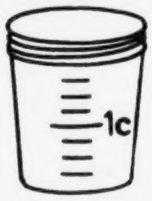

Full
80 gal.
Empty
0

______ gal.

Name ______________________________

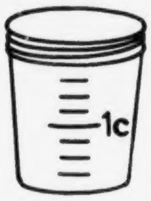
Shade in the indicated level in each of these peanut butter jars to show the part filled with peanut butter.

9.

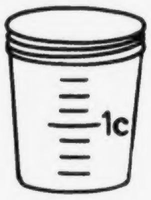


$\frac{1}{2}$ cup

10.



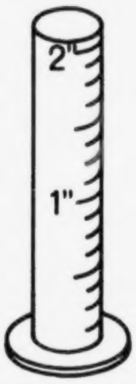
$\frac{1}{4}$ cup



$1\frac{1}{4}$ cup

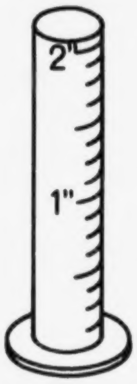
Shade each rain gauge pictured below to show the amount of rainfall indicated directly below it.

12.



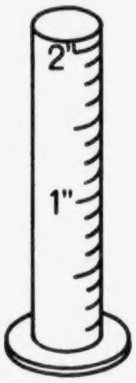
Mon. $\frac{1}{2}$″

13.



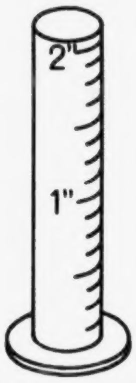
Tues. $\frac{5}{8}$″

14.



Wed. $1\frac{1}{2}$″

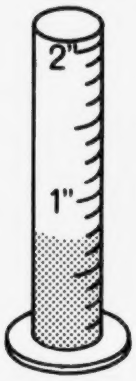
15.



Thurs. $1\frac{3}{8}$″

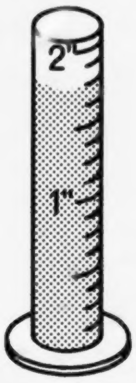
Read each gauge as carefully as possible and write the level of water below it.

16.



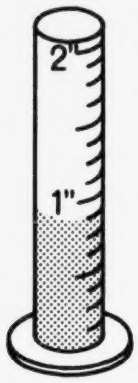
____

17.



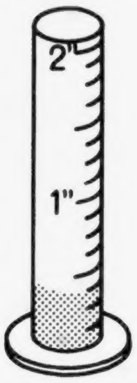
____

18.



____

19.



____

Teacher Materials

A

1. Take a piece of paper from this envelope.
2. Cut off $\frac{1}{4}$ of this piece of paper.
3. Write "$\frac{1}{4}$" on it.
4. Put it in your envelope and throw the other $\frac{3}{4}$ of the piece in the wastebasket.

B

1. Take a piece of paper from this envelope.
2. Cut off $\frac{1}{8}$ of this piece of paper.
3. Write "$\frac{1}{8}$" on it.
4. Put it in your envelope and throw the other $\frac{7}{8}$ of the piece in the wastebasket.

C

1. Take a piece of paper from this envelope.
2. Cut off $\frac{3}{4}$ of this piece of paper.
3. Write "$\frac{3}{4}$" on it.
4. Put it in your envelope and throw the other $\frac{1}{4}$ of the piece in the wastebasket.

Answers. The student's large envelope should contain three pieces of graph paper as follows:

(From Envelope A)	8 squares labeled ¼
(From Envelope B)	4 squares labeled ⅛
(From Envelope C)	24 squares labeled ¾

Teacher Materials

D

1. Take one of the pieces of string from this envelope.
2. Cut off $\frac{1}{2}$ of this piece of string.
3. Put it in your envelope.
4. Put the other $\frac{1}{2}$ of the piece of string in the wastebasket.

E

1. Take one of the pieces of string from this envelope.
2. Cut off $\frac{1}{3}$ of this piece of string.
3. Put it in your envelope.
4. Put the other $\frac{2}{3}$ of the piece of string in the wastebasket.

F

1. Take one of the pieces of string from this envelope.
2. Cut off $\frac{3}{4}$ of this piece of string.
3. Put it in your envelope.
4. Put the other $\frac{1}{4}$ of the piece of string in the wastebasket.

Answers. The student's large envelope should contain three pieces of string of the following lengths:

(From Envelope D)	6 inches
(From Envelope E)	4 inches
(From Envelope F)	9 inches

Teacher Materials

G

1. Take $\frac{1}{3}$ of the lima beans from this envelope.
2. Place these lima beans in the jar.
3. Put this envelope with the remaining $\frac{2}{3}$ of the lima beans in your envelope.

H

1. Take $\frac{3}{4}$ of the lima beans from this envelope.
2. Place these lima beans in the jar.
3. Put this envelope with the remaining $\frac{1}{4}$ of the lima beans in your envelope.

I

1. Take $\frac{3}{5}$ of the lima beans from this envelope.
2. Place them in the jar.
   What fraction of the lima beans is left? ______
   How many lima beans are left? ______
3. Put these lima beans in your envelope.

Answers. The student's large envelope should contain the following:

Envelope G containing 4 lima beans
Envelope H containing 2 lima beans
Envelope I containing 4 lima beans, with
answers "⅖" and "4" written on the outside

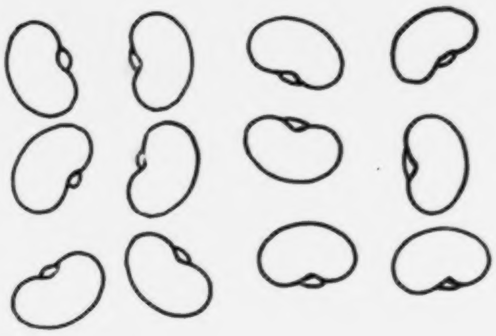
FOCUS ON "ONE"

UNIT 5, EXPERIENCE 3

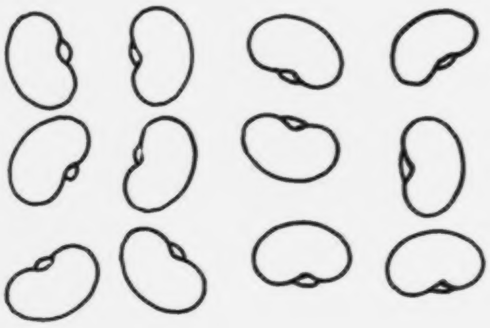
Name ______________________________

Put this sheet in your envelope when you have finished it.

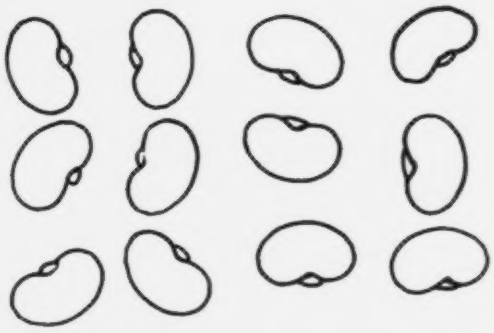
1. Draw a ring around $\frac{1}{2}$ of the lima beans pictured here.



2. Draw a ring around $\frac{1}{4}$ of the lima beans pictured here.

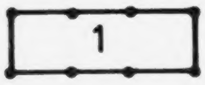


3. Draw a ring around $\frac{1}{3}$ of the lima beans pictured here.

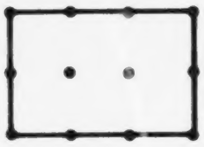


Teacher Materials

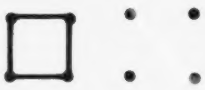
If we agreed to name this figure "1,"



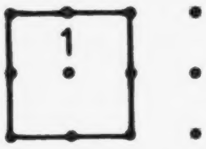
what name would we give to this?



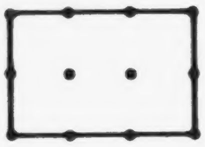
To this?



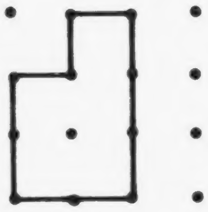
Now suppose we name *this* figure "1."



What would we name this?

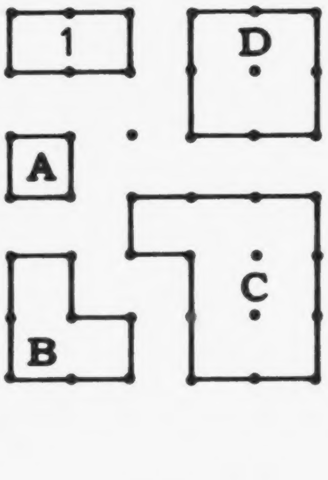


And this?



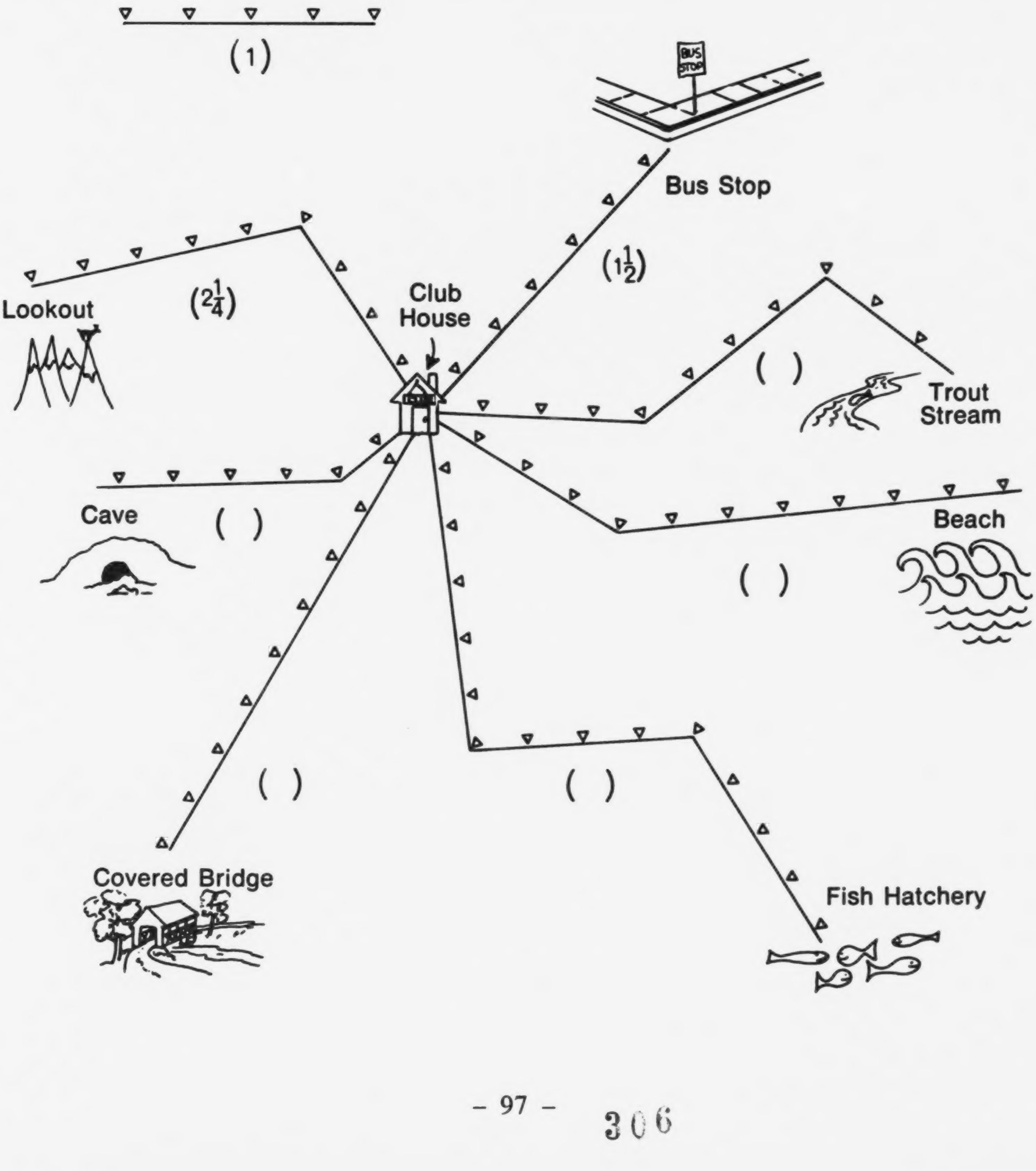
Suppose we name this figure "1."

Write down on a piece of scratch paper the name we would give to *A*; to *B*; to *C*; to *D*.



Activity 1 Name ________________________________

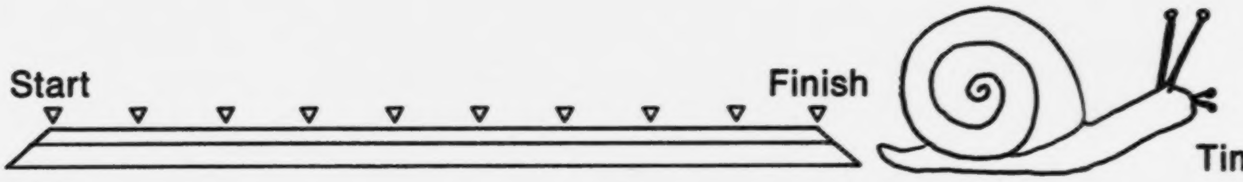
The Paul Bunyan Hiking Club names its hikes with numbers that indicate how many hours they are expected to take. A 4-mile hike is named "1" because it takes about 1 hour. This system is used for all hikes of 20 miles or less. The map below shows popular hikes with the mileposts marked ▽. Two of the hikes are already named. Name the others.


(1)
BUS STOP
Bus Stop
Lookout
$(2\frac{1}{4})$
Club House
$(1\frac{1}{2})$
( )
Trout Stream
Cave
( )
( )
Beach
( )
( )
Covered Bridge
Fish Hatchery

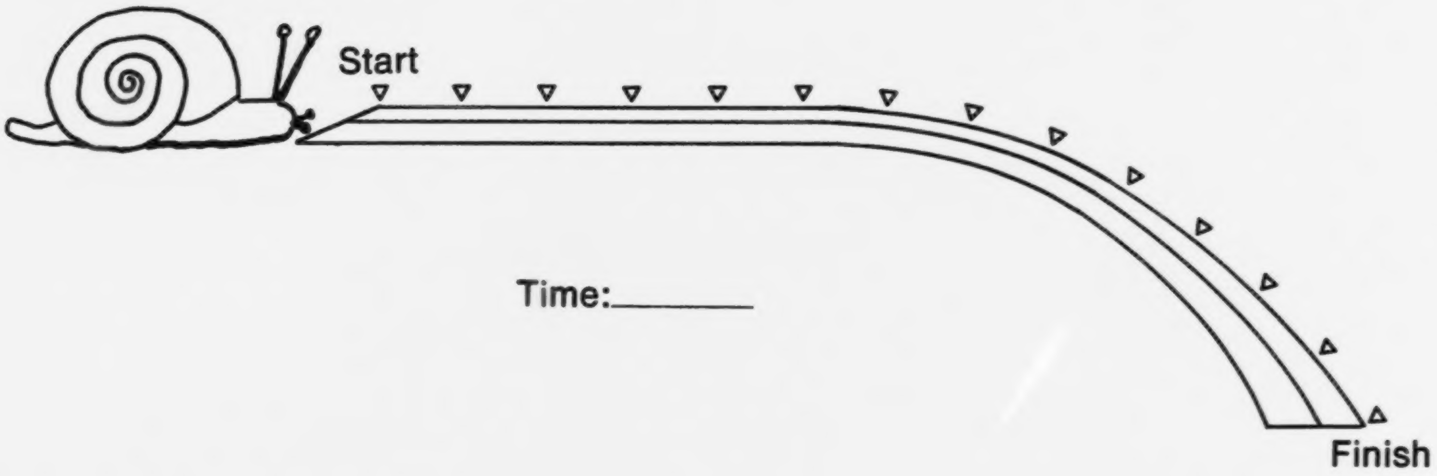
Activity 2

Name ______________________________

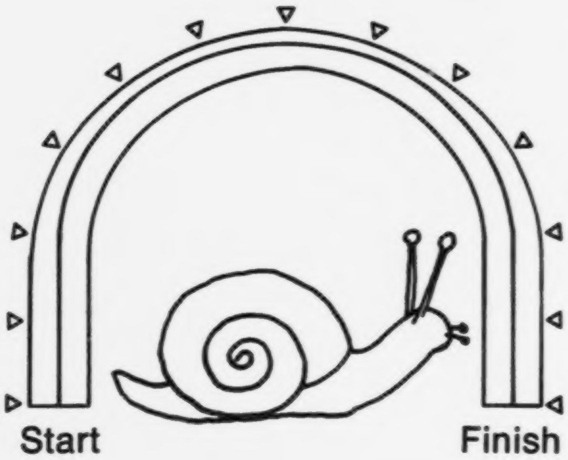
Samson, the trained racing snail, crawled at the rate of 5 centimeters in 1 minute. Four of the racetracks he crawled on are shown below. Centimeters are marked ∇. Write his time for each track on the blank provided.


Start
Finish

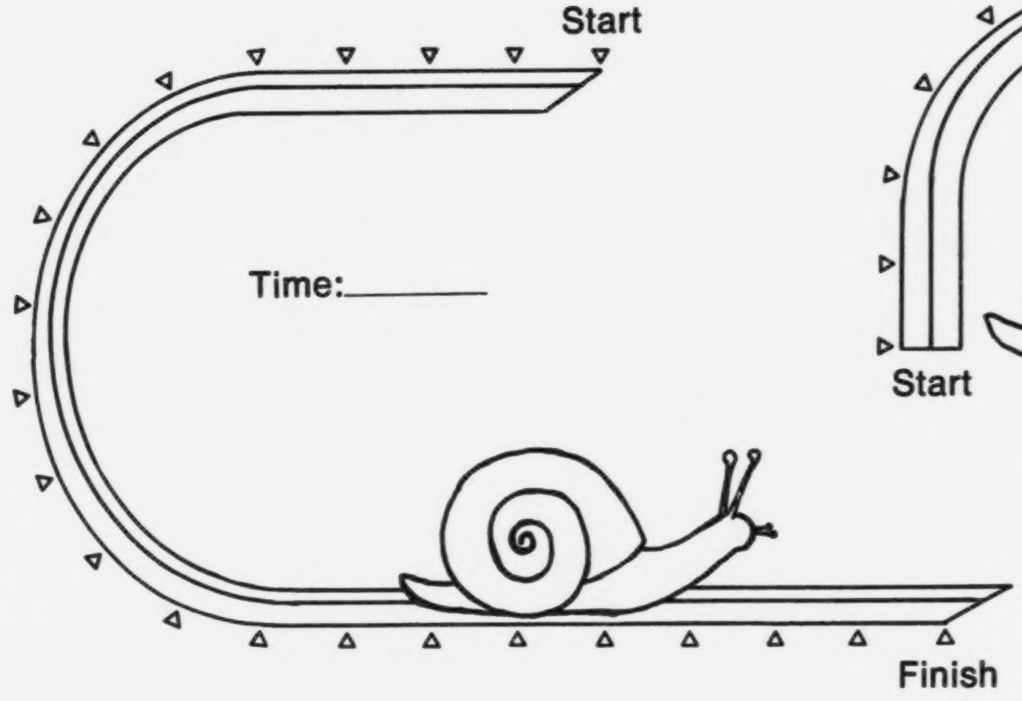
Time: ______


Start
Finish

Time: ______


Start
Finish

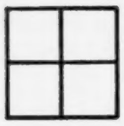
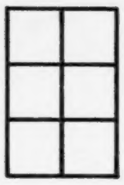
Time: ______


Start
Finish

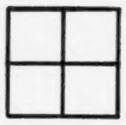
Time: ______

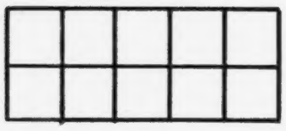
307

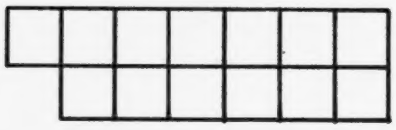
Activity 3 Name ____________________

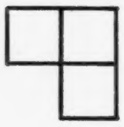
If  is 1, then  is $1\frac{1}{2}$.

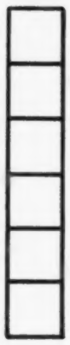
Complete:

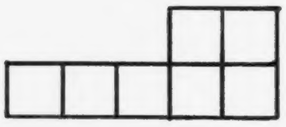
If  is 1, then

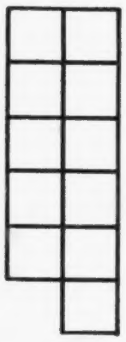
 is ______

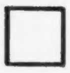
 is ______

 is ______

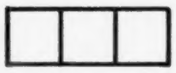
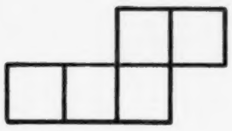
 is ______

 is ______

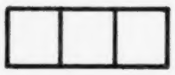
 is ______

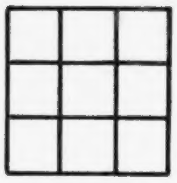
 is ______

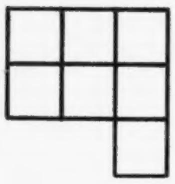
Activity 4 Name ______________________________

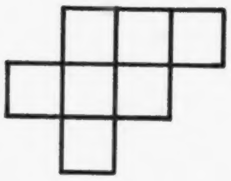
If  is 1, then  is $1\frac{2}{3}$.

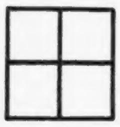
Complete:

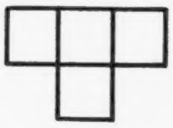
If  is 1, then

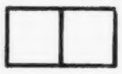
 is ________

 is ________

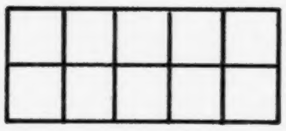
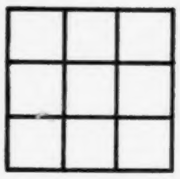
 is ________

 is ________

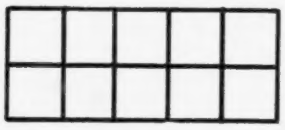
 is ________

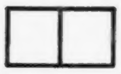
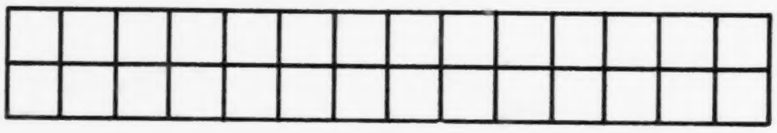
 is ________

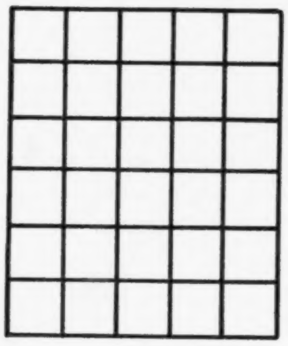
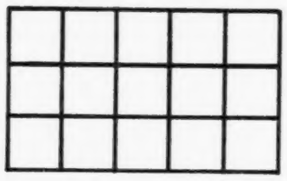
Activity 5 Name ________________________________

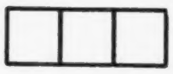
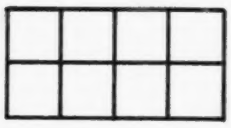
If  is 1, then  is $\frac{9}{10}$.

Complete:

If  is 1, then

 is ________  is ________

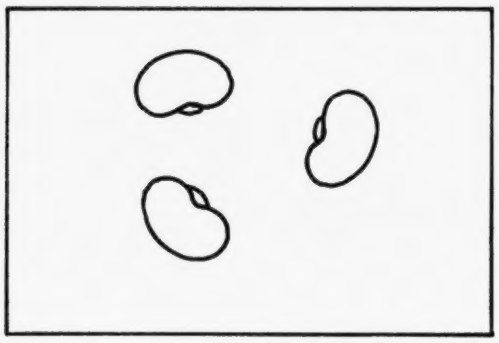
 is ________  is ________

 is ________  is ________

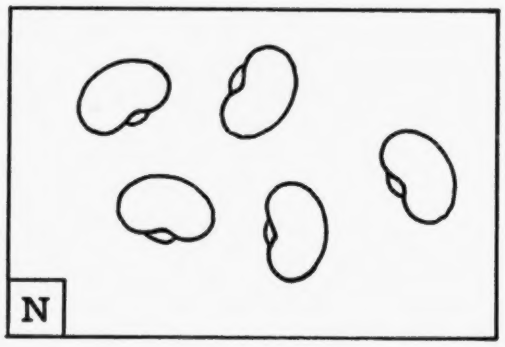
Activity 6

Name ______________________________

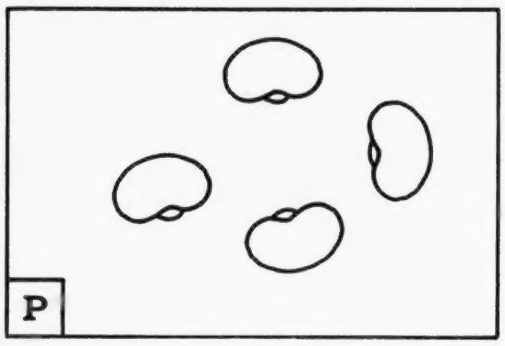
If



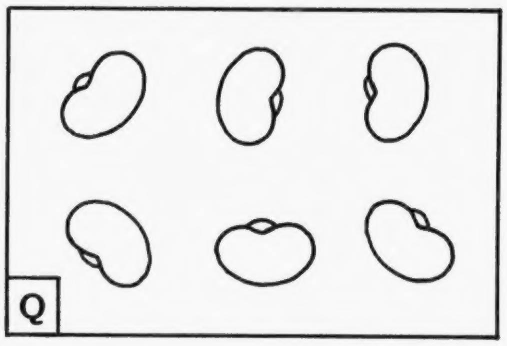
is 1, then



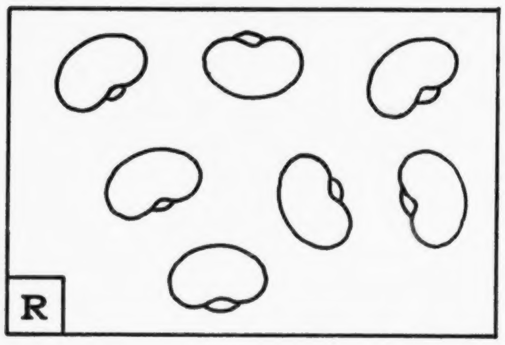
is ________



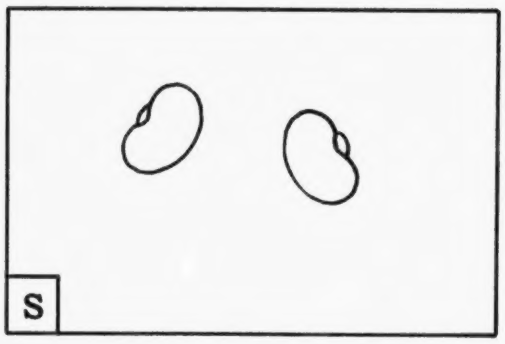
is ________



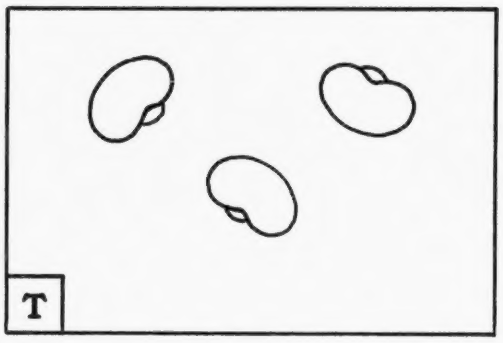
is ________



is ________



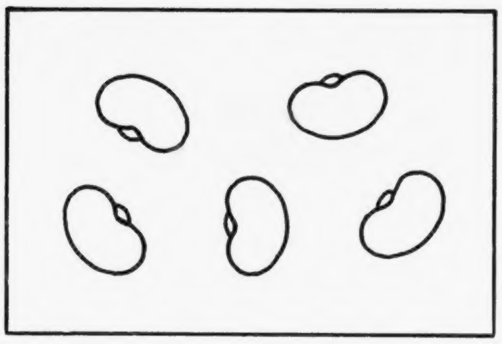
is ________



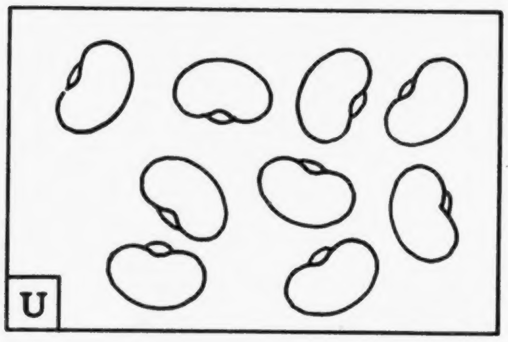
is ________

Activity 7 Name ______________________________

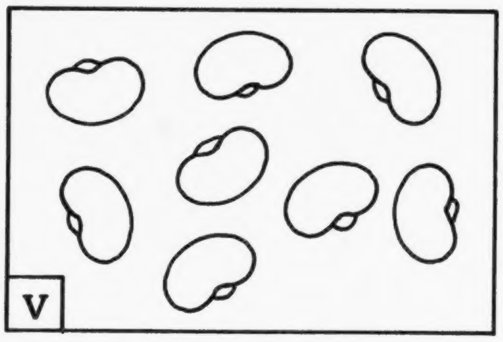
If



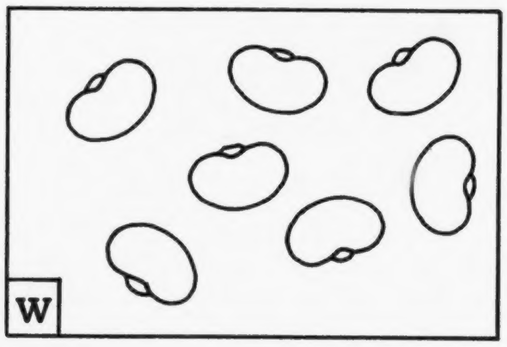
is 1, then



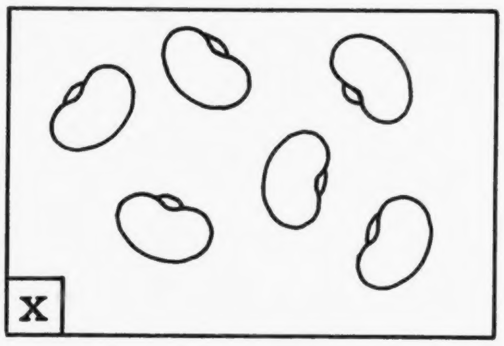
U is ________



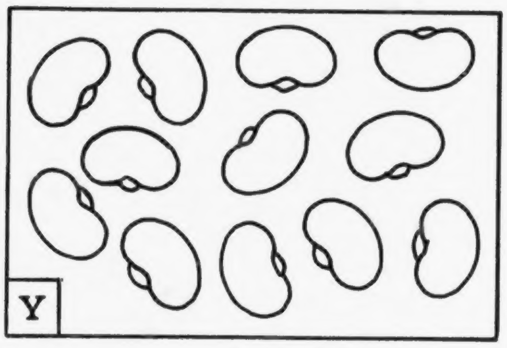
V is ________



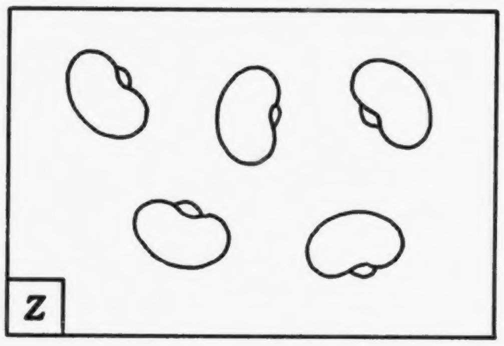
W is ________



X is ________



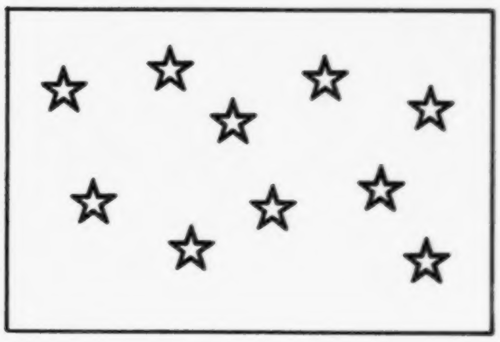
Y is ________



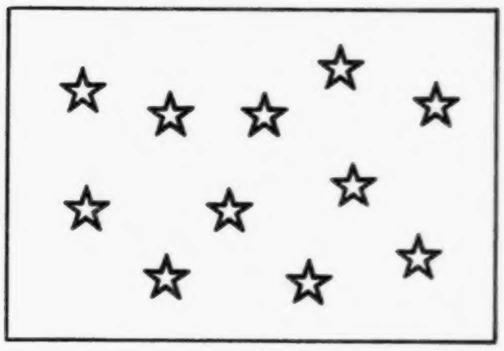
Z is ________

Activity 8 Name ______________________

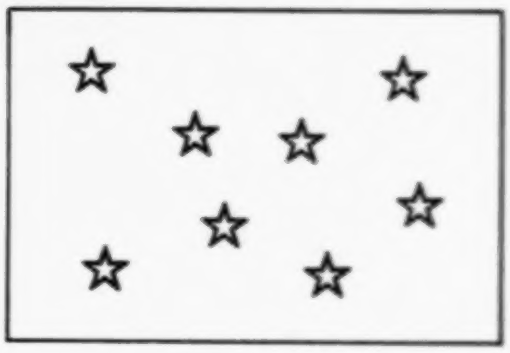
If



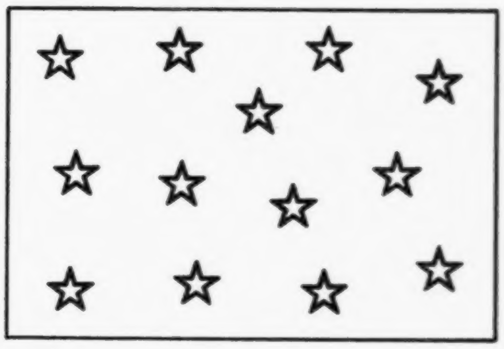
is 1, then



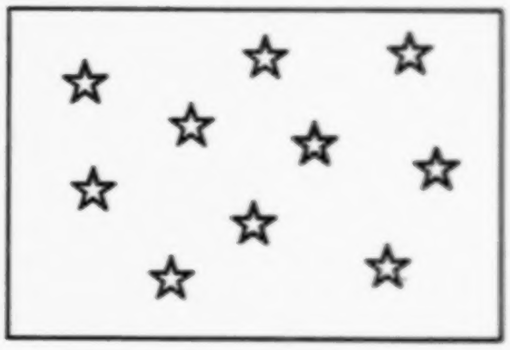
is ______



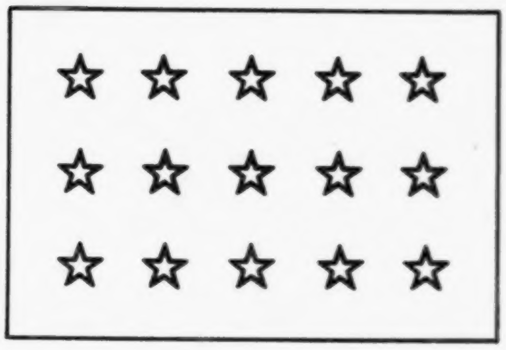
is ______



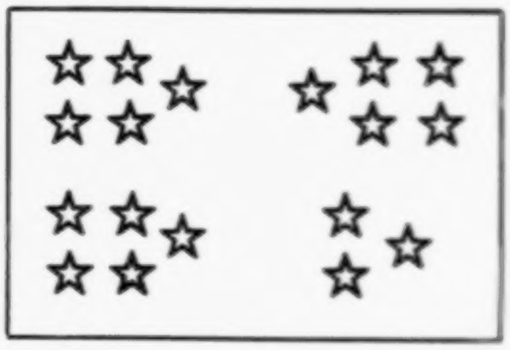
is ______



is ______



is ______

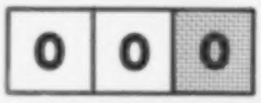


is ______

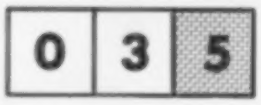
Teacher Materials

Tod got a new bicycle with a mileage gauge, an odometer, on it. He decided to find out how far it was from his home to the city park.

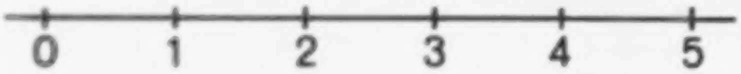
He set his gauge to look like this when he started:



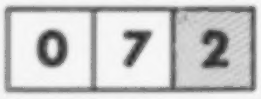
When he got to the park his gauge looked like this:



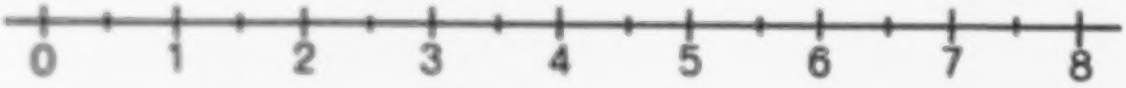
How many miles had he gone? Where would we find that number on this number line:



Tod took a longer route home. When he got there, his gauge looked like this:



How many miles had he gone? Where would the number be on this number line:

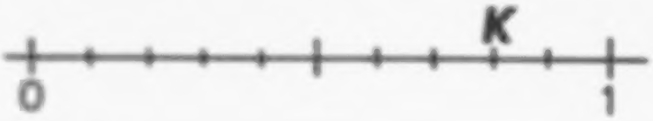


Bob and Jack have delivery routes after school. They argued about whose route was the longer. To find out, each set his odometer at zero when he started his route and wrote down the reading when he finished.

Bob's gauge read 6.2 when he finished, and Jack's read 5.9.

Find each of these numbers on the number line above. Which boy's route was longer? Jack said the difference was less than a half a mile. Was he correct? How do you know?

Kim said she checked the distance from her home to school. She showed the class the distance by marking *K* on this number line:

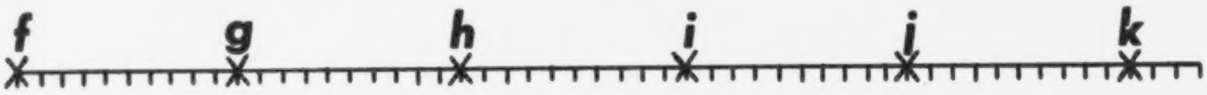


How far did she live from school?

READING AND USING TENTHS — UNIT 5, EXPERIENCE 5

Activity 1 Name ______________________________

In Central City 10 blocks equal 1 mile. The "map" below shows 5.3 miles of Main Street. The mileposts are marked with an × and given a letter name. Each block is shown by the marks between the mileposts.

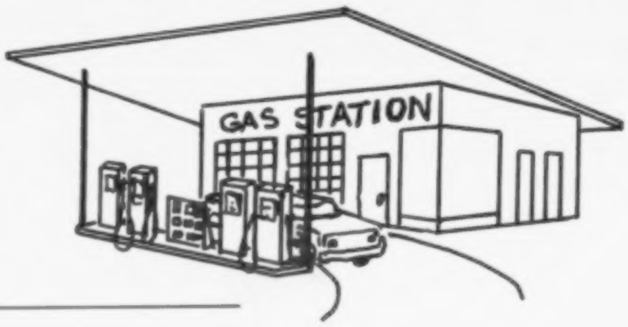


1. Duke's Hobby Shop is 2.3 miles from milepost *k*. Show where his shop is by putting a D on the map.

DUKE'S
HOBBY SHOP

How far is Duke's from milepost *f*? ________________

2. Bob's service station is 3.8 miles from milepost *f*. Show where it is by putting a B on the map.



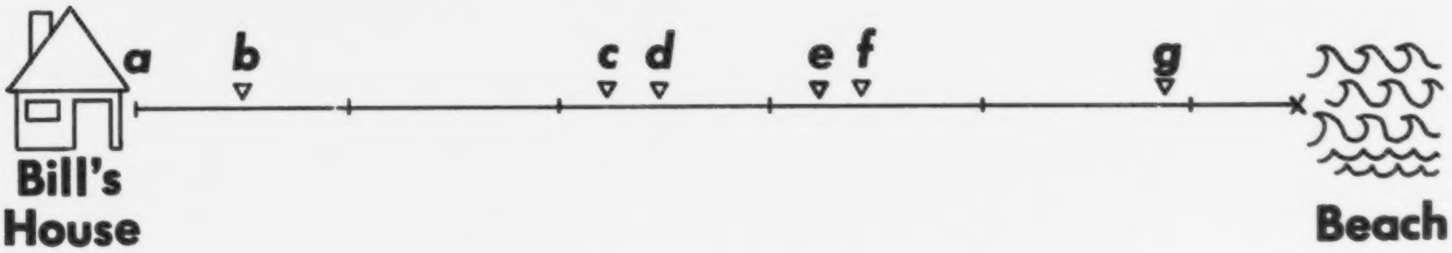
How far is Bob's station from milepost *k*? ________________
How far is Bob's station from Duke's shop? ________________

3. Cora's Beauty Shop is 1.7 miles from milepost *j* and 0.3 miles from milepost *h*. Show where her shop is by putting a C on the map.

BEAUTY SHOP

How far is Cora's shop from Duke's? ________________
How far is Cora's shop from Bob's station? ________________

Activity 2 Name ____________________



The "map" shows that the beach is more than 5 blocks from Bill's house, which is marked *a*. Jack's house is marked *b*. Jack's house is $\frac{1}{2}$ block from Bill's. Tom's house is between Bill's and the beach. It is $2\frac{1}{2}$ blocks from Bill's.

Which point locates Tom's house? ________

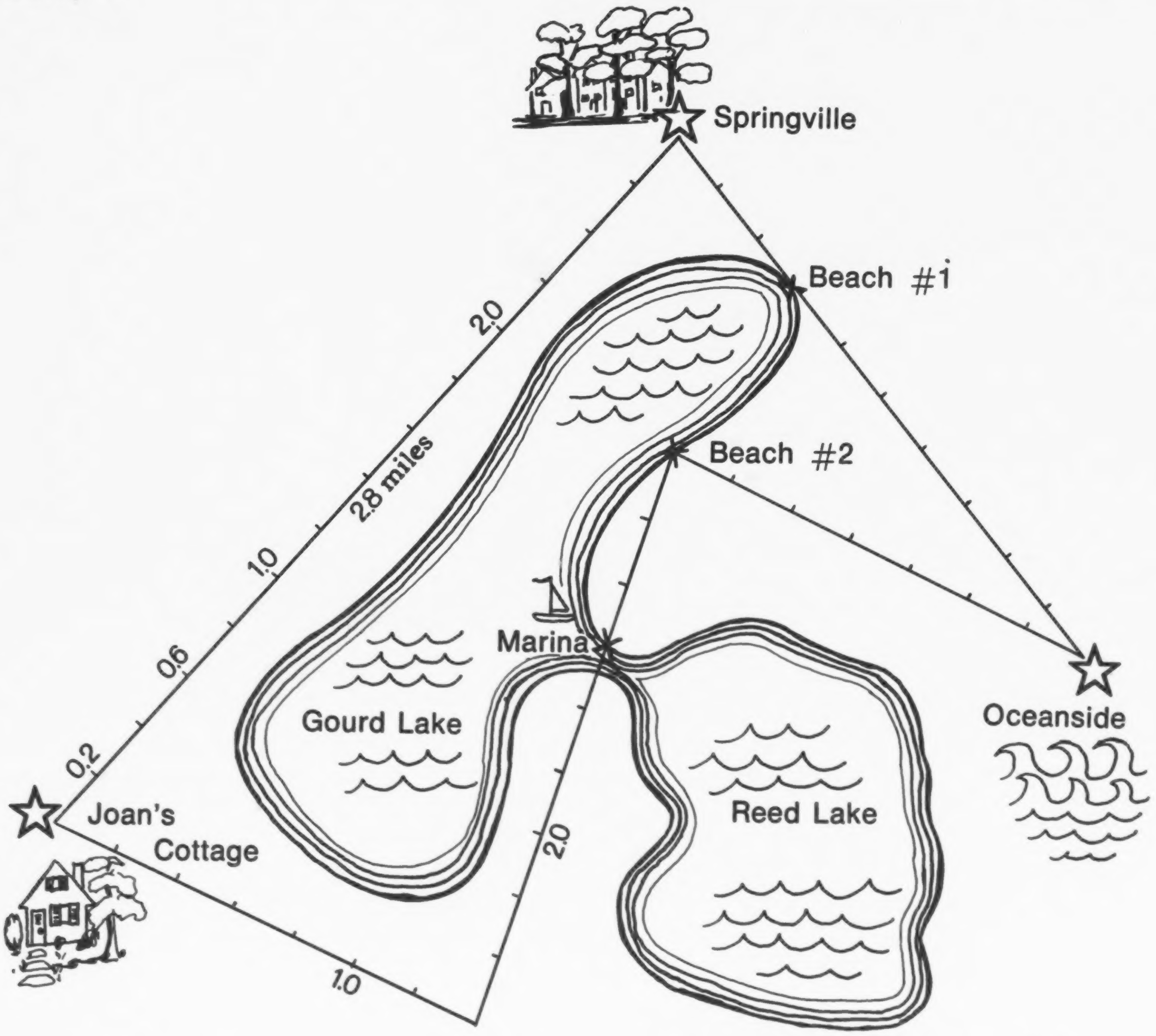
The Candy Kitchen, a favorite hangout, is $3\frac{1}{4}$ blocks from Bill's (on the way to the beach).

Which of the points shown on the map is nearest the Candy Kitchen? ________

Which point shown on the map is 3 blocks from Jack's house? ________

Activity 3

Name ______________________________


Springville
Beach #1
Beach #2
Marina
Oceanside
Gourd Lake
Reed Lake
Joan's Cottage
0.2
0.6
1.0
2.8 miles
2.0
2.0
1.0

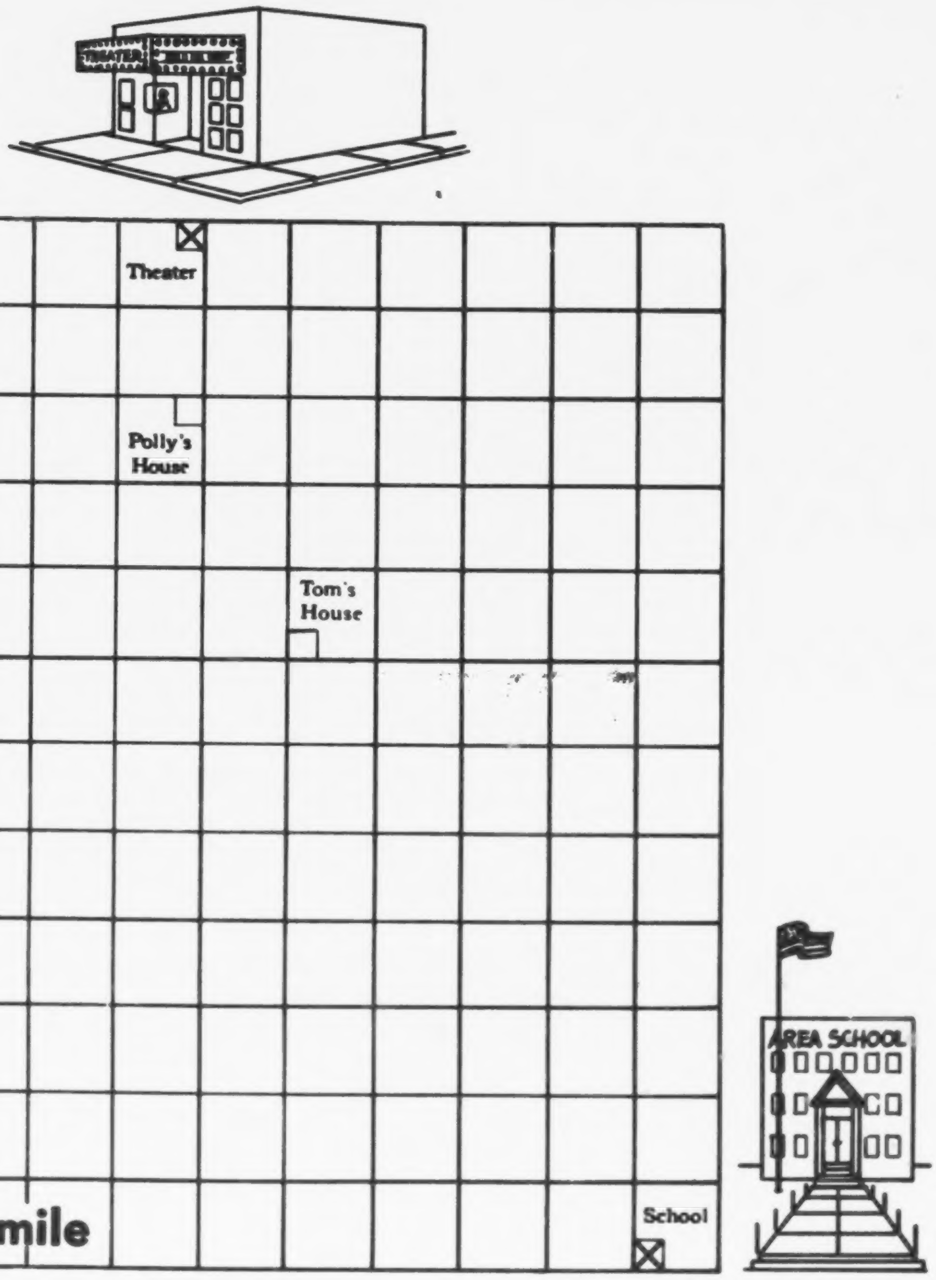
The map shows the lake area near the summer cottage Joan's family owns. The distance between marks on the road is 0.2 mile. In answering questions, "Keep to the roads!"

1. Which beach is closer to Joan's cottage? ______________
2. How far is it? ______________
3. How far is it from the Marina to Oceanside? ______________
4. How far is it from Springville to the Marina? ______________
5. How far is Oceanside from Joan's cottage? ______________
6. Joan left the cottage on her bicycle and rode 4.2 miles. At this point she was 1 mile from one of the beaches. Did she go by way of Springville or by way of the Marina?

____

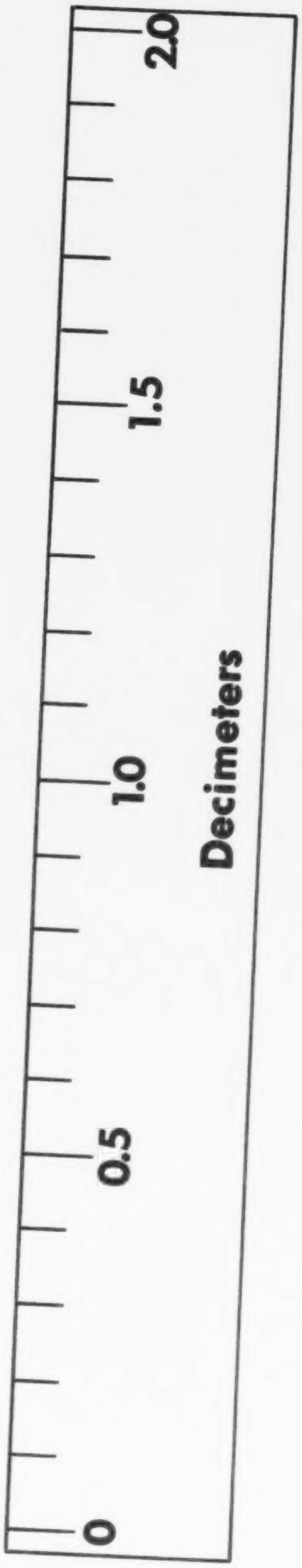
Activity 4

Name ______________________

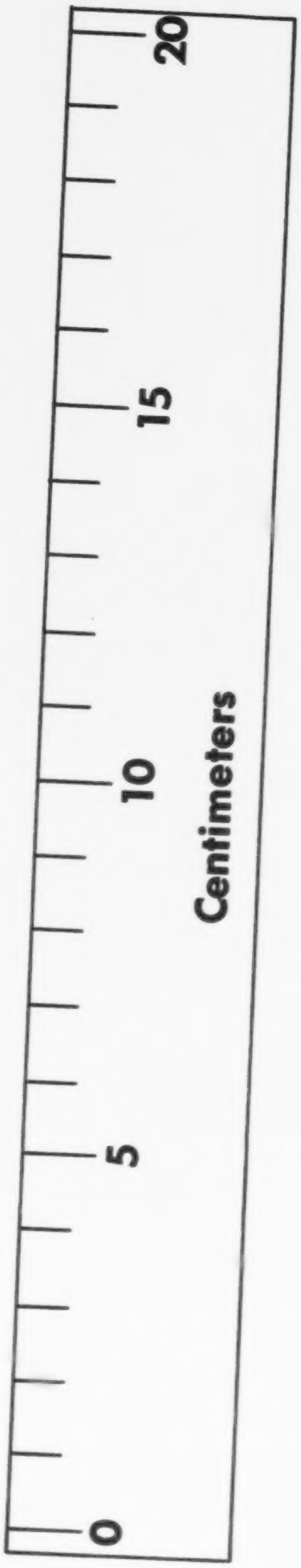

THEATER
Jim's House
Theater
Polly's House
Mary's House
Tom's House
Bill's House
10 blocks =1.0 mile
School
AREA SCHOOL

1. Who lives closer to school, Mary or Polly? ______________
2. How many miles does Polly live from school? ______________
3. Polly's father drove the five teenagers to the theater one night. He took the shortest route, which was 2.0 miles. Who did he pick up first? ______________
4. Jim started out for school. When he had gone 1.1 miles he was 0.4 mile from Tom's house, and 0.6 mile from Bill's. Put a J on the map to show where he is.
5. Mr. Morse, the school principal, said that anyone who lives more than 1.5 miles from school can ride the school bus. Which of these students can ride the bus? ______________
6. Jim's father picked up the other three for a pizza party at Tom's. Who should he pick up first if he is to take the shortest route? ______________
How far did he drive (one way)? ______________

Teacher Materials

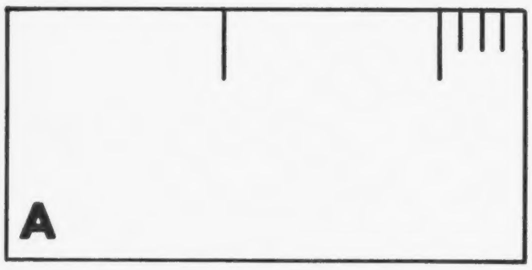

2.0
1.5
1.0
0.5
0
Decimeters

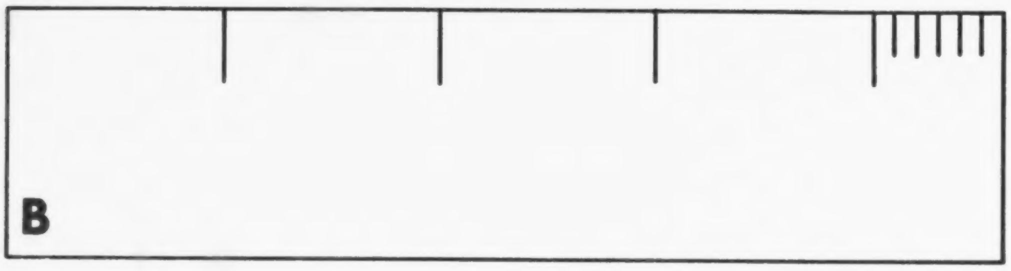
Copies of these rulers should be made for each student for Activity 1.

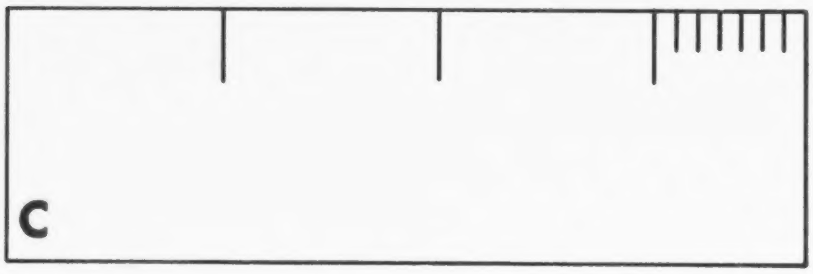

20
15
10
5
0
Centimeters

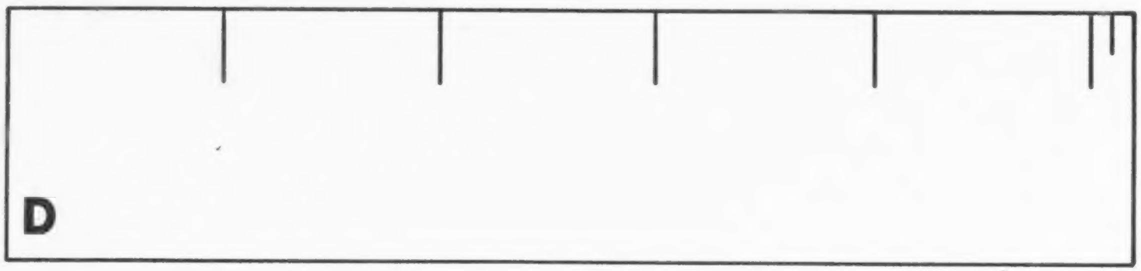
Teacher Materials

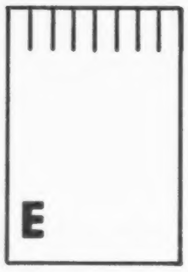
The following patterns may be used in cutting the sticks required for Station A. The patterns are actual size. A measures 2.4 inches; B, 4.6 inches; C, 3.7 inches; D, 5.2 inches; E, .8 inch.







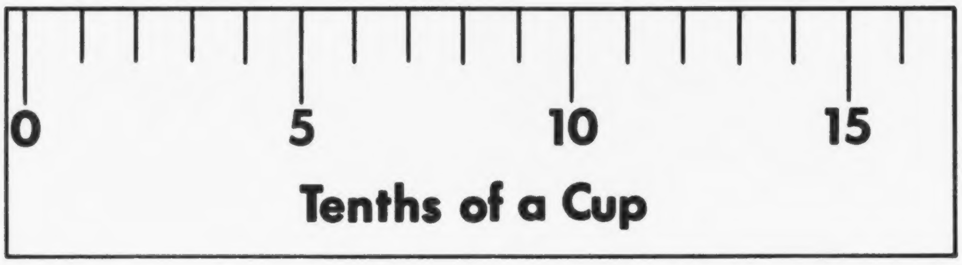


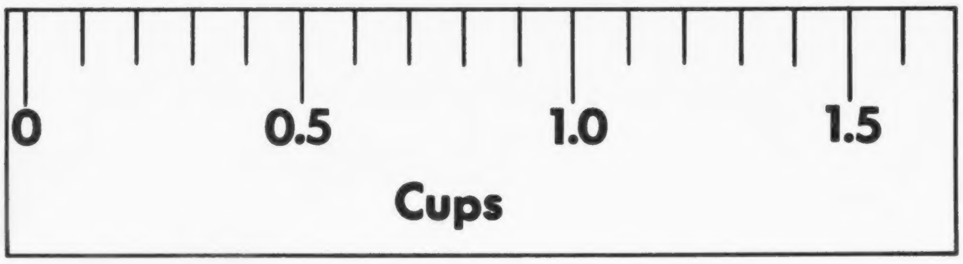


320

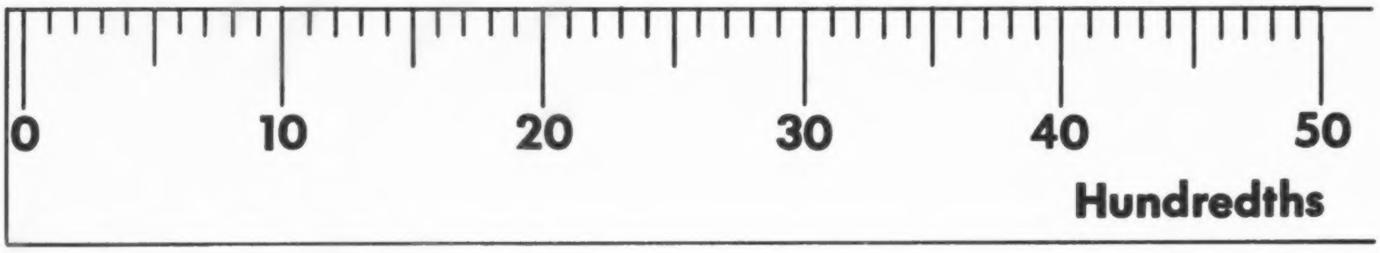
Teacher Materials

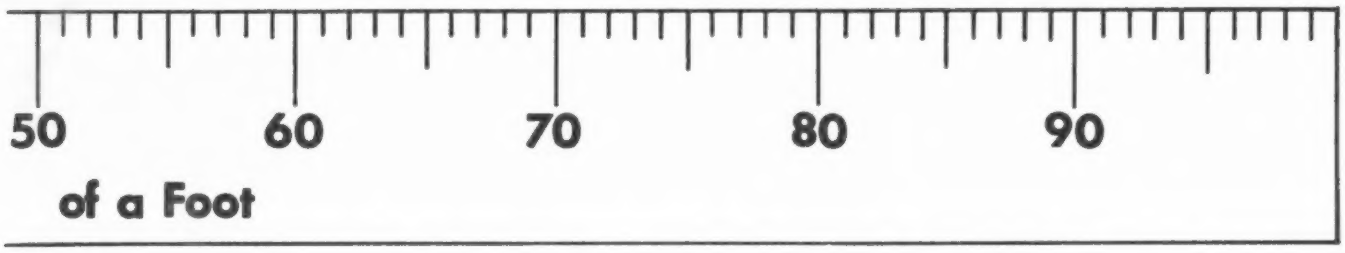
Copies of the following two scales should be made for each student for Activity 3, Station B.


0
5
10
15
Tenths of a Cup


0
0.5
1.0
1.5
Cups

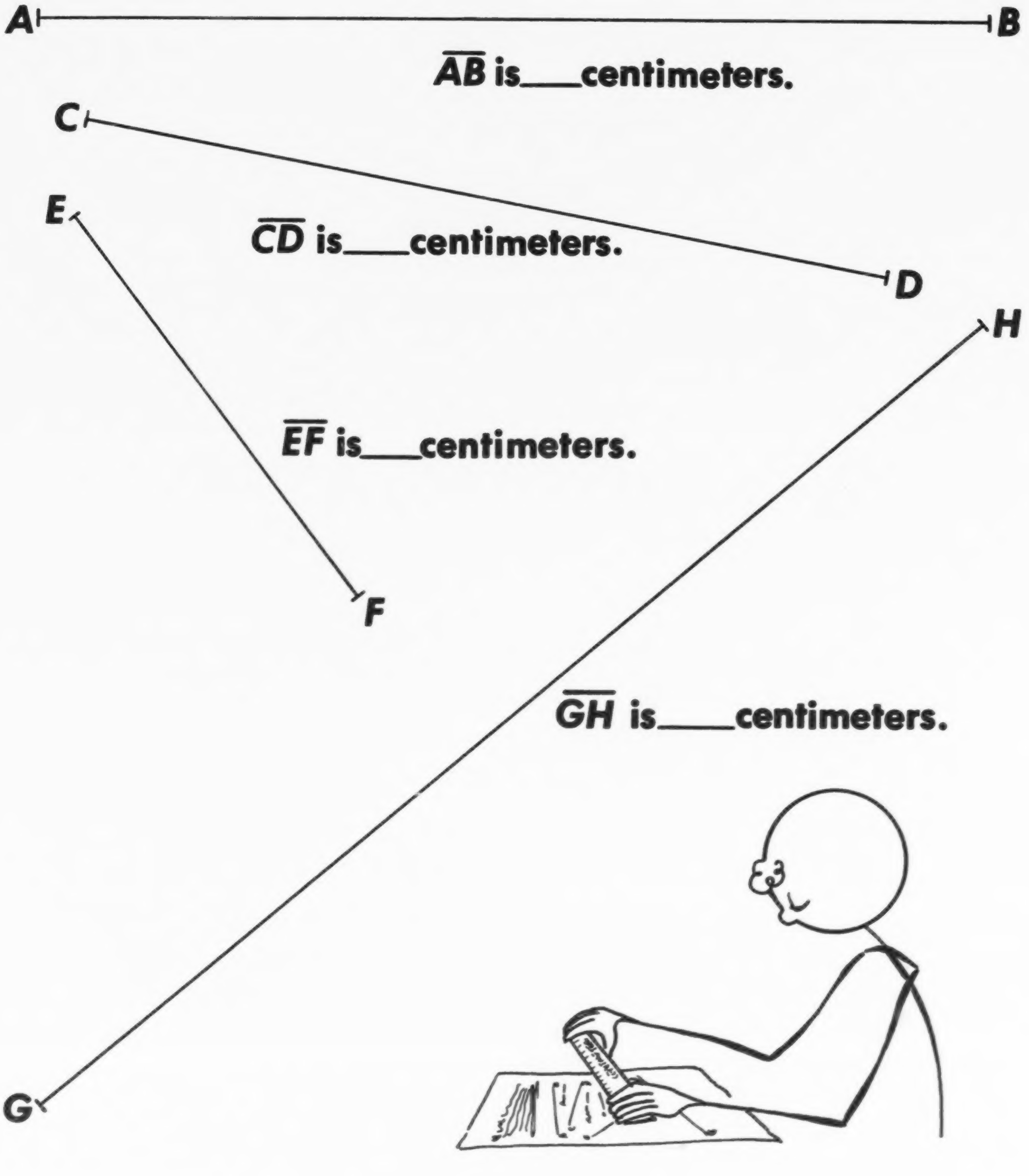
Make a copy of this ruler for each student to use in Activity 4. The pattern is in two parts and must be taped together.


0
10
20
30
40
50
Hundredths


50
60
70
80
90
of a Foot

Activity 1, Part 1 Name ______________________________

The ruler for this activity is marked off in centimeters. Use it to measure the length of each of these line segments. Write the lengths in the blanks.



$\overline{AB}$ is___centimeters.

$\overline{CD}$ is___centimeters.

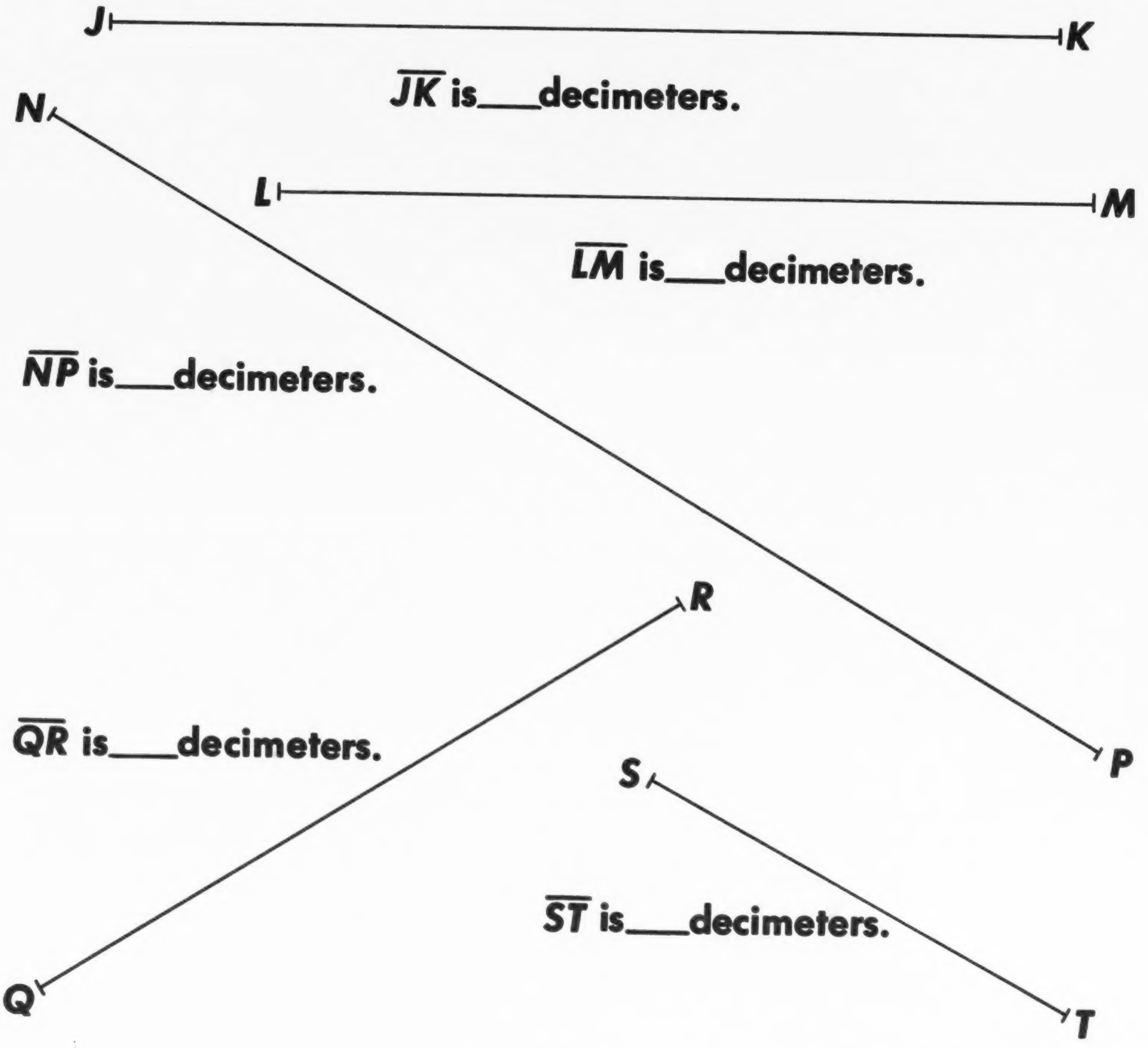
$\overline{EF}$ is___centimeters.

$\overline{GH}$ is___centimeters.

Activity 1, Part 2

Name ______________________________

The ruler for this activity is marked off in decimeters. Use it to measure these line segments.



$\overline{JK}$ is___decimeters.

$\overline{LM}$ is___decimeters.

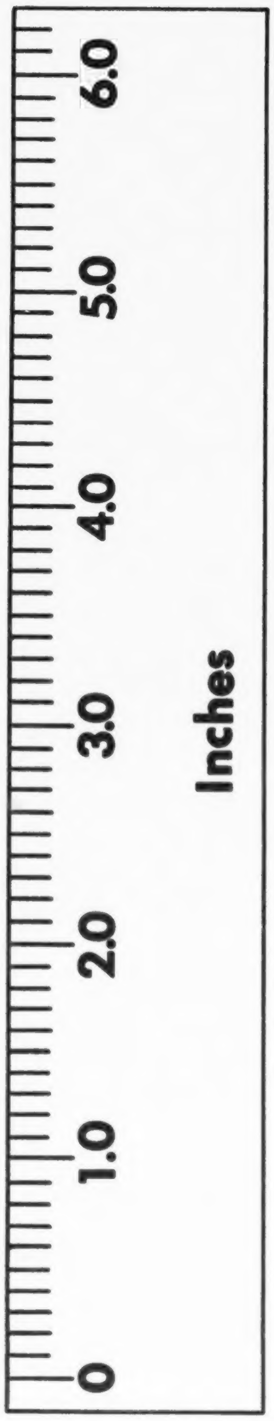
$\overline{NP}$ is___decimeters.

$\overline{QR}$ is___decimeters.

$\overline{ST}$ is___decimeters.

Activity 2, Station A

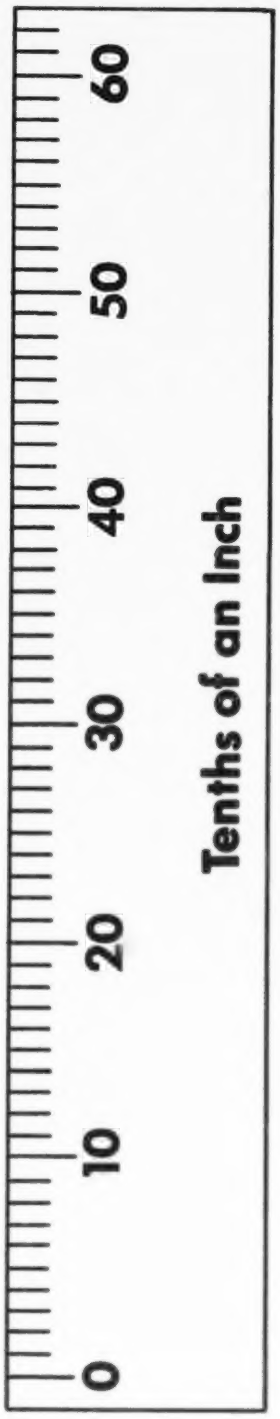
Name ______________________________



Part 1

The scale at the right is marked in tenths of an inch. Use it to measure the length of each stick in the envelope for Part 1. Write the length of each below. Then replace the sticks in the envelope.

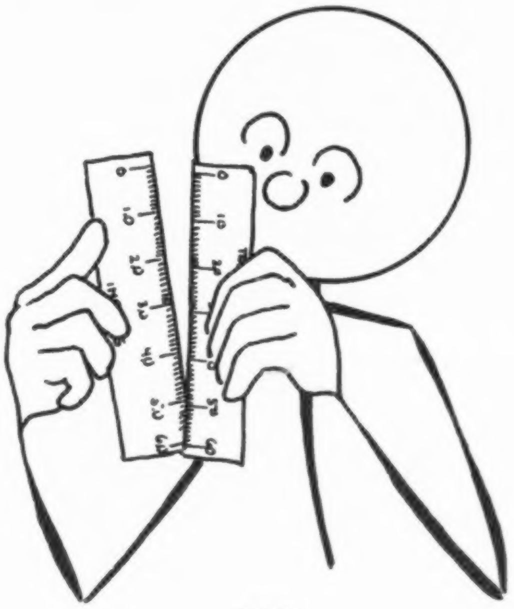
Length of A 24 tenths of an inch
Length of B ______ tenths of an inch
Length of C ______ tenths of an inch
Length of D ______ tenths of an inch
Length of E ______ tenths of an inch



Part 2

Use the scale at the left to measure each of the five sticks in the envelope for Part 2. Record your results below, then replace the sticks in the envelope.

Length of A 2.4 inches
Length of B ______ inches
Length of C ______ inches
Length of D ______ inches
Length of E ______ inches



Activity 3, Station B Name ______________________

Part 1

Use the ruler for tenths of a cup to determine the number of tenth-cups in each jar at this station. Write your results below.

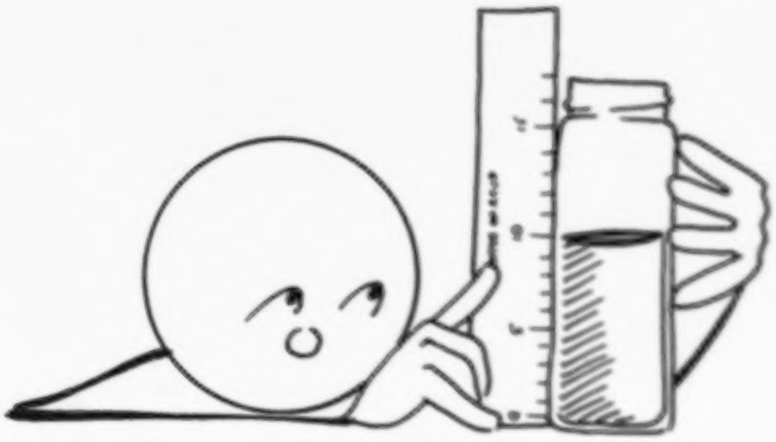
Jar A ______ tenth-cups

Jar B ______ tenth-cups

Jar C ______ tenth-cups

Jar D ______ tenth-cups

Jar E ______ tenth-cups



Part 2

Use the scale for cups to determine the number of cups in each jar at this station. Write your results below.

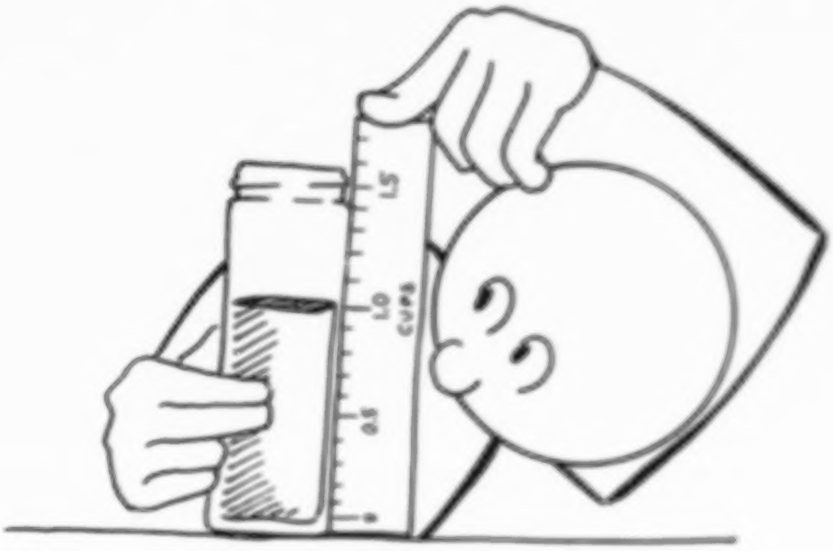
Jar A ______ cups

Jar B ______ cups

Jar C ______ cups

Jar D ______ cups

Jar E ______ cups

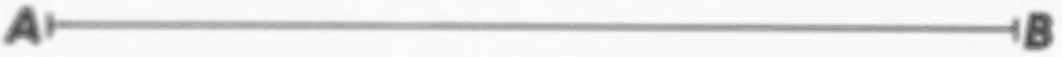


Activity 4

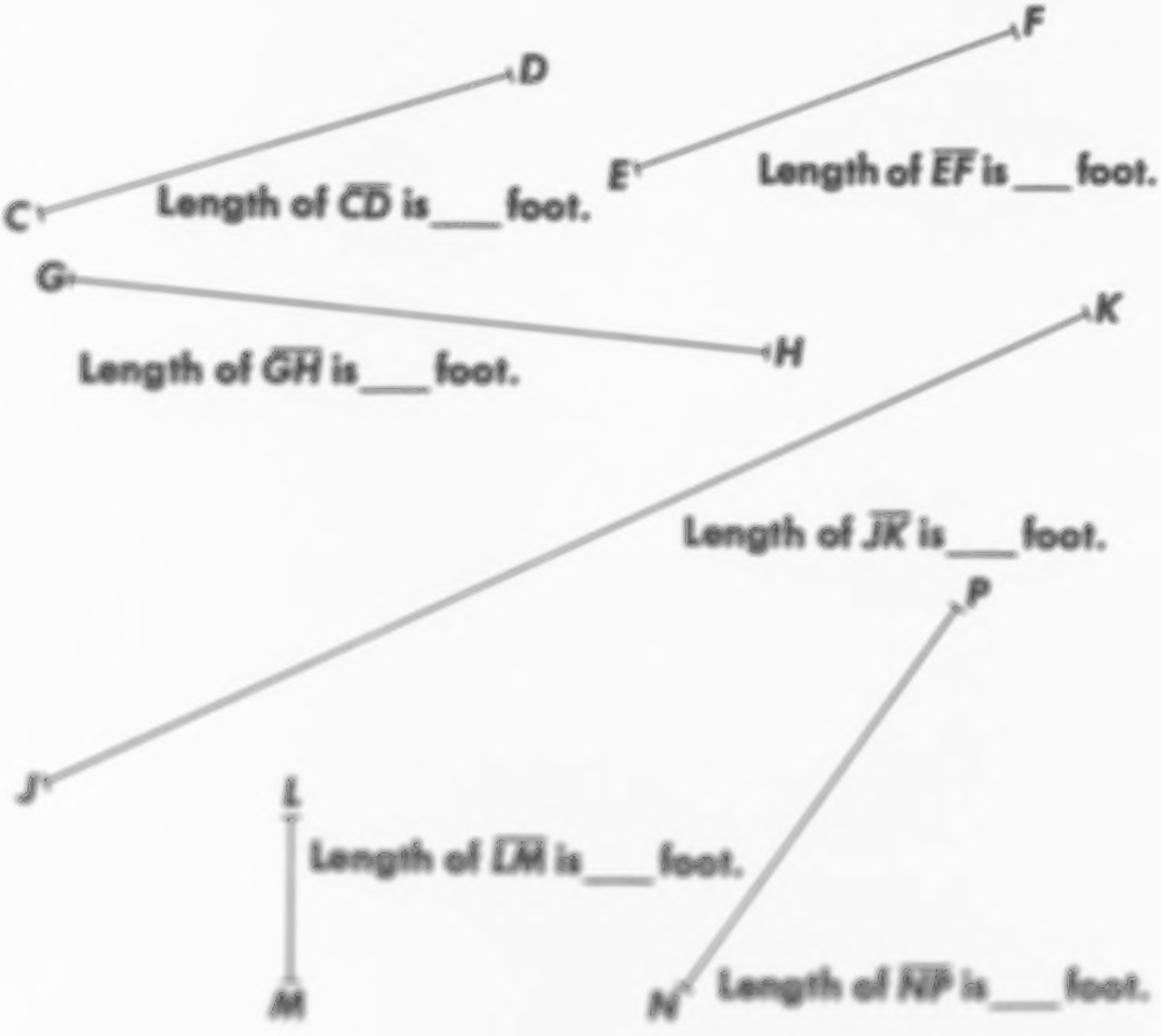
Name ______________________________

The ruler is marked in hundredths of a foot. Use it to measure these line segments, following the example below, which measures 37 hundredths of a foot.

Example:



Length of $\overline{AB}$ is .37 foot.



Length of $\overline{CD}$ is___foot.

Length of $\overline{EF}$ is___foot.

Length of $\overline{GH}$ is___foot.

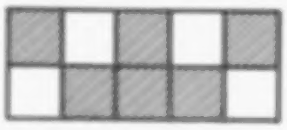
Length of $\overline{JK}$ is___foot.

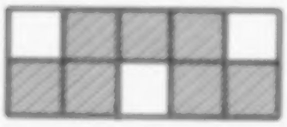
Length of $\overline{LM}$ is___foot.

Length of $\overline{NP}$ is___foot.

Activity 1 Name ______________________

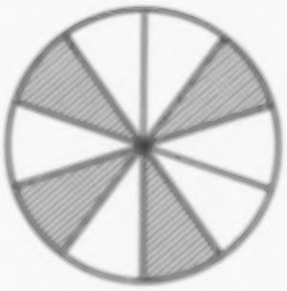
Answer the questions in the spaces provided.





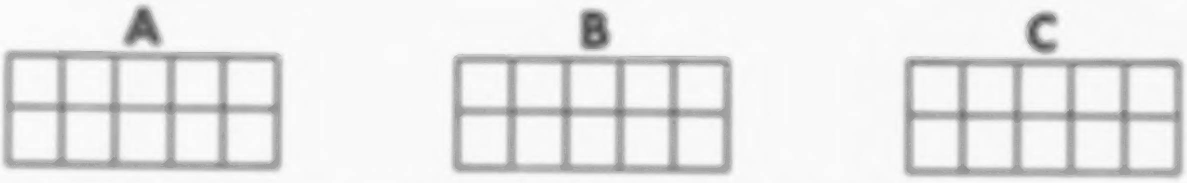
1. Six-tenths (.6) of the first rectangle is shaded. What fractional part of the rectangle is unshaded? ______
2. What fractional part of the second rectangle is unshaded? ______

Shade 4 more parts of this circle green.



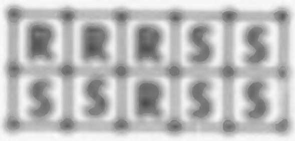
3. What fractional part of the circle is shaded either black or green? ______
4. What fractional part of the circle is not shaded? ______

Color 3 small squares of A, 4 small squares of B, and 7 small squares of C.



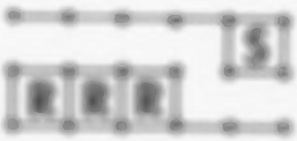
5. If A, B, and C are each considered to be 1.0, what fraction describes the 14 colored parts? ______
6. What fraction describes the uncolored parts? ______

Rita and Sally played a game of Complete the Square with the result shown here.



7. What fractional part of the squares did Rita complete? ______
8. What fractional part of the squares did Sally complete? ______

In another game, the results looked like this after each one had played 9 times:

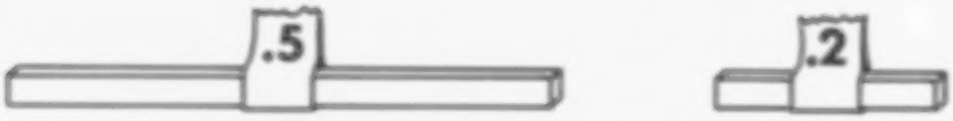


9. What fractional part of the squares has been completed? ______

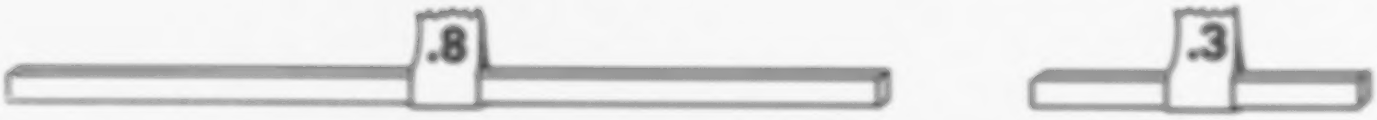
Teacher Materials

The patterns shown here may be used in making the measuring sticks for this experience. The patterns are actual size, using 5 inches as the unit for the first three stations and 2½ inches for the fourth. Labels can be made from masking tape.

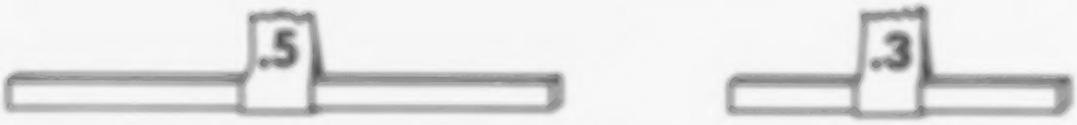
In each case, the two strips should be placed in an envelope marked with the station identification.



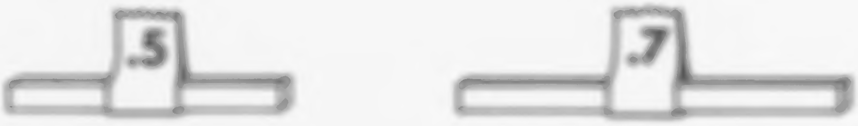
Station A, 2½ inches and 1 inch



Station B, 4 inches and 1½ inches



Station C, 2½ inches and 1½ inches



Station D, 1¼ inches and 1¾ inches

Name ______________________________

Activity 2, Station A

Use the sticks inside Envelope A for the following activities:

Mark Point *G* on this line so that the length of $\overline{AG}$ is .7.

A

Mark Point *Y* on this line so that the length of $\overline{XY}$ is 1.0 (ten tenths).

X

Mark Point *D* on this line so that the length of $\overline{CD}$ is .9.

C

Replace the sticks in Envelope A.

Activity 3, Station B

Use the sticks inside Envelope B for the following activities:

Mark Point *R* on this line so that the length of $\overline{QR}$ is .5.

Q

Mark Point *V* on this line so that the length of $\overline{NV}$ is .9.

N

Mark Point *E* on this line so that the length of $\overline{AE}$ is .2.

A

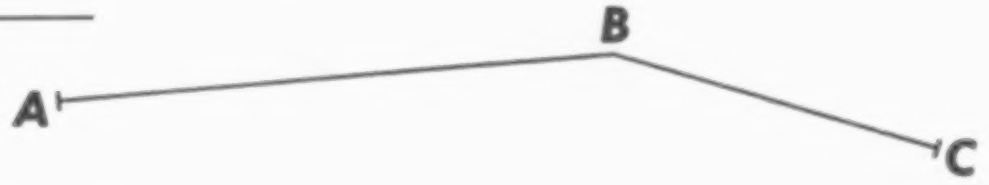
Replace the sticks in Envelope B.

Activity 4, Station C

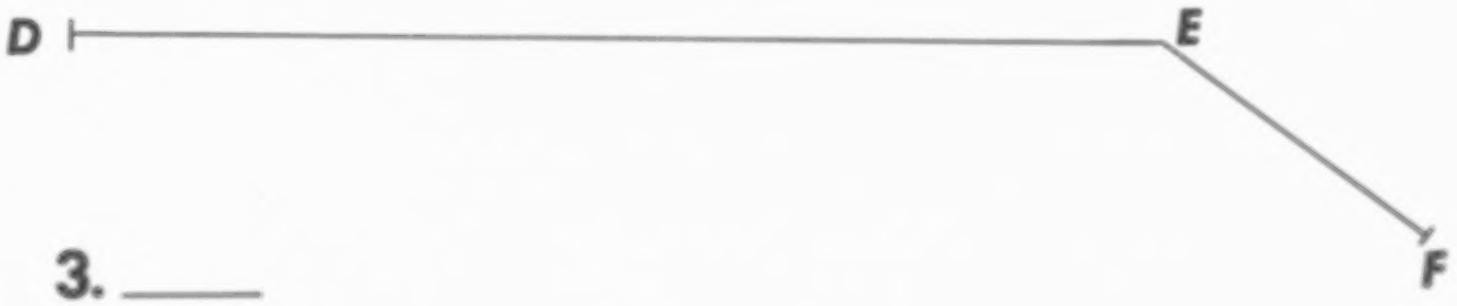
Name ______________________________

Use the sticks inside Envelope C to measure each of these "paths." Record your measure for each one in the space provided.

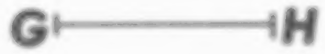
1. _____



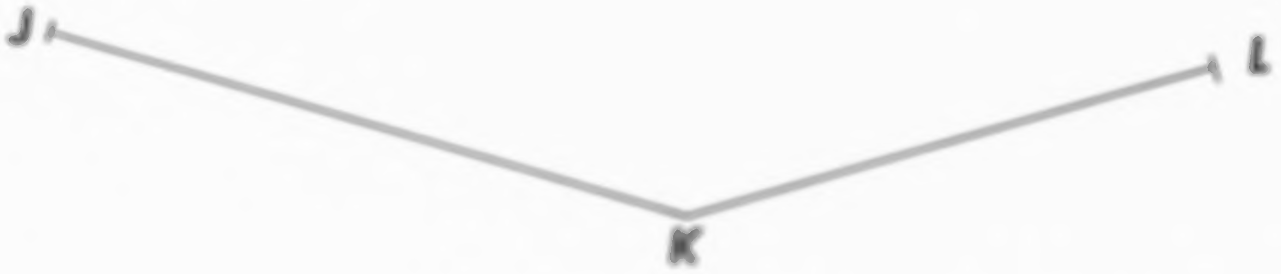
2. _____



3. _____



4. _____



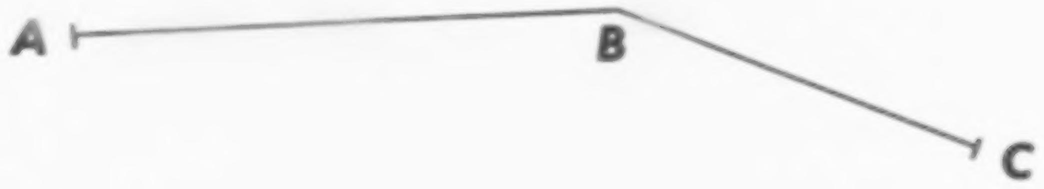
Replace the sticks in Envelope C.

Activity 5, Station D

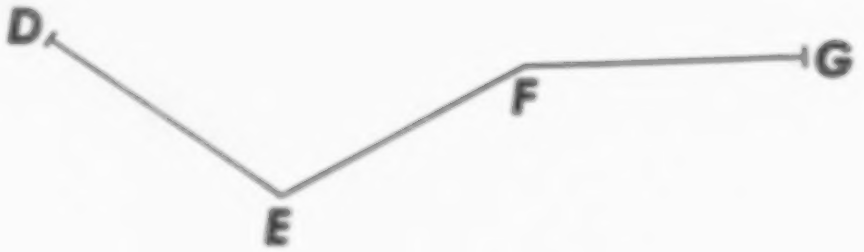
Name ____________________________

Use the sticks inside Envelope D to measure each of the "paths." Record your measure for each one in the space provided.

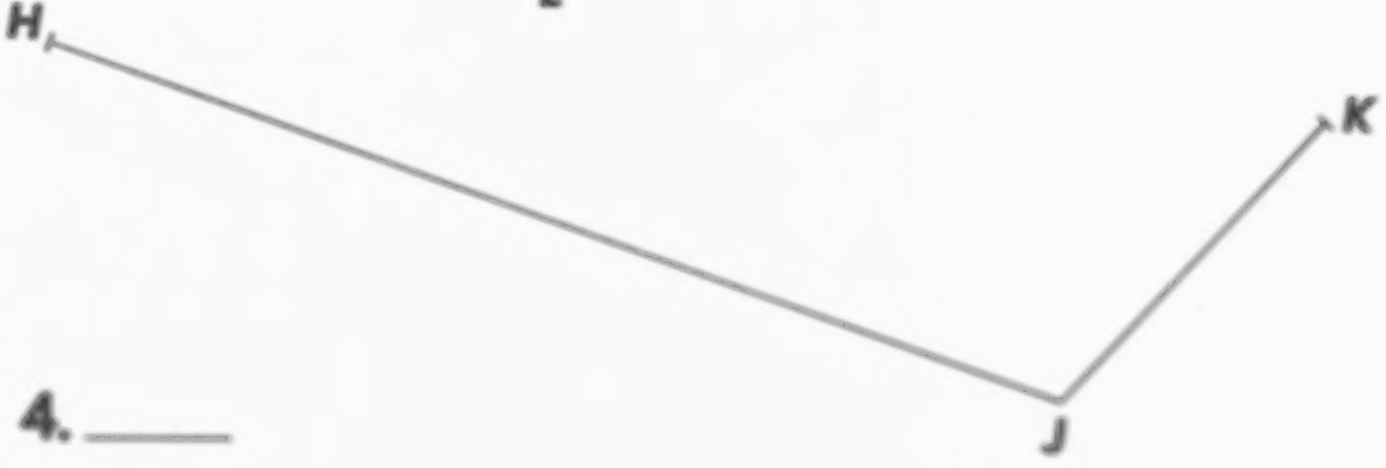
1. _____



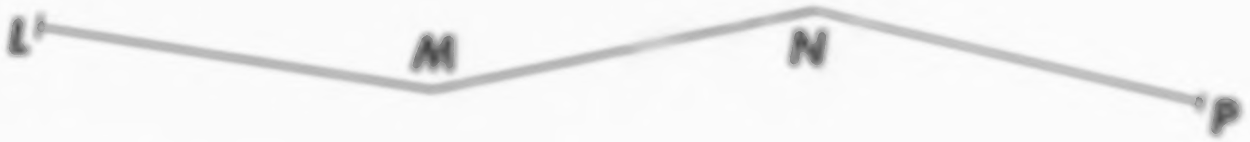
2. _____



3. _____



4. _____



Replace the sticks in Envelope D.

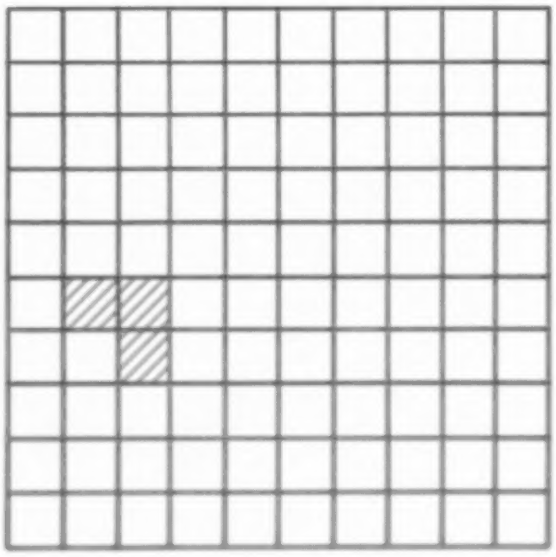
Activity 6 Name ______________________

Answer the questions for all three parts of this activity. Part 3 is on another sheet.

Part 1

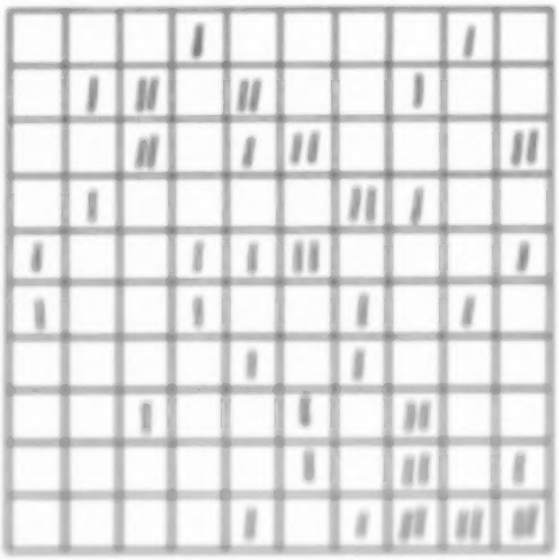
Three-hundredths (.03) of the big square is colored black.

1. Color 16 of the small squares green.
2. What fractional part of the big square is colored green? ______
3. What fractional part is colored either green or black? ______
4. Color 35 of the little squares red.
5. What fractional part of the big square is colored green, black, or red? ______
6. What fractional part of the big square is not colored? ______



Part 2

1. How many little squares have only 1 mark? ______
2. What fractional part of the big square is this? ______
3. How many little squares have 2 marks in them? ______
4. What fractional part of the big square is this? ______
5. How many little squares have either 1 or 2 marks in them? ______
6. What fractional part of the big square has no marks? ______



[Continued]

Activity 6–*Continued* Name ______________________________

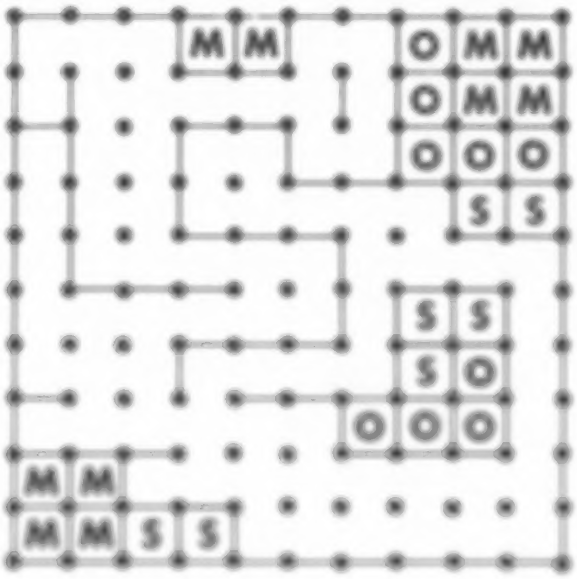
Part 3

Sam, Max, and Oscar played a game of Complete the Square, with the results shown here.

1. What fractional part of the squares did Sam complete? ______
2. What fractional part of the squares did Max complete? ______
3. Which one completed more squares? ______

S	S	S	O	O	O	O	O	O	O
O	O	S	M	M	M	M	M	M	O
M	O	S	O	O	O	O	S	M	O
M	O	S	S	S	S	S	S	M	O
M	O	O	O	O	O	O	O	O	O
M	M	M	M	M	M	S	S	S	S
S	S	S	S	S	S	S	M	O	O
O	O	S	M	M	M	M	M	O	S
M	O	S	M	O	S	S	S	S	S
M	S	S	M	O	O	O	O	O	O

They began another game, and in a few minutes the results looked like this:

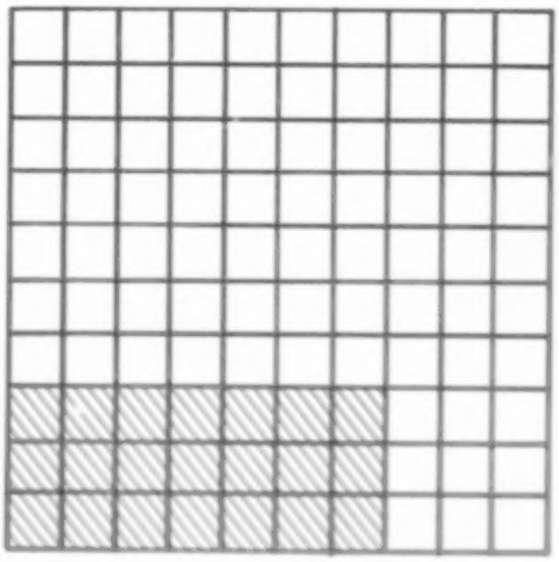


4. What fractional part of the squares has been completed? ______
5. What fractional part of the squares has not been completed? ______

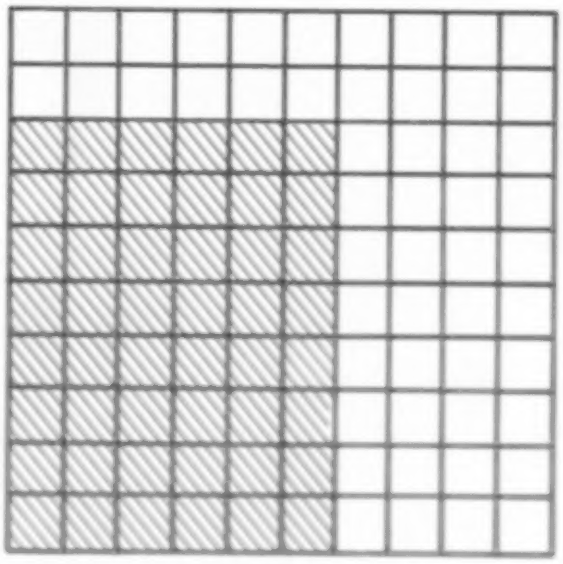
AREA MODEL FOR HUNDREDTHS　　UNIT 6, EXPERIENCE 3

Activity 1　　Name ______________________________

For each grid, tell what fractional part is shaded.

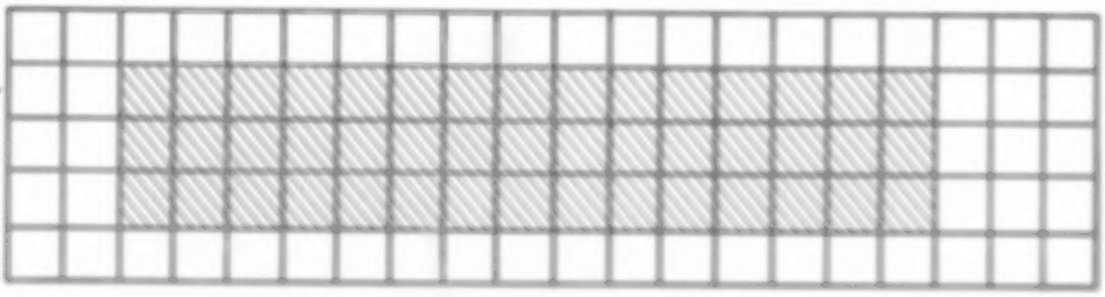


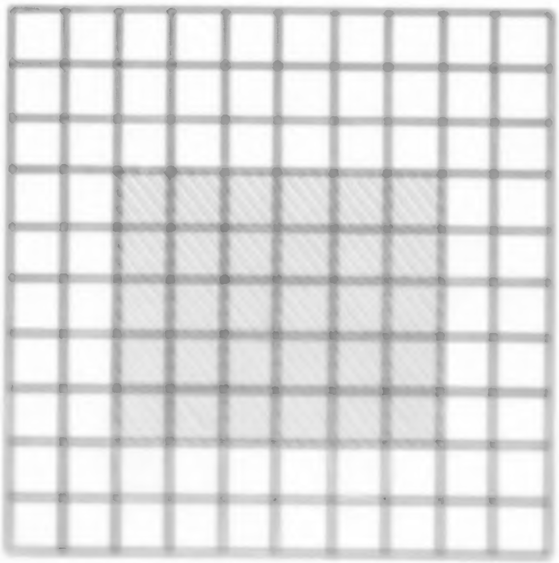
1.____



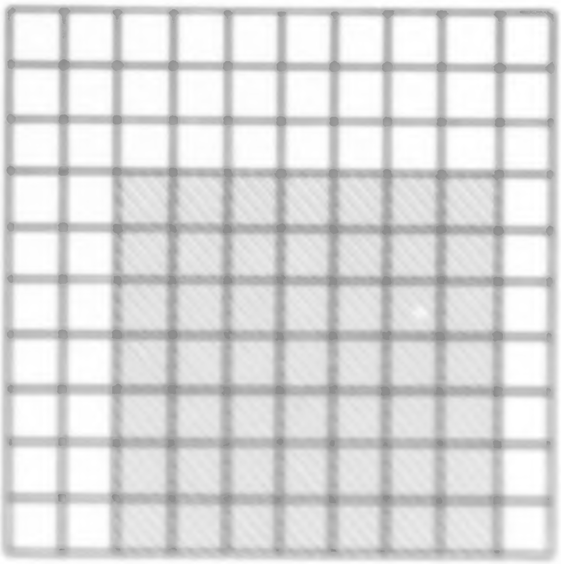
2.____

3.______





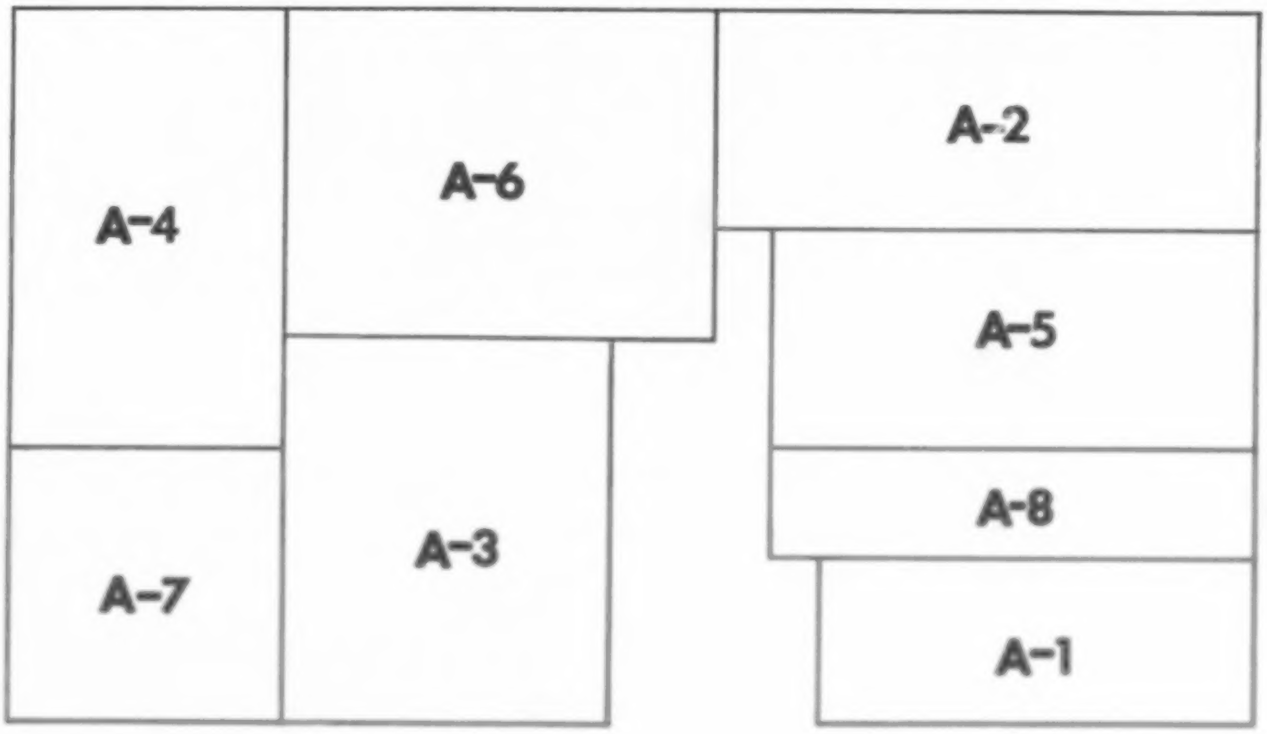
4. ______

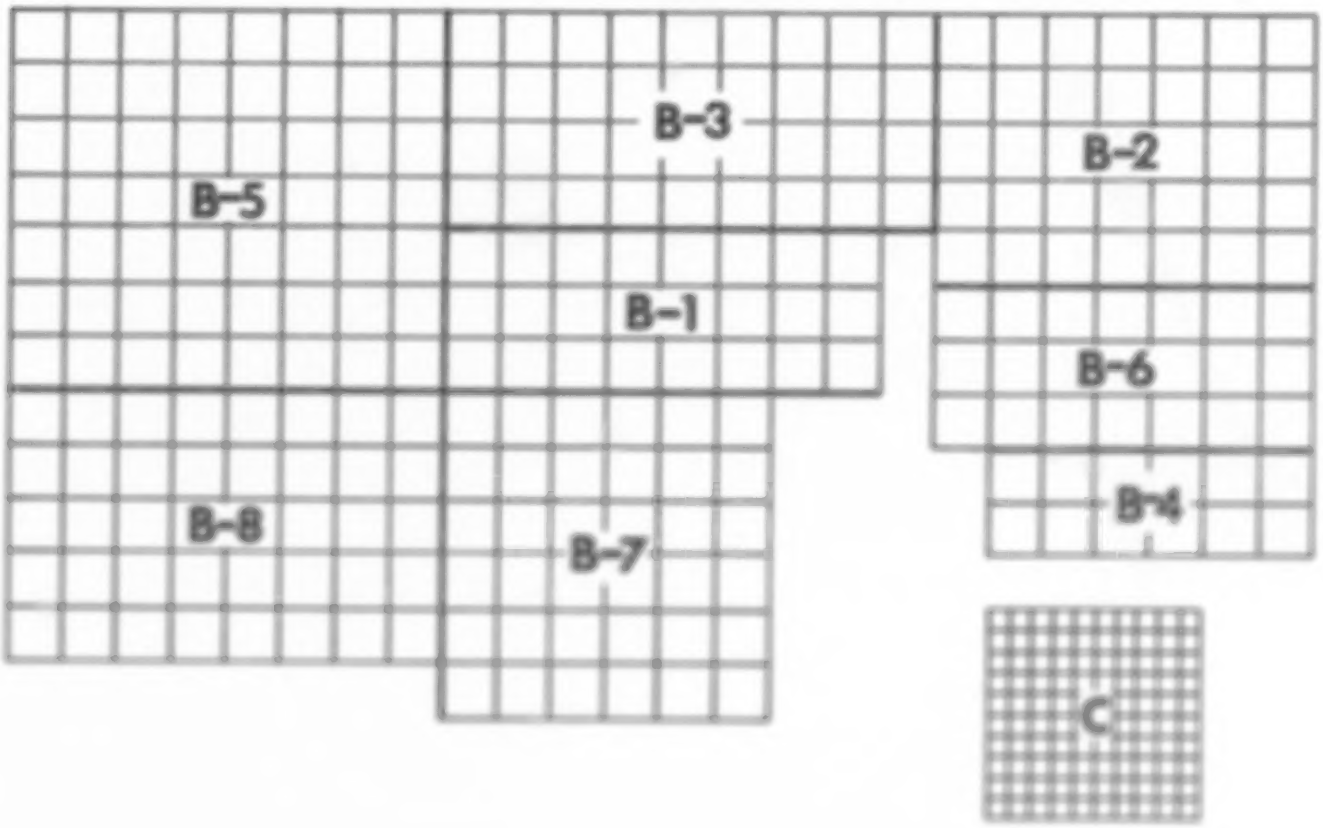


5. ______

Teacher Materials

Student material shown below should be reproduced for Activities 2, 3, and 4 at Stations A, B, and C. The white areas shown for Activity 2 should be cut apart and placed in Envelope A, the grilled ones for Activity 3 should be cut apart and placed in Envelope B. The grid for Activity 4 should be trimmed and placed in Envelope C.

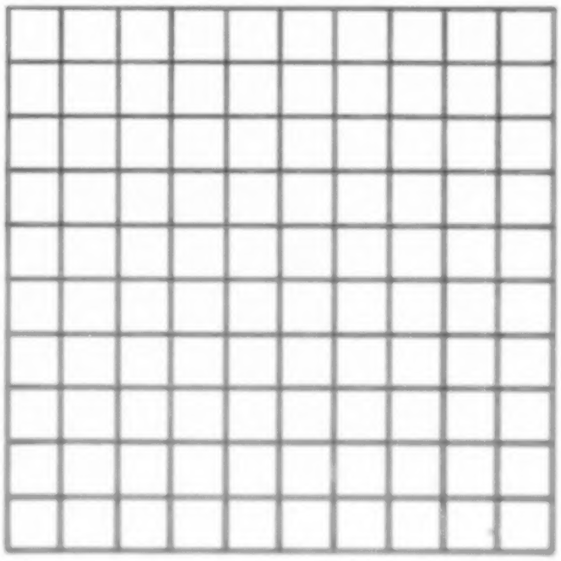




Name ______________________________

Activity 2, Station A

If we assign the number 1.00 to this rectangle, we can assign a number to each piece of paper in the envelope at this station. Write your answers in the spaces below.



A-1 ______

A-2 ______

A-3 ______

A-4 ______

A-5 ______

A-6 ______

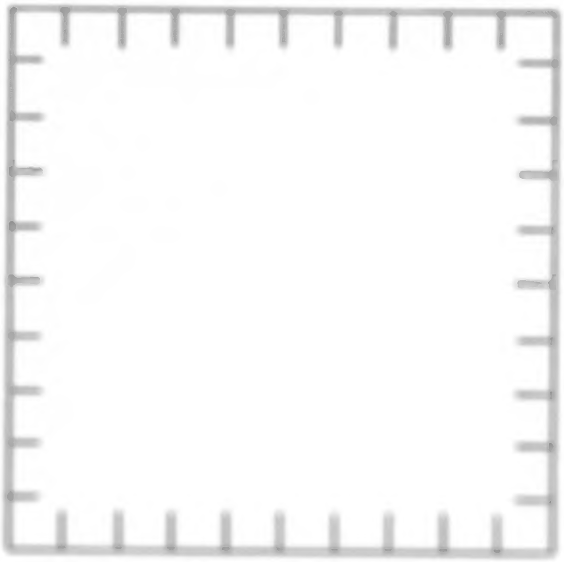
A-7 ______

A-8 ______

Return the pieces of paper to Envelope A.

Activity 3, Station B

If we assign the number 1.00 to this rectangle, we can assign a number to each piece of paper in the envelope at this station. Write your answers in the spaces below.



B-1 ______

B-2 ______

B-3 ______

B-4 ______

B-5 ______

B-6 ______

B-7 ______

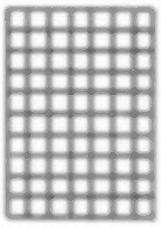
B-8 ______

Return the pieces of paper to Envelope B.

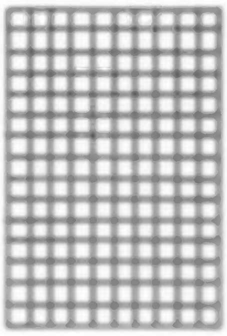
AREA MODEL FOR HUNDREDTHS UNIT 6, EXPERIENCE 3

Activity 4, Station C Name ______________________________

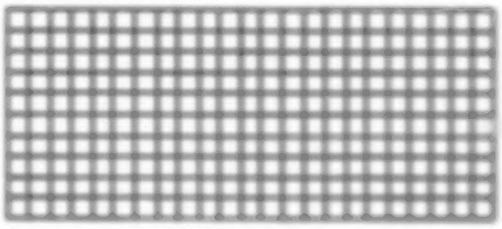
If we assign the number 1.00 to the square in Envelope C, we can use it to assign a number to each of the rectangles below. Write your answers in the spaces.



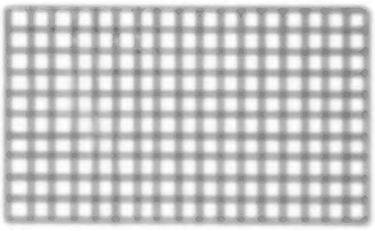
C-1 ______



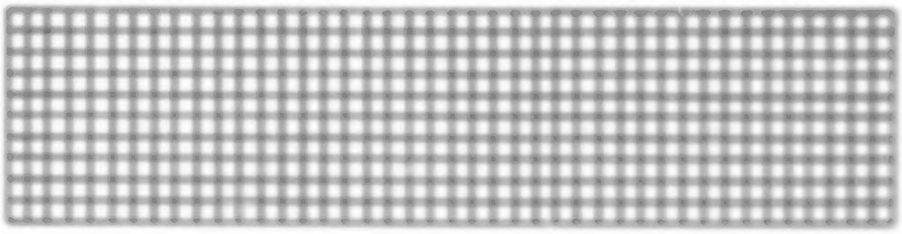
C-2 ______



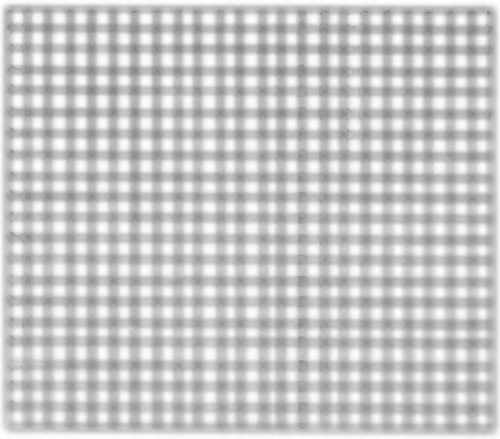
C-3 ______



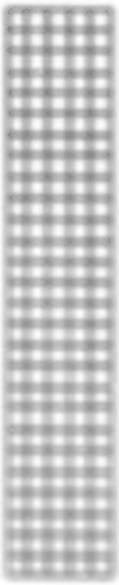
C-4 ______



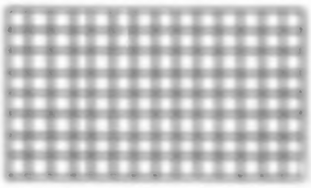
C-5 ______



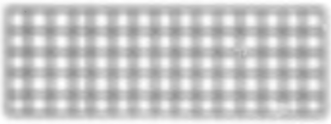
C-6 ______



C-7 ______



C-8 ______

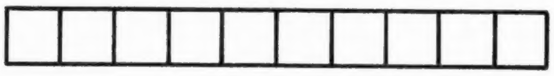


C-9 ______

Activity 5 Name ______________________________

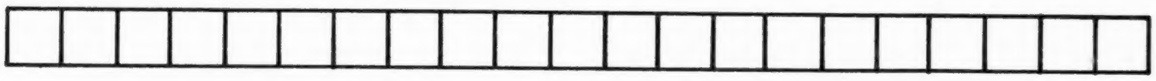
Part 1

If

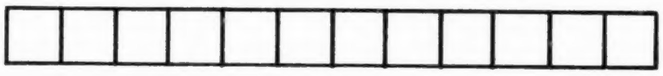


represents 1.0, assign a number to each of the lengths shown below.

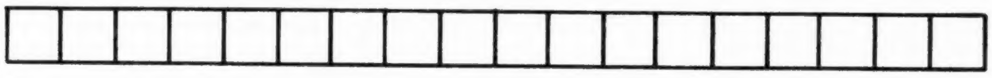
1.____



2.____

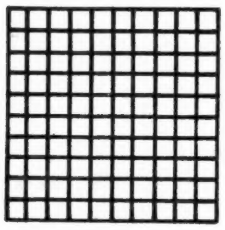


3.____



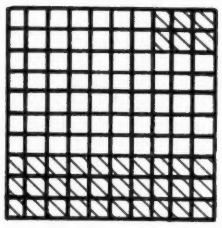
Part 2

If

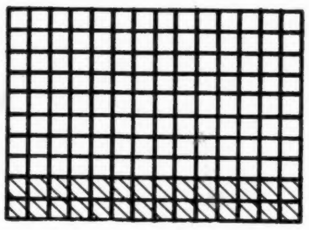


represents 1.00, assign a number to the total of the shaded areas in each of the figures shown below.

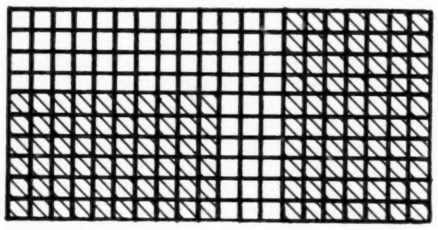
1.____



2.____



3.____



6

Name ______________________________

Station A

1. Take the books marked R, S, T, and U and look at pages 21 through 30 in each of them. What decimal fraction of these 10 pages has pictures?

   _____ in R _____ in S _____ in T _____ in U

2. Take any *one* of the books marked V, W, X, and Y and look at pages 101 through 200.

   What decimal fraction of these 100 pages has pictures? _____

   There are pictures on _____ of these pages in book _____.

3. Take another book marked V, W, X, or Y and look at pages 101 through 200.

   What decimal fraction of these 100 pages has pictures? _____

   There are pictures on _____ of these pages in book _____.

Station B

Take one of the newspapers at this station and write your name on it.

1. Draw a ring around the first 10 words in the first column at the left on page 1. Underline every *the, a,* and *and* inside this ring. What decimal fraction of the 10 words is underlined?

   _____

2. Repeat for each of the other columns on that page.

   _____ Col. 2 _____ Col. 3 _____ Col. 4 _____ Col. 5

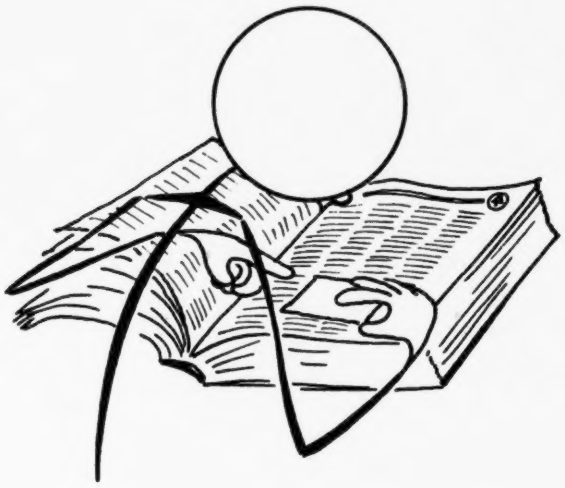
   _____ Col. 6 _____ Col. 7 _____ Col. 8

3. Draw a ring around the first 100 words in the first column on page 2. Underline all words that start with the letter *t*. What fraction of the 100 words is underlined? _____

Name ______________________________

Station C

Take one of the telephone books at this station.
Turn to the first full page of telephone numbers.



1. What decimal fraction of the first 10 telephone numbers ends in 2, 4, 6, 8, or 0? ______
2. What fraction of the first 100 telephone numbers ends in 5? ______
3. What fraction of the first 100 telephone numbers has a 5 as 1 of the last 4 digits? ______

Station D

1. What fraction of the file cards in Box D-1 has been used? ______
2. What fraction of the file cards in Box D-1 has *not* been used? ______
3. What fraction of the washers in Box D-2 is bent? ______

4. What fraction of the washers in Box D-2 is *not* bent? ______
5. What fraction of the paper clips in Box D-3 is of the larger size? ______